Social Networks 34 (2012) 645–657

Contents lists available at SciVerse ScienceDirect

Social Networks

journal homepage: www.elsevier.com/locate/socnet

Univariate and multivariate models of positive and negative networks: Liking, disliking, and bully–victim relationships

Gijs Huitsing[a,*], Marijtje A.J. van Duijn[a], Tom A.B. Snijders[a,b], Peng Wang[c], Miia Sainio[d], Christina Salmivalli[d,e], René Veenstra[a,d]

[a] University of Groningen and Interuniversity Center for Social Science Theory and Methodology (ICS), The Netherlands
[b] University of Oxford, England, United Kingdom
[c] University of Melbourne, Australia
[d] University of Turku, Finland
[e] University of Stavanger, Norway

ARTICLE INFO

Keywords:
Bully–victim relationships
Dislike
Like
Exponential random graph models

ABSTRACT

Three relations between elementary school children were investigated: networks of general dislike and bullying were related to networks of general like. These were modeled using multivariate cross-sectional (statistical) network models. Exponential random graph models for a sample of 18 classrooms, numbering 393 students, were summarized using meta-analyses. Results showed (balanced) network structures with positive ties between those who were structurally equivalent in the negative network. Moreover, essential structural parameters for the univariate network structure of positive (general like) and negative (general dislike and bullying) tie networks were identified. Different structures emerged in positive and negative networks. The results provide a starting point for further theoretical and (multiplex) empirical research about negative ties and their interplay with positive ties.

1. Introduction

Traditionally, the focus of social network analysis has mainly been on relations with a positive meaning, such as friendship, exchange, or cooperation. Although negative relations are important in classical theories such as balance theory (Heider, 1946) and its representation by signed graphs (Cartwright and Harary, 1956), overall they have been less frequently analyzed than positive relations. Our goal was to investigate the network of negative relations simultaneously with that of positive relations within the same group using a bivariate or multiplex approach. This is important because we expected to increase our understanding of positive and, especially, negative networks by investigating them simultaneously. For example, in bullying research, interventions are proposed based on the assumption that positive ties protect against bullying. Moreover, this is an alternative approach to analyzing signed digraphs, which are formed by combining a binary positive and binary negative digraph, following De Nooy (1999). The multivariate structure of negative and positive relations was analyzed in a classroom setting of social networks of Finnish elementary school students, using a bivariate network modeling ("ERGM") approach with parameters for positive, negative, and mixed-tie configurations (Robins et al., 2009).

1.1. Aims of the study

In this study, we aimed to gain insight into the typical structural patterns observable in both positive and negative relations. In performing a multivariate social network analysis of positive and negative ties, we investigated whether networks of positive and negative relations were meaningfully related, and whether the multivariate structures provided further insight into the interdependence between positive and negative networks. It was necessary first to study the network structures of the positive and negative ties on their own (i.e., univariately). Thus, we aimed to set a starting point for further empirical research about negative ties (univariately and multivariately).

To this end, we identified the network structure of positive and negative social networks to determine which structural parameters would be sufficient to model the network data of positive and negative networks. We presumed that this would be more revealing for the negative than for the positive relations, as less is known about the former, but it was important for both types of relations in preparation for the multivariate analysis. The findings of these univariate analyses would also enable us to establish

* Corresponding author. Tel.: +31 50 3636197.
E-mail address: g.e.huitsing@rug.nl (G. Huitsing).

0378-8733/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.socnet.2012.08.001

differences – if any – between the structural network patterns that apply to positive and negative relations.

To address these research aims, we investigated classroom networks consisting of a positive relation of "general like" and two negative relations of "general dislike" and bullying. Several classrooms were considered, with the aim of obtaining results that would go beyond the network structure in just one particular classroom. The negative networks of general dislike and bullying were both included in this study to achieve greater generalizability in representing negative relations. We investigated the combination of general like and only one of the negative ties, because examination of two different networks simultaneously is currently the maximum for the available software.

1.2. Approaches to investigating negative and positive networks

The foundations of the simultaneous study of positive and negative tie networks were laid in the work on structural balance theory. Structural balance theory has a long and rich history; Hummon and Doreian (2003) provide a concise overview (see also Wasserman and Faust, 1994). If relations between actors create tension or "imbalance", a social process is triggered by which actors change their relations in order to reach a "balanced" state. Structural balance theory can be used as a set of dynamic mechanisms to explain such tie formation in networks (at the micro level, see Heider, 1946) as well as the existence and evolution of group structures (at the macro level, see Newcomb, 1961). Hummon and Doreian (2003) argue that both micro- and macro-level processes should be incorporated to investigate balance theoretic processes. Cartwright and Harary (1956) formalized the ideas on balance theory, and they proposed use of signed graphs to represent the multiple structural relations of actors, making balance theory applicable to social networks.

Structural or social balance is regarded as a fundamental social process, and can account for the structure of affective relations (Heider, 1946; Newcomb, 1961). Usually, affective relations are measured using dichotomous sociometric data; relations between actors are present or absent. Such binary networks can be considered incomplete signed (directed) graphs, because they do not distinguish positive, negative, or neutral (absent) relations (De Nooy, 1999). For example, the absence of a relation between actors in a positive network does not inform us whether the null dyad is neutral or negative. More knowledge can be gained by including negative network relations.

In the present study, we used a bivariate approach in which we investigated positive and negative networks simultaneously. The network structures to be investigated were composed of a network of positive ties and a network of negative ties on a common node set, and are directed signed graphs; a tie in the positive network expresses positive affect, a tie in the negative network expresses negative affect, and the absence of a tie in both networks expresses neutral affect. In such a bivariate network approach, it is possible to observe whether the non-present positive tie is null or negative. One might even observe both a positive and a negative tie from an actor *i* to another actor *j*; although such "love–hate" relationships were rare in the combined networks in our analysis, they do occur.

1.3. Networks under investigation

Before investigating positive and negative networks simultaneously, we needed to examine the network structure of single positive and negative networks. Below, we introduce the positive and negative networks investigated and describe possible structural characteristics. Typical network structures of negative relations are expected to differ from those of positive relations. An example of these differences was already given by Robins et al. (2009). In an ERGM analysis of a network of "work difficulties" in a business organization, they found that isolates and one-sided isolates (sinks and sources) are significant network configurations, which is not usually seen for positive relations.

1.3.1. Positive networks

Positive relations are usually characterized by persistent affective bonds between two individuals; therefore, it is natural to assume that reciprocity is one of the main components that drive the formation of such relations. Moreover, transitivity is frequently observed in networks of positive relations (e.g., Feld and Elmore, 1982; Veenstra and Steglich, 2012). Sharing a common friend makes the establishment of a friendship more likely (Davis, 1970; Holland and Leinhardt, 1971). The precondition for triadic closure is being linked at distance two, a *two-path*. In our study of the positive network of general like, we expected that reciprocity and transitive closure would be localized social processes characterizing the preferences of children (e.g., children prefer to reciprocate nominations of general like), and that these micro-preferences would combine to form the larger (global) network structure of such positive relations.

1.3.2. Negative networks

Negative relationships are persistent and repeated negative judgments, feelings, or behaviors toward another person (Labianca and Brass, 2006). Labianca and Brass (2006) distinguished four different characteristics of negative relations: strength (or behavioral intensity) of the relation, reciprocity (mutuality of the negative relation), cognition (whether ego knows how alter evaluates him/her), and social distance (whether the negative tie is direct or indirect, the latter meaning that someone is positively connected to a person who is involved in negative ties).

Social network analyses of empirical data on negative relations are relatively rare, but have recently received more attention, with topics such as work difficulties (Robins et al., 2009), negative gossip at the workplace (Ellwardt et al., 2012; Grosser et al., 2010), or bully–victim relations (Sijtsema et al., 2009; Veenstra et al., 2007; Zijlstra et al., 2008). Some examples of research about negative relations outside of the signed graph tradition are studies on the relations of aggressive students in schools (Cairns et al., 1988; Farmer and Xie, 2007) and research on relational problems in organizations (Labianca and Brass, 2006).

A reason for the scarcity of studies on negative tie networks might be that those networks are often relatively sparse (compared with positive tie networks) and, therefore, more difficult to model as there is less information on the network structure. Generally, networks with fewer ties will have less structure. Another possible explanation is that negative networks are more difficult to observe, because this involves asking sensitive questions, which is not always easy.

In this study, we investigated two different negative tie networks: networks of general dislike and bully–victim relations, which are both important to children in everyday peer interactions (e.g., Rubin et al., 2009). Regarding relations of general dislike in classrooms, it is not unusual for a child to dislike at least one classmate or to be disliked by one or more classmates. For example, it was found in a sample of 2000 sixth-grade students that about two-thirds of the students received at least one dislike nomination from a peer (Witkow et al., 2005). Moreover, general dislike is often reciprocal: in a meta-analysis of mutual antipathies in 26 studies, Card (2010) showed that about one third of children in classrooms had at least one *mutual* relationship of general dislike. Bully–victim relationships have a different nature and structure. They are typically defined as dyads in which there is an imbalance of power between bullies and victims, and where negative actions of the bully toward the victim are intended and repeated over time (Olweus, 1993). Bully–victim relations are much less common than general dislike: about 5–10% of the relations in a classroom are usually bully–victim relations (Sijtsema et al., 2009; Veenstra et al., 2007). The differences between these relations of general dislike and bullying are their prevalence, or average degree, the rare mutuality of bully–victim ties, and the larger behavioral intensity (strength) of the bullying relation.

1.4. Multivariate networks: general like, general dislike, and bully–victim relations

When networks of positive and negative ties are investigated simultaneously, their interrelatedness can occur in different network configurations. We considered dyadic, degree-level, and triadic/higher-order dependencies (see also Table 1).

At the dyadic level, actors can report having both positive and negative relations with the same peers. Mixed reciprocity can also occur; for instance, when $i \rightarrow j$ is positive (e.g., like) whereas $j \rightarrow i$ is negative (e.g., dislike).

At the degree-level, the number of nominations received (indegrees) by children for positive ties may be correlated with the number of nominations they received for negative ties. The outdegrees (number of nominations given) of children for positive ties may also be related to their outdegrees for negative ties; this may be interpreted as a general response tendency in nominating others. The number of times that children are nominated for a positive tie can also be related to their tendency to nominate others negatively; and conversely.

The third level represents several actors in triads or higher-order combinations. Various combined triadic patterns have been proposed (Lazega and Pattison, 1999; Robins et al., 2009); we elaborate on these below.

As noted, combined patterns of positive-negative relations are related to structural balance theory (see, for example, Doreian and Krackhardt, 2001; Heider, 1946; Newcomb, 1961). Doreian and Krackhardt (2001, p.48) elaborated for signed triples ($i \rightarrow k$, $k \rightarrow j$, $i \rightarrow j$) that an even number of negative dyads is necessary to obtain a balanced subgraph (see also Wasserman and Faust, 1994, chapter 6). For networks of positive and negative relations, it appears that triads are balanced when all three dyads between three actors are positive or negative, or when two of the three dyads are negative and one positive. A group of three or more actors is considered to be structurally balanced if two people have a positive relation and they are consistent in their relations with other people (either positive or negative).

Doreian and Krackhardt (2001) found in the classical data of Newcomb (1961), however, that some of the triple types predicted by structural balance theory were not found in the observed data. Doreian and Krackhardt hypothesized that this inconsistency might be due to individual characteristics of actors, competing mechanisms for the attention of (popular) actors (i.e., children dislike each other but are friends with the same popular classmate, leading to an unbalanced structure), or patterns at the group level (such as peer group rejection). To investigate such mechanisms, we formulated hypotheses for triadic and higher-order dependence patterns in multiplex positive–negative relations; this is described in the next section.

1.4.1. Multivariate triangular configurations

The starting point for our elaboration is the configuration mentioned above: children (actors) agree about the (negative) evaluation of others. This is illustrated in the triangular configuration in Fig. 1a. When i and j agree about the (multiple) actors k they dislike, they are expected to be positively tied because they are balanced: they share the children whom they do not like. This pattern is also applicable to bully–victim relations, when using reports on the question: “By which classmates are you victimized?” When i and j perceive that they are being victimized by the same bullies k, they might seek each other for comfort and support against the bullies (Fox and Boulton, 2006). Such relational patterns can be considered examples of structural equivalence or structural homophily (e.g., Wasserman and Faust, 1994), where actors i and j have a similar network position. In this case, actors with structurally similar ties in the negative network have a positive tie. Thus, we expected that children would have positive relations when they shared negative out-ties.

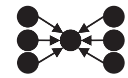
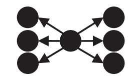
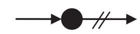
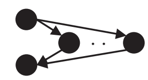
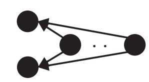
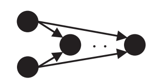
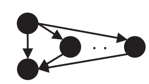
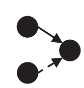
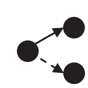
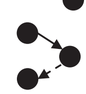
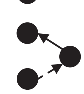
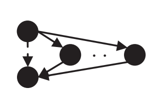
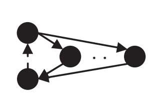
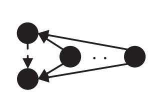
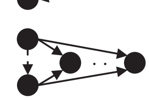
Another form of structural equivalence applies to children who receive negative nominations, either for general dislike or for bullying. In line with the argument of peer group rejection (Doreian and Krackhardt, 2001), receiving negative ties is a form of being rejected. Receiving negative ties from multiple others may lead to a vicious cycle in which a lack of fit with the group and being rejected enhance each other, making it hard to return to the peer group once rejected (Juvonen and Gross, 2005; Mikami et al., 2010). Moreover, once children are rejected by a substantial number of peers, the pool of peers from which they can choose friends is limited. This leads to default selection (Sijtsema et al., 2010): children have difficulties realizing the friendships they want to have and are forced to choose friends they initially would not have chosen. The social distance (cf. Labianca and Brass, 2006) in the negative network is small: these children are rejected themselves and they are positively tied to other rejected peers. This is shown in Fig. 1b. When children i and j are rejected by the same (multiple) peers k, they are more likely to form a positive relation. Children i and j might have preferred other friends, but due to their rejected position, they end up befriending other rejected (and structurally equivalent) peers (Mikami et al., 2010). This may also apply to bullying, because some aggressive children lack the skills to provide emotional and practical support and are, therefore, unattractive to form friendships with (Sijtsema et al., 2010). Thus, it is possible that bullies i and j form a friendship when they bully many of the same peers k. In sum, we expected that children would have positive relations when they shared negative in-ties.

In addition to the patterns of structural equivalence, other triangular patterns may occur: children may have opinions about the enemies of the children with whom they have difficult relations. We explored these configurations in our empirical analysis, as they are interrelated, though not yet theoretically underpinned. This may further developments in explaining the interplay of positive and negative ties. In Fig. 1c, child i dislikes child k, who, in turn, dislikes child j. Because both i and j have a negative relation with peers k, i and j may have a positive relation (enemies of enemies are friends), in either direction (from i to j or from j to i). The first is a depiction of transitive closure (see Fig. 1c); the second can be seen as cyclic closure (see Fig. 1d). For bully–victim relations, the pattern might be different because if i is being bullied by k, who is being bullied by j, a social dominance hierarchy may ensue, with child j at the top. Because low social dominance typically leads to rejection (Hawley, 1999; Mikami et al., 2010), it may also be the case that child j bullies child i, who is the lowest in the hierarchy, such that a positive relation between i and j is unlikely.

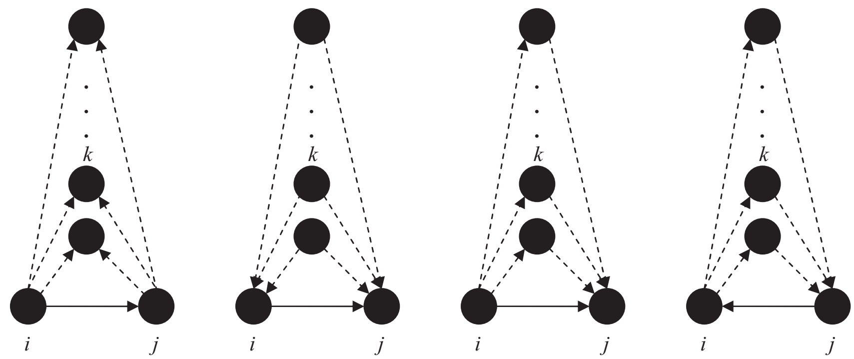
1.5. The present study: exponential random graph models

We used exponential random graph models (ERGMs, also called p^* models) to model the network structure of positive and negative relations. ERGMs are probability models for complete networks of a given set of actors that are used to estimate parameters of dyadic, triadic, and higher-order level effects (see, Robins et al., 2007a, for an introduction). We considered these models suitable

Table 1
Summary of the parameters in the univariate and multivariate exponential random graph models for directed networks.

Parameter (statistic)	Description	Graphical representation
Univariate parameters		
Dyadic parameters		
1 Reciprocity	Occurrence of mutual ties	
Degree-level parameters		
2 In-ties spread (A-in-S)	Dispersion of in-ties distribution (if positive, distribution is dispersed: some actors receive more nominations than others). Also indication of indegree-centrality	
3 Out-ties spread (A-out-S)	Dispersion of out-ties distribution (if positive, the distribution is dispersed: some actors give more nominations than others)	
4 Isolates	Occurrence of isolated actors (zero indegree and zero outdegree)	
5 Sinks	Occurrence of actors with zero outdegree and at least one indegree	
Multiple connectivity and closure parameters		
6 Multiple two-paths (A2P-T)	Occurrence of (multiple) out-ties and in-ties, or being linked at distance two	
7 Shared in-ties (A2P-D)	In-ties-based structural equivalence (being nominated by the same actors)	
8 Shared out-ties (A2P-U)	Out-ties-based structural equivalence (nominating the same actors)	
9 Transitive closure (AT-T)	Closure of (multiple) two-paths	
Multivariate parameters		
Dyadic parameters		
10 Multiplex arc (Arc-AB)	Occurrence of nominating others for both A and B	
Degree-level parameters		
11 Multiplex in-2-stars (In-2-star-AB)	Number of nominations received for A in correlation with number of nominations received for B	
12 Multiplex out-2-stars (Out-2-star-AB)	General tendency to nominate others (for A and B)	
13 Multiplex mixed-stars-AB (Mixed-2-star-AB)	Number of times nominated for A and nominating others for B	
14 Multiplex mixed-stars-BA (Mixed-2-star-BA)	Number of times nominated for B and nominating others for A	
Multivariate triangles		
15 Multiple two-paths of A with transitive closure of B (TKT-ABA)	Transitive closure of two-paths A by relation B	
16 Multiple two-paths of A with cyclic closure of B (CKT-ABA)	Cyclic closure of two-paths A by relation B	
17 Closure of B for shared in-ties of A (DKT-ABA)	Actors with shared in-ties for A tend to form tie B (structural equivalence)	
18 Closure of B for shared out-ties of A (UKT-ABA)	Actors with shared out-ties for A tend to form tie B (structural equivalence)	

Note. Solid lines indicate relations of A, and dotted lines indicate relations of B in the graphical representations of the multiplex parameters



a. Shared negative out-ties (UK-2-path-A) with positive closure (solid line, AKT-ABA)

b. Shared negative in-ties (DK-2-path-A) with positive closure (solid line, DKT-ABA)

c. Multiple negative two-paths (TK-2-path-A) with transitive path closure (solid line, TKT-ABA)

d. Multiple negative two-paths (TK-2-path-A) with cyclic path closure (solid line, CKT-ABA)

Note. Characters between brackets indicate the names of the configurations as they are named in *XPNet*, where *A* refers to the negative ties (general dislike, bully-victim relations) and *B* to the positive ties (general like).

Fig. 1. Triangulation in multivariate networks: Configurations of alternating triadic closure for directed graphs. Dotted lines represent negative nominations of general dislike ("Whom do you like the least?") or bullying ("By which classmates are you victimized?"); solid lines represent positive nominations of general like ("Whom do you like the most?").

to use in examining our research questions about the network structure, as the associated methodology provides ways for obtaining good representations of the network structure. Single tie networks of general like, general dislike, and bullying can be investigated simultaneously in multivariate ERGMs (Lazega and Pattison, 1999; Pattison and Wasserman, 1999).

We first investigated the structures of the networks for general like, general dislike, and bullying separately. To identify which structural parameters represented the relational structures of the single networks, we searched for a parsimonious model that could be applied to all classrooms; our aim was to select the lowest possible number of structural parameters, while achieving well-converged and reliable estimations for each classroom. Therefore, we focused on the structural patterns in the networks, and neglected possible actor-level (such as gender) differences in the structure of negative and positive networks. Although we acknowledge that gender might be an important factor in the three networks investigated for the question *who nominates whom* (Card et al., 2008; Dijkstra et al., 2007; Maccoby, 1998), it may be only one of many factors that drive nominations. For example, children are selective in whom they dislike, and one aspect that plays a role in that mechanism is gender, among a great deal of other factors. The univariate estimations of the networks of the 18 classrooms were summarized using a meta-analytic procedure, which describes the occurrence of (need for) the various structural parameters and their means and variability over the different networks. This enabled us to know if and how the general network patterns of different types of ties were different. Following the same procedures, we investigated the structures of the networks of positive and negative ties simultaneously, using multivariate exponential random graph models for positive (general like) and negative (general dislike or bullying) tie networks.

2. Data and method

2.1. Participants

Data stem from the Finnish *KiVa bullying intervention program*, a representative sample of elementary schools in all five provinces of mainland Finland (for an extensive description, see Kärnä et al., 2011). The data used in the present study come from the third wave, collected in May 2008, involving classroom networks where children were allowed to nominate an unlimited number of their classmates for negative as well as positive relations. We selected three schools from the total sample that had (1) more than three classrooms, (2) more than 70% participating students per classroom, and (3) more than ten students in each classroom. This subsample had 18 classrooms and 393 students, which was sufficient for the purposes of the present study. This arbitrary number of classrooms enabled us to investigate variation across classrooms, while having a sample for which performing time-consuming estimations was not too demanding. On average, there were 21.8 students per classroom (range 12–31).

2.2. Procedure

Students filled out Internet-based questionnaires in the schools' computer labs during regular school hours. The process was administered by the teachers, who were given detailed instructions concerning the procedure about 2 weeks prior to the data collection. The order of questions, individual items, and scales used in this study were randomized so that the order of presentation of the questions would not have any systematic effect on the results. The students, whose parents provided active consent for them to participate in the study, were assured that their answers would remain strictly confidential and not be revealed to teachers or parents.

The term *bullying* was defined to the students in the way formulated in Olweus' Bully/Victim questionnaire (Olweus, 1996). Several examples covering different forms of bullying were given, followed by an explanation emphasizing the intentional and repetitive nature of bullying and the power imbalance (see also Huitsing et al., in press).

2.3. General like, general dislike, and bully–victim relationships

Students were asked "Whom do you like the most?" and "Whom do you like the least?" to measure networks of general like and general dislike, respectively. To obtain bully–victim networks, all students who (1) indicated on any of eleven self-reported Olweus bully/victim items (concerning several forms of victimization) that they were victimized at least once and (2) reported being bullied by classmates, were presented with a roster including the names of all their classmates, and asked "By which classmates are you victimized?" (see also Veenstra et al., 2007). For general like, general dislike, and bully–victim relationships, unlimited same-sex as well as cross-sex nominations were allowed.

2.4. Analytical strategy

For estimations of the models, we used the univariate *PNet* and multivariate *XPNet* programs (Wang et al., 2009), available at www.sna.unimelb.edu.au. These programs use the Monte Carlo maximum likelihood methods of Snijders (2002). When ERGMS are used for social networks, parameter estimates of graph *configurations* are obtained. Configurations are specific patterns of ties between subsets of actors, for which parameter statistics are included in the model. The structure of the observed social network can be interpreted as the combination of the configurations, and the corresponding parameters can be interpreted as the outcome of structural processes in the network. The configurations used in this study were based on ERGM specifications introduced by Snijders et al. (2006) and Robins et al. (2007a,b, 2009), including alternating in- and out-stars and alternating triangles of various forms (Robins et al., 2009), and their multivariate extensions. The alternating star parameters, for example, model the entire indegree (A-in-S; alternating-in-star) and outdegree (A-out-S) distribution. These parameters model the dispersion of the indegrees and outdegrees by taking into account all star effects simultaneously (i.e., 2-stars, 3-stars, etc.). All parameters used in this study are described in Table 1.

The significance of the parameter estimates was tested using a *t*-ratio (the estimated parameter divided by its standard error); the distribution was approximately standard normal in the null hypothesis of no effect (Snijders et al., 2006). Convergence of the estimation algorithm was checked by ensuring that the absolute values of the "*t*-ratios for convergence" (Robins et al., 2007b) were less than 0.10.

Once the model converged for each classroom, the Goodness of Fit for all implemented graph statistics in (X)PNet (including the ones not directly estimated) was assessed through simulation of the networks with the estimated parameters. Not explicitly modeled statistics had acceptable Goodness of Fit when the deviations between observed and average simulated statistics, divided by the standard deviation of the simulated values, were less than 2 in absolute value (Robins et al., 2009).

2.4.1. Model selection procedure

We fixed the graph density in all models because this improves model convergence considerably (Lubbers and Snijders, 2007). To address the goal of obtaining a parsimonious model, we estimated ERGMs for each classroom separately and inspected the Goodness of Fit (GoF) statistics presented in PNet to look for network statistics that were possibly poorly estimated. When network statistics were not fitted in a satisfactory manner, we included extra parameters to obtain good GoF statistics. We also removed statistically insignificant parameters and looked at their impact on the GoF. In this back-and-forth process, we ended up with different parsimonious models for each classroom, with the parameters that were needed for obtaining an estimated (through simulation) network structure resembling the observed structure in each classroom. In the final step, we estimated a full model that incorporated all parameters found in the various classrooms and obtained an acceptable model fit for these larger models in all classrooms. The GoF statistics for all analyses are available on request. In the multivariate models we tested the triangular patterns (see Fig. 1) while controlling for multivariate degree-level effects and univariate configurations. To estimate directed signed graphs by estimating positive–negative network combinations, we estimated the networks of general like combined with general dislike, and general like combined with bully–victim relations.

2.4.2. Meta-analysis

The results of the network models for the 18 classrooms were combined in a meta-analytic procedure as described in Lubbers and Snijders (2007). This assumes a model in which each network has a true parameter, which is estimated with some estimation error; the true parameters are distributed across networks according to a normal distribution, while the estimation errors are independently and normally distributed, with a mean of zero and a standard deviation equal to the estimated standard error. Estimation of this model was carried out using the program MLwiN (Rasbash et al., 2000). The obtained estimated mean parameter represents an unstandardized aggregated estimate across classrooms (along with its standard error); the accompanying standard deviation represents the degree to which estimates vary across classrooms. The statistical significance of the mean parameters was tested by dividing the estimate by its standard error; this ratio was tested using a *t*-ratio, which has approximately a normal distribution. The significance of the parameters for the standard deviations was tested using a chi-square difference test with 1 degree of freedom.

3. Results

3.1. Univariate analyses

3.1.1. General like

In total, we analyzed 18 classrooms for general like; see the descriptive statistics in Table 2. On average, children liked about four to five classmates, and 43% of the nominations were reciprocated. There were only eight isolated children, who were neither liked by classmates nor liked classmates themselves. Fifty-four children were liked but did not nominate peers themselves (actors with at least one indegree but with zero outdegree: the so-called '*sinks*').

The structure found in the networks of general like, based on the meta-analyses of the final models, is given in Table 3. Children tended to reciprocate nominations for liking (1.27, $p < 0.01$), and tended to like the friends of friends (the transitive closure parameter, 0.85, $p < 0.01$). Moreover, the multiple two-paths parameter was negative (-0.32, $p < 0.01$). To obtain acceptable GoF statistics, it was necessary to include the shared in-ties parameter in the models of eight classrooms. When this effect was left out in these classrooms, the outdegree distribution and the structure of the two-paths were not modeled satisfactorily. The overall mean of the shared in-ties for the 18 classrooms was positive (0.10, $p < .01$), with significant variation over the classrooms (0.09, $p < 0.01$).

3.1.2. General dislike

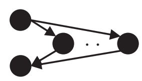
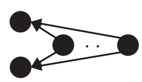
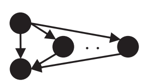
In the 18 networks of general dislike, children disliked on average about four classmates, and about a quarter of the nominations were reciprocated (see Table 2). As expected, many children were in some way involved in relations of general dislike. Only 8 out of the 393 children were neither disliked by classmates nor disliked classmates themselves (isolates). A considerable number of children did not report disliking classmates although peers reported disliking them (sinks).

The outcomes of the meta-analyses of the ERGMs for the final network models of general dislike are given in Table 4. Four parameters appeared to be necessary to obtain good GoF statistics in all classrooms. The in-ties spread was estimated to be significantly

Table 2
Descriptive statistics for general like, general dislike, and bully–victim relationships.

	General like	General dislike	Bully–victim relationships
Number of classrooms analyzed	18	18	17
Total number of nominations	1741	1570	412
Total number of possible nominations	8628	8628	8208
Average density (*standard deviation*)	21.3% (4.8%)	18.3% (5.0%)	5.7% (3.3%)
Average in/outdegree	4.43	4.02	1.11
Standard deviation outdegree	3.76	4.54	2.86
Standard deviation indegree	2.66	3.48	1.39
Reciprocity (*standard deviation*)	43.1% (8.9%)	24.0% (9.8%)	4.7% (8.1%)
Number of sinks (actors with zero outdegree)	54	88	167
Number of sources (actors with zero indegree)	16	35	43
Number of isolates	8	8	107

Table 3
"Whom do you like the most?" Exponential random graph models for network structure of friendships.

Parameter	Statistic	Mean parameter		Standard deviation	
		Estimate	Std. err.	Estimate	χ^2
1 Reciprocity		1.27**	(0.10)	0.000	0.00
6 Multiple two-paths (A2P-T)		−0.32**	(0.02)	0.032	0.39
7 Shared in-ties (A2P-D)		0.10**	(0.02)	0.089	40.08**
9 Transitive closure (AT-T)		0.85**	(0.04)	0.105	1.40

Note.
** $p < .01$.
The degree of freedom for the χ^2-test is 1. The mean parameter is an unstandardized aggregated estimate across classrooms. The standard deviation represents the degree to which estimates vary across classrooms ($N = 18$, with 393 students).

positive (1.49, $p < 0.01$). This parameter indicates a dispersed distribution of the indegrees (or indegree-centralization): some children were disliked by many classmates and others by few. A dispersed distribution was also found for the outdegrees (out-ties spread, 1.84, $p < 0.01$). This tendency to nominate classmates varied significantly over the classrooms (0.89, $p < 0.01$). It was also found that when children were disliked in a classroom, they were usually disliked by the same classmates (the shared in-ties parameter, 0.27, $p < 0.01$). Furthermore, the significant positive shared out-ties parameter (0.27, $p < 0.01$) modeled the tendency of children to dislike classmates who were also disliked by others. Interpreting these parameters together, some children disliked many peers and some children were disliked by many peers, but this goes along with shared in-ties and shared out-ties: children tended to agree about whom to dislike.

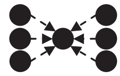
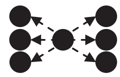
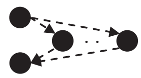
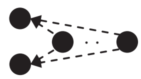
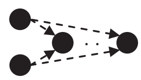
In addition to these parameters, estimation of the reciprocity parameter was necessary in half of the classrooms in order to obtain acceptable GoF statistics. In the other classrooms, the number of reciprocal nominations for general dislike was well fitted with the other parameters (i.e., convergence *t*-ratios were below 2), though we still included reciprocity to obtain comparable models. The overall effect for reciprocity in networks of general dislike was 0.76 ($p < 0.01$). In four classrooms, there were some children who disliked (almost) all their classmates. Therefore, all children in those classrooms had an indegree of at least one. For these classrooms we considered it necessary to include the multiple two-paths parameter to obtain well-fitted models, although over all classrooms it was estimated to be negative (−0.06, $p < 0.01$). This parameter models the tendency of children to have (multiple) outgoing as well as ingoing ties. Almost a quarter of the children in our sample had only in-ties for general dislike (88 sinks; see Table 2), but we did not have to specify this in our model specifications. The number of sinks was well fitted with the model given in Table 4.

3.1.3. *Bully–victim relationships*

We analyzed the bully–victim relationships of children in 17 classrooms. We had to exclude one classroom of 21 children in which there were only three nominations and, thus, no network structure to estimate. Children who were victimized nominated on average one classmate for bullying them (see Table 2 for descriptive statistics). The low number of nominations was accompanied by a low percentage of reciprocity: about 5% of the bully–victim nominations were reciprocated. This is also reflected in the large number of children who were only nominated as bullies (167 sinks, which is more than 40% of the sample). About a quarter of the children were isolated from bullying: they neither indicated being victimized nor were reported as bullies.

The meta-analyses for the final network models of bully–victim relations (see Table 5) show that the in-ties spread was estimated to be significantly positive overall (1.76, $p < 0.01$), implying that there was variation in how frequently children were nominated as bullies. The variation in the frequency of being nominated as a bully also varied significantly over the classrooms (1.02, $p = 0.03$). The shared in-ties parameter models the agreement of children to nominate the same bullies (0.29, $p < 0.01$). Because the shared in-ties parameter was necessary to obtain good GoF statistics in six classrooms, its strength varied over the classrooms (0.10, $p < 0.01$). Regarding uninvolved children, the parameter for isolates had a

Table 4
"Whom do you like the least?" Exponential random graph models for network structure of general dislike.

Parameter	Statistic	Mean parameter		Standard deviation	
		Estimate	Std. err.	Estimate	χ^2
1 Reciprocity		0.76^{**}	(0.13)	0.000	0.00
2 In-ties spread (A-in-S)		1.49^{**}	(0.26)	0.000	0.00
3 Out-ties spread (A-out-S)		1.84^{**}	(0.15)	0.886	11.27^{**}
6 Multiple two-paths (A2P-T)		-0.06^{*}	(0.02)	0.000	0.16
7 Shared in-ties (A2P-D)		0.27^{**}	(0.04)	0.110	3.55
8 Shared out-ties (A2P-U)		0.27^{**}	(0.03)	0.077	1.07

Note.
** $p < .01$.
The degree of freedom for the χ^2-test is 1. The mean parameter is an unstandardized aggregated estimate across classrooms. The standard deviation represents the degree to which estimates vary across classrooms ($N = 18$, with 393 students).

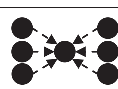
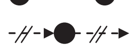
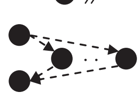
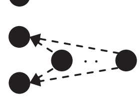
positive parameter estimate in all classrooms (4.47, $p < 0.01$). Also the sinks parameter had a positive estimate (4.17, $p < 0.01$), which is in line with the descriptive statistics.

The isolates parameter could not be estimated in two classrooms in which one child nominated all classmates for bullying, with the consequence that there were no isolated children. When the isolates parameter was included in the model estimations for these classrooms, the model estimation did not converge or the effect for the isolates was very small with an inflated standard error. Omitting this parameter did not affect the estimation of the other parameters, so we excluded it for those particular classrooms. The same applied for multiple two-paths in some classrooms. We found in some classrooms that children nominated many but not all classmates; therefore, we had to include the multiple two-paths parameter to model the tendency for children to have (multiple) outgoing as well as ingoing ties (0.07, $p < 0.01$). This appeared to be necessary to adjust our models to represent children who actively nominated classmates for bullying. In four other classrooms, however, this effect could not be estimated because (multiple) outgoing as well as ingoing ties did not occur.

3.2. Multivariate analyses: general dislike and general like

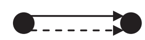
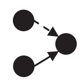
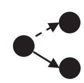
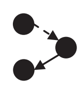
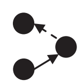
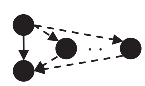
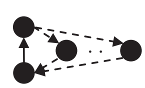
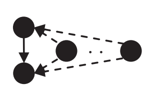
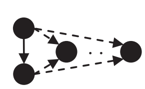
In the multivariate analyses, we included for each classroom the parameters that were modeled in the univariate approach to obtain a good estimation of the networks of general dislike and general like separately. The outcomes of the meta-analyses of the multivariate ERGMs for general dislike and general like are given in Table 6.

Table 5
"By which classmates are you victimized?": exponential random graph models for network structure of bullying.

Parameter	Statistic	Mean parameter		Standard deviation	
		Estimate	Std. err.	Estimate	χ^2
2 In-ties spread (A-in-S)		1.76^{**}	(0.36)	1.015	4.79^{*}
4 Isolates[a]		4.47^{**}	(0.45)	0.000	0.00
5 Sinks		4.17^{**}	(0.59)	1.614	3.18
6 Multiple two-paths (A2P-T)[b]		0.07^{**}	(0.02)	0.000	0.00
7 Shared in-ties (A2P-D)		0.29^{**}	(0.04)	0.100	8.05^{**}

Note.
* $p < .05$.
** $p < .01$.
The degree of freedom for the χ^2-test is 1. The mean parameter is an unstandardized aggregated estimate across classrooms. The standard deviation represents the degree to which estimates vary across classrooms ($N = 17$, with 372 students).
[a] $N_{classrooms} = 15$, with 322 students.
[b] $N_{classrooms} = 13$, with 287 students.

Table 6
“Whom do you like the least” and “Whom do you like the most?”: multivariate exponential random graph models for general dislike and general like.

Parameter	Statistic	Mean parameter		Standard deviation	
		Estimate	Std. err.	Estimate	χ^2
General dislike					
1 Reciprocity		0.55**	(0.14)	0.000	0.00
2 In-ties spread (A-in-S)		1.61**	(0.31)	1.080	15.75**
3 Out-ties spread (A-out-S)		1.58**	(0.16)	0.307	0.55
6 Multiple two-paths (A2P-T)		−0.02	(0.02)	0.045	0.52
7 Shared in-ties (A2P-D)		0.07	(0.05)	0.141	8.17**
8 Shared out-ties (A2P-U)		0.12*	(0.06)	0.173	11.13**
General like					
1 Reciprocity		1.30**	(0.12)	0.235	0.47
6 Multiple two-paths (A2P-T)		−0.28**	(0.02)	0.032	0.28
7 Shared in-ties (A2P-D)		0.15**	(0.02)	0.055	3.23
9 Transitive closure (AT-T)		0.76**	(0.05)	0.148	3.10
Multivariate relations					
10 Arc dislike and like (Arc-AB)		−3.12**	(0.37)	1.017	4.73**
11 In-ties dislike and like (In-2-star-AB)		−0.11**	(0.02)	0.032	0.90
12 Out-ties dislike and like (Out-2-star-AB)		0.08**	(0.01)	0.032	3.94*
13 In-ties dislike and out-ties like (Mixed-2-star-AB)		0.00	(0.02)	0.045	3.04**
14 In-ties like and out-ties dislike (Mixed-2-star-BA)		0.02*	(0.01)	0.000	0.00
15 Multiple two-paths of dislike with transitive liking closure (TKT-ABA)		0.10*	(0.05)	0.045	0.02
16 Multiple two-paths of dislike with cyclic liking closure (CKT-ABA)		−0.16**	(0.05)	0.000	0.00
17 Liking closure for shared in-ties of dislike (DKT-ABA)		0.50**	(0.08)	0.224	6.08**
18 Liking closure for shared out-ties of dislike (UKT-ABA)		−0.04	(0.04)	0.055	0.05

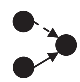
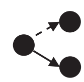
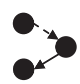
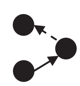
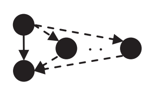
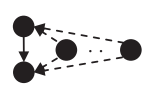
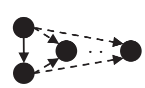
Note.
* $p < .05$.
** $p < .01$.
The degree of freedom for the χ^2-test is 1. Dotted lines indicate relations of general dislike, and solid lines indicate relations of general like in the graphical representations of the multiplex parameters. The mean parameter is an unstandardized aggregated estimate across classrooms. The standard deviation represents the degree to which estimates vary across classrooms ($N = 18$, with 393 students).

The parameters in the univariate approach had approximately the same parameter estimates in the multivariate approach, although some were smaller for general dislike. For instance, the estimates for shared in-ties and shared out-ties for general dislike dropped considerably in the multivariate analyses. These lower-order univariate configurations are included in the higher-order multivariate triangles (see below).

It was very unlikely that children would like the children they also disliked, as can be seen in the negative parameter for the arc of co-nominations of dislike and like (−3.12, $p < 0.01$). This parameter not only modeled the co-nominations of dislike and like, but also the mixed reciprocity effects (e.g., child *i* likes child *j* whereas child *j* dislikes child *i*). Such mixed reciprocity effects were hardly present in the classrooms under investigation, but were included already by the combination of direct reciprocity and the negative parameter for the dislike–like combination. When we did not constrain the co-nominations of dislike and like, it turned out that the presence of such mixed reciprocity nominations was overestimated. Moreover, at the degree-level, there was a negative association between being liked and disliked (−0.11, $p < .01$), and a positive association in the number of nominations given for like and dislike (0.08, $p < .01$).

Next, we turn to the higher-order multivariate triangles. In line with our expectations about positive liking closure for shared negative in-ties of general dislike, we found this parameter estimate to be positive in the classrooms under investigation (0.50, $p < .01$), though there was some variation (0.22, $p < .01$). This means that children who were disliked by the same classmates tended to like each other. We also expected that children who disliked the same classmates would be more inclined to like each other, but this was not supported by the data (−0.04, *n.s.*). Thus, the parameter for liking closure for shared out-ties of dislike was not significant over the sampled classrooms. We also found the estimate for the

Table 7
"By which classmates are you victimized?" and "Whom do you like the most?": multivariate exponential random graph models for bullying and general like.

Parameter	Statistic	Mean parameter		Standard deviation	
		Estimate	Std. err.	Estimate	χ^2
Bullying					
2 In-ties spread (A-in-S)		2.26^{**}	(0.73)	2.571	20.96^{**}
4 Isolates[a]		3.49^{**}	(0.53)	0.549	0.07
5 Sinks		4.05^{**}	(0.55)	1.428	2.52
6 Multiple two-paths (A2P-T)[b]		0.12^{*}	(0.05)	0.089	1.88
7 Shared in-ties (A2P-D)		0.05	(0.06)	0.134	3.29
8 Shared out-ties (A2P-U)		-0.79^{*}	(0.30)	0.820	12.08^{**}
General like					
1 Reciprocity		1.37^{**}	(0.10)	0.071	0.01
6 Multiple two-paths (A2P-T)		-0.30^{**}	(0.02)	0.055	2.31
7 Shared in-ties (A2P-D)		0.10^{**}	(0.02)	0.077	10.98^{**}
9 Transitive closure (AT-T)		0.79^{**}	(0.04)	0.045	0.03
Multivariate relations					
11 In-ties bullying and like (In-2-star-AB)		-0.13^{**}	(0.03)	0.000	0.00
12 Out-ties bullying and like (Out-2-star-AB)		−0.01	(0.02)	0.032	0.53
13 In-ties bullying and out-ties like (Mixed-2-star-AB)		-0.06^{*}	(0.02)	0.055	1.31
14 In-ties like and out-ties bullying (Mixed-2-star-BA)		−0.03	(0.02)	0.032	0.37
15 Multiple bullying two-paths with transitive liking closure (TKT-ABA)[c]		0.11	(0.11)	0.000	0.00
17 Liking closure for shared in-ties of bullying (DKT-ABA)		0.48^{**}	(0.06)	0.000	0.00
18 Liking closure for shared out-ties of bullying (UKT-ABA)[d]		0.21	(0.11)	0.000	0.00

Note.
* $p < .05$.
** $p < .01$.
The degree of freedom for the χ^2-test is 1. Dotted lines indicate bully–victim relations, and solid lines indicate relations of general like in the graphical representations of the multiplex parameters. The mean parameter is an unstandardized aggregated estimate across classrooms. The standard deviation represents the degree to which estimates vary across classrooms ($N = 17$, with 372 students).
[a] $N_{classrooms} = 13$, with 275 students.
[b] $N_{classrooms} = 13$, with 287 students.
[c] $N_{classrooms} = 9$, with 202 students.
[d] $N_{classrooms} = 14$, with 302 students.

transitive closure for multiple two-paths of dislike to be positive (0.10, $p = .05$), meaning that children liked the enemies of their enemies. The cyclic closure for multiple two-paths of dislike was, however, estimated to be negative (−0.16, $p < .01$). Children were unlikely to like children who disliked their enemies.

3.3. Multivariate analyses: bullying and general like

The meta-analyses of the multivariate ERGMs for bullying and general like are given in Table 7. For the multivariate parameters, we found that it was unlikely for children to be liked and to be nominated as a bully at the degree-level (−0.13, $p < .01$). Moreover, children who were nominated as bullies were less likely to like classmates (−0.06, $p = .02$). The shared out-ties parameter (A2P-U) for liking was included in these models because this univariate configuration is contained in the multivariate closure for shared out-ties (UKT-ABA) configuration, and was required to obtain good fit, and to counterbalance the multivariate effect. The shared out-ties parameter was not necessary for a well-fitted model in the univariate approach and was, therefore, not included there.

For the higher-order multivariate triangles, we found a positive effect for liking closure for shared in-ties of bullying (0.48, $p < .01$). This means that children who were nominated as bullies by the same victims tended to like each other. The joint networks of bullying and general like indeed exhibited balance. Because bully–victim relationships are relatively rare, we were not able to estimate the effects of other seldom observed higher-order multivariate triangles in all classrooms. In the fourteen classrooms for which we estimated the liking closure for shared out-ties of bullying, we found this parameter estimate to be marginally significantly positive (0.21, $p = .06$). Children who were victimized by the same bullies had the tendency to like each other. In nine other classrooms, we estimated the parameter for multiple bullying two-paths with transitive liking closure. This parameter, however, was not statistically significant in any of the classrooms. The cyclic variant of friendship closure for bullying two-paths hardly occurred in our sampled classrooms and was, therefore, not included as a parameter in the models.

4. Discussion

The results of our study point toward essential configurations for modeling the network structure of positive and negative relations. Whereas positive networks have been modeled often, few researchers have empirically investigated the network structure of negative networks. In this study of 18 classrooms with 393 students, we have shown that negative tie networks of children's relations of general dislike and bully–victim relations can be meaningfully modeled. The findings of this study also show that these relations differ noticeably with regard to the fundamental configurations that form the larger network structure. Further, they demonstrate that positive and negative networks can be brought together in the tradition of signed graphs; general dislike and bullying are meaningfully related to networks of general like when investigated using a multivariate approach, the results of which can be summarized as tendencies toward positive ties between those who are structurally equivalent in the negative network.

4.1. Multivariate network models of general like, general dislike, and bullying

In a bivariate approach, we modeled one of the negative networks simultaneously with the positive network. The combination of these binary networks are directed signed graphs, because there is information on whether a dyad is positive, negative, or neutral (dyads where both positive and negative ties are absent). In doing so, we connect with the long history of research on structural balance in networks (Cartwright and Harary, 1956; Heider, 1946; Hummon and Doreian, 2003). Structural balance theory posits that social processes operate by which actors form ties to obtain balanced relations. Actors are expected to have a positive relation when they agree about, or are structurally equivalent in, their relations with other people.

In line with structural balance theory, we expected that structurally equivalent children (i.e., those who have negative relations with the same peers) would have a higher probability of liking each other, thus forming a balanced triad. This was expected for both general dislike and bullying. Indeed, we found that children tended to like each other when they were victimized by the same bullies (though this was marginally significant). For networks of general dislike, however, we did not find structural effects of children liking each other when sharing a dislike of a number of peers. A possible explanation for this difference might be that, contrary to bullying, disliking a person is not threatening, and cannot be considered as rejection by others. For example, disliking a peer for withdrawn behavior does not give children an urge to become friends with others who also have negative feelings toward such children. Being victimized, however, can harm children's social status and can lead to a range of maladjustment outcomes, such as depression or low self-esteem (Arseneault et al., 2009; Hawker and Boulton, 2000). To prevent or ease such negative outcomes, children might search for peers to provide them with support to stand stronger against bullies (Fox and Boulton, 2006; Hodges et al., 1999). The peers these children form a positive tie with might also perceive a serious threat from these bullies, and are, thus, also inclined to search for support.

Victims of the same bullies can be positively tied for comfort and support, and it has been shown in a previous study that children are often friends with their defenders (Sainio et al., 2011). It is also possible, however, that at some point bullies will target the friends of their victims as well (Card and Hodges, 2006). Longitudinal analyses are needed to disentangle these causal relations and to determine what comes first: bullying of two children that leads to a positive relation between these victims (support-hypothesis), victimization of one child in a friendship dyad that is followed by victimization of the other (dual offense of the bully-hypothesis), or that victims have no choice in whom to choose as friend (default selection-hypothesis, Sijtsema et al., 2010).

We also explored expectations regarding the default selection hypothesis (Sijtsema et al., 2010), which states that children end up being befriended with peers they initially would not have chosen. As a consequence of a rejected position in the classroom, they end up with other outcasts of the classroom (Juvonen and Gross, 2005; Mikami et al., 2010). For general dislike, we expected and found balance with the network of general like: children liked each other when they were rejected by the same peers. These rejected children may have formed positive relations with each other because others did not want to affiliate with them. We also found balanced triads of friendships between bullies: bullies were likely to be friends. This is in line with default selection (children do not want to be friends with bullies), but it can also be a strategy of bullies to benefit from each other, in terms of power or status. It has been found that bullying children group together (Espelage et al., 2007; Salmivalli et al., 1997; Witvliet et al., 2010). Possible mechanisms for the formation of such subgroups of bullies are peer contagion (Dishion and Tipsord, 2011) and the finding that bullies are often supported by assistants and reinforcers (Salmivalli, 2010; Salmivalli et al., 1996). The support of followers means that bullies have a stronger position in the classroom and are motivated to continue their negative behaviors.

It was also found that children had opinions about the enemies of the children they disliked. Children liked the children who were disliked by the children they disliked, as was modeled using multiplex transitive path closure (see Fig. 1c). The parameter for the cyclic variant (see Fig. 1d), however, was estimated to be negative. This phenomenon is possibly better understood by taking into account the actors sending and receiving nominations for general dislike and general like. For transitive path closure, it is actor i who dislikes actor k and likes actor j (see Fig. 1c). Thus, both nominations stem from the same actor (i). Children might know which peers dislike the classmates with whom they have a negative relation (the cognition characteristic, Labianca and Brass, 2006). For cyclic path closure, however, the nomination for general like from actor j to actor i is accompanied by a received dislike nomination for actor j from actor k (see Fig. 1d). When the nominations do not come from the same actor, they might, therefore, be harder to observe.

In these multivariate analyses, it was found that several of the parameters for the negative relations were considerably smaller than in the univariate analyses (see also the next section). This applies specifically to the connectivity parameters of shared in-ties and shared out-ties, which are contained in the multivariate configurations (see also Fig. 1). This seems to indicate that the network structure of the negative ties is partially explained by their association with the liking network. In contrast to the negative relations, we found that the strength of the parameters for general like hardly changed in the multivariate approach.

4.2. Univariate network models of general like, general dislike, and bullying

In addition to the multivariate modeling of networks, we also applied a univariate approach. These univariate analyses were intended to prepare for the multivariate approach, but the "byproduct" is that we gained knowledge about their single network structures. In the following sections, we describe these single network structures of general like, general dislike, and bully–victim relations, the differences in the structural network patterns of these relations, and the generalizability of these models.

4.2.1. General like

The results for the network models of general like were in line with the findings of previous studies in which positive tie networks were modeled. A large proportion of the nominations for general like were reciprocated, and children often liked the friends of their friends (the *transitivity* configuration). For the global network structure, this might lead to patterns of *dense clustering* where (small) groups of children form tight-knit networks. In these subgroups within classrooms, many relations are reciprocated and many children are befriended with each other. Another configuration had to be included in the statistical modeling to obtain well-converged models: the *shared in-ties* configuration. This configuration can be seen as an indication of agreement about whom to like, and also points toward subgroups of children who like each other.

4.2.2. General dislike

Although many nominations for general dislike were given in the sampled classrooms, there appears to be some consistency among these nominations. Some children disliked more classmates than others (*out-ties spread*) and some children were disliked more than others (*in-ties spread*). However, children tended to agree about whom to dislike (as indicated by the *shared out-ties* and *shared in-ties* configurations), which is in line with the mechanism of peer group rejection (Doreian and Krackhardt, 2001). The reasons for being disliked are well known. In general, rejected children have low levels of prosocial and cognitive skills, and they display problem behaviors, like disrupting others or being aggressive or withdrawn (Asher and McDonald, 2009; Newcomb et al., 1993). For the global network structure this may lead to patterns of *selectivity*: in some classrooms there exists a core of highly rejected children who are disliked by many classmates. Also, Robins et al. (2009) found a strong in-ties spread in their analysis of a network of work difficulties. In line with the findings of the meta-analysis of Card (2010), we also found that a substantial part of the children in our classrooms reciprocally disliked each other.

4.2.3. Bully–victim relations

Many children were not involved in bully–victim relations; these were modeled using the *isolates* configuration. It also appeared that some children received more bully nominations than others, as can be seen using the *in-ties spread* configuration (comparable to the networks of general dislike). However, agreement between nominators was less relevant in bully–victim networks; the shared in-ties configuration was significantly positive in the meta-analysis, but it varied across the classrooms. We found that many children who were nominated as bully did not nominate classmates for bullying them, as indicated by the positive *sinks* configuration. An interpretation of a combination of these configurations leads to a global network structure that is *centralized*. Some children are central bullies who target many peers (see also Huitsing et al., in press), but do not report being victimized themselves. We found that the out-ties spread configuration was not necessary to obtain well-fitted models for bullying. When estimated to be positive, this configuration could be an indication of central victims, who are victimized by many bullies (Huitsing et al., in press).

4.2.4. Differences in network structure of positive and negative ties

When we compared the networks of general like, general dislike, and bully–victim relations, we found that there were some similarities but many more differences. For all networks, it was found that there was some degree of agreement about whom to like, dislike, or nominate as bully (*shared in-ties*). Only *shared in-ties*, *multiple two-paths*, and *reciprocity* were fundamental in the network models for general dislike and general like, but other configurations were needed to complete the model estimation for these networks (for general like: *transitive closure*; for general dislike: *in-ties* and *out-ties spread*, *shared out-ties*). This is an indication that relations of general like and general dislike are quite different (see also Dijkstra et al., 2007; László and Pál, 2010), which is in line with signed graphs.

The negative networks of general dislike and bullying are comparable in that a few children received many nominations (*in-ties spread*). For general dislike, however, this is in the context of a relatively dense network where some children nominated more peers than others (*out-ties spread*), whereas for bullying there was a large number of uninvolved children (either completely or one-sided: isolates or sinks, respectively). Bullying networks can be characterized by a more centralized structure, where bullies themselves often rarely nominate others for bullying them.

4.2.5. Generalizability of network models

In addition to essential configurations in network models, it appeared that some individual children influenced the network models by nominating a large number of classmates. For example, for bully–victim relations, we had to exclude the isolate configuration in some classrooms because one or more children nominated all their classmates. Similarly, we found that in some classrooms models fitted well only when the multiple two-paths parameter was included for modeling the networks of general dislike and bullying, because some children nominated a lot of children for these behaviors. This implies that our "basic" models are sensitive to the presence of one or a few children who nominate many more classmates than their peers do.

The solution we implemented in this study was to adjust models for each classroom specifically (i.e., parsimonious models), and estimate the complete models in a final step. The advantage of this approach was that every estimated network of a classroom resembled the observed network sufficiently; however, it was a time-consuming procedure that required searching in a back-and-forth process for an optimal model with the adequate parameters. In further studies the proposed models may be used to see whether they sufficiently capture the network structure. Recent analyses of bullying and defending networks in 27 Dutch classrooms of 9- to 12-year-olds (Huitsing and Veenstra, in press) as well as bullying networks in 67 Swiss kindergartens with 5- to 7-year-olds showed highly comparable results (available on request). These findings are promising, and indicate that there may be similar patterns in negative relations in different age-groups and different countries. Whether the structures found for general dislike and bully–victim relations also apply to other negative tie networks in other settings (e.g., outside schools), or change when actor-covariates are controlled for (such as gender), remains a question for further investigation.

Acknowledgements

This research was partly supported by Toptalent Grant (021.002.022) from the Netherlands Organization for Scientific Research (NWO) to the first author, by grants from the Academy of Finland (121091, 134843) to the sixth author, and the Dutch Ministry of Education (Onderwijs Bewijs) to the last author.

References

Arseneault, L., Bowes, L., Shakoor, S., 2009. Bullying victimization in youths and mental health problem: 'much ado about nothing'? Psychological Medicine 40, 717–729.

Asher, S.R., McDonald, K.L., 2009. The behavioral basis of acceptance, rejection, and perceived popularity. In: Rubin, K.H., Bukowski, W.M., Laursen, B. (Eds.), Handbook of Peer Interactions, Relationships, and Groups. Guilford, New York, pp. 232–248.
Cairns, R.B., Cairns, B.D., Neckerman, H.J., Gest, S.D., Gariépy, J.L., 1988. Social networks and aggressive behavior: peer support or peer rejection. Developmental Psychology 24, 815–823.
Card, N.A., 2010. Antipathetic relationships in child and adolescent development: a meta-analytic review and recommendations for an emerging area of study. Developmental Psychology 46, 516–529.
Card, N.A., Hodges, E.V.E., 2006. Shared targets for aggression by early adolescent friends. Developmental Psychology 42, 1327–1338.
Card, N.A., Stucky, B.D., Sawalani, G.M., Little, T.D., 2008. Direct and indirect aggression during childhood and adolescence: a meta-analytic review of gender differences, intercorrelations, and relations to maladjustment. Child Development 79, 1185–1229.
Cartwright, D., Harary, F., 1956. Structural balance: a generalization of Heider's theory. Psychological Review 63, 277–293.
Davis, J.A., 1970. Clustering and hierarchy in interpersonal relations: testing two graph theoretical models on 742 sociomatrices. American Sociological Review 35, 843–851.
De Nooy, W., 1999. The sign of affection: balance-theoretic models and incomplete signed digraphs. Social Networks 21, 269–286.
Dijkstra, J.K., Lindenberg, S., Veenstra, R., 2007. Same-gender and cross-gender peer acceptance and peer rejection and their relation to bullying and helping among preadolescents: comparing predictions from gender-homophily and goal-framing approaches. Developmental Psychology 43, 1377–1389.
Dishion, T.J., Tipsord, J.M., 2011. Peer contagion in child and adolescent social and emotional development. Annual Review of Psychology 62, 189–214.
Doreian, P., Krackhardt, D., 2001. Pre-transitive balance mechanisms for signed networks. Journal of Mathematical Sociology 25, 43–67.
Ellwardt, L., Labianca, G., Wittek, R., 2012. Who are the objects of positive and negative gossip at work? A social network perspective on workplace gossip. Social Networks 34, 193–205.
Espelage, D.L., Green, H.D., Wasserman, S., 2007. Statistical analysis of friendship patterns and bullying behaviors among youth. New Directions for Child and Adolescent Development 118, 61–75.
Farmer, T.W., Xie, H., 2007. Aggression and school social dynamics: the good, the bad, and the ordinary. Journal of School Psychology 45, 461–478.
Feld, S.L., Elmore, R., 1982. Patterns of sociometric choices: transitivity reconsidered. Social Psychology Quarterly 45, 77–85.
Fox, C.L., Boulton, M.J., 2006. Friendship as a moderator of the relationship between social skills problems and peer victimisation. Aggressive Behavior 32, 110–121.
Grosser, T.J., Lopez-Kidwell, V., Labianca, G., 2010. A social network analysis of positive and negative gossip in organizational life. Group & Organization Management 35, 177–212.
Hawker, D.S.J., Boulton, M.J., 2000. Twenty years' research on peer victimization and psychosocial maladjustment: a meta-analytic review of cross-sectional studies. Journal of Child Psychology and Psychiatry 41, 441–455.
Hawley, P.H., 1999. The ontogenesis of social dominance: a strategy-based evolutionary perspective. Developmental Review 19, 97–132.
Heider, F., 1946. Attitudes and cognitive organization. Journal of Psychology 21, 107–112.
Hodges, E.V.E., Boivin, M., Vitaro, F., Bukowski, W.M., 1999. The power of friendship: protection against an escalating cycle of peer victimization. Developmental Psychology 35, 94–101.
Holland, P.W., Leinhardt, S., 1971. Transitivity in structural models of small groups. Comparative Group Studies 2, 107–124.
Huitsing, G., Veenstra, R. Bullying in classrooms: participant roles from a social network perspective. Aggressive Behavior, http://dx.doi.org/10.1002/ab.21438, in press.
Huitsing, G., Veenstra, R., Sainio, M., Salmivalli, C. It must be me or it could be them. The impact of the social network position of bullies and victims on victims' adjustment. Social Networks, http://dx.doi.org/10.1016/j.socnet.2010.07.002, in press.
Hummon, N.P., Doreian, P., 2003. Some dynamics of social balance processes: bringing Heider back into balance theory. Social Networks 25, 17–49.
Juvonen, J., Gross, E.F., 2005. The rejected and the bullied: lessons about social misfits from developmental psychology. In: Williams, K.D., Forgas, J.P., Von Hippel, W. (Eds.), The Social Outcast: Ostracism, Social Exclusion, Rejection, and Bullying. Psychology Press, New York, pp. 155–170.
Kärnä, A., Voeten, M., Little, T.D., Poskiparta, E., Kaljonen, A., Salmivalli, C., 2011. A large-scale evaluation of the KiVa anti-bullying program. Child Development 82, 311–330.
Labianca, G., Brass, D.J., 2006. Exploring the social ledger: negative relationships and negative asymmetry in social networks in organizations. Academy of Management Review 31, 596–614.
László, Z., Pál, J., 2010. How negative networks are forming and changing in time? Theoretical overview and empirical analysis in two high-school classes. Review of Sociology 20, 69–95.
Lazega, E., Pattison, P.E., 1999. Multiplexity, generalized exchange and cooperation in organizations: a case study. Social Networks 21, 67–90.
Lubbers, M.J., Snijders, T.A.B., 2007. A comparison of various approaches to the exponential random graph model: a reanalysis of 102 student networks in school classes. Social Networks 29, 489–507.
Maccoby, E.E., 1998. The Two Sexes. Growing up Apart, Coming Together. Belknap, Cambridge, MA.
Mikami, A.Y., Lerner, M.D., Lun, J., 2010. Social context influences on children's rejection by their peers. Child Development Perspectives 4, 123–130.
Newcomb, T.M., 1961. The Acquaintance Process. Holt, Rinehard, and Winston, New York.
Newcomb, A.F., Bukowski, W.M., Pattee, L., 1993. Children's peer relations: a meta-analytic review of popular, rejected, neglected, controversial, and average sociometric status. Psychological Bulletin 113, 99–128.
Olweus, D., 1993. Bullying at School: What We Know and What We Can Do. Blackwell, Malden, MA.
Olweus, D., 1996. The Revised Olweus Bully/Victim Questionnaire. Research Center for Health Promotion (HEMIL Center), University of Bergen, Bergen, Norway.
Pattison, P., Wasserman, S., 1999. Logit models and logistic regressions for social networks: II. Multivariate relations. British Journal of Mathematical and Statistical Psychology 52, 169–193.
Rasbash, J., Browne, W., Goldstein, H., Yang, M., Plewis, I., Healy, M., et al., 2000. A User's Guide to MLwiN. Institute of Education, London.
Robins, G., Pattison, P., Kalish, Y., Lusher, D., 2007a. An introduction to exponential random graph (p^*) models for social networks. Social Networks 29, 173–191.
Robins, G.L., Snijders, T.A.B., Wang, P., Handcock, M., Pattison, P., 2007b. Recent developments in exponential random graph (p^*) models for social networks. Social Networks 29, 192–215.
Robins, G., Pattison, P., Wang, P., 2009. Closure, connectivity and degree distributions: exponential random graph (p^*) models for directed social networks. Social Networks 31, 105–117.
Rubin, K.H., Bukowski, W.M., Laursen, B., 2009. Handbook of Peer Interactions, Relationships, and Groups. Guilford Press, New York.
Sainio, M., Veenstra, R., Huitsing, G., Salmivalli, C., 2011. Victims and their defenders: a dyadic approach. International Journal of Behavioral Development 35, 144–151.
Salmivalli, C., 2010. Bullying and the peer group: a review. Aggression and Violent Behavior 15, 112–120.
Salmivalli, C., Lagerspetz, K., Björkqvist, K., Österman, K., Kaukiainen, A., 1996. Bullying as a group process: participant roles and their relations to social status within the group. Aggressive Behavior 22, 1–15.
Salmivalli, C., Huttunen, A., Lagerspetz, K.M.J., 1997. Peer networks and bullying in schools. Scandinavian Journal of Psychology 38, 305–312.
Sijtsema, J.J., Veenstra, R., Lindenberg, S., Salmivalli, C., 2009. Empirical test of bullies' status goals: assessing direct goals, aggression, and prestige. Aggressive Behavior 35, 57–67.
Sijtsema, J.J., Lindenberg, S.M., Veenstra, R., 2010. Do they get what they want or are they stuck with what they can get? Testing homophily against default selection for friendships of highly aggressive boys. The TRAILS study. Journal of Abnormal Child Psychology 38, 803–813.
Snijders, T.A.B., 2002. Markov chain Monte Carlo estimation of exponential random graph models. Journal of Social Structure 3, 1–41.
Snijders, T.A.B., Pattison, P.E., Robins, G.L., Handcock, M.S., 2006. New specifications for exponential random graph models. Sociological Methodology 36, 99–153.
Veenstra, R., Steglich, C.E.G., 2012. Actor-based model for network and behavior dynamics: a tool to examine selection and influence processes. In: Laursen, B., Little, T.D., Card, N.A. (Eds.), Handbook of Developmental Research Methods. Guilford, New York, pp. 598–618.
Veenstra, R., Lindenberg, S., Zijlstra, B.J.H., De Winter, A.F., Verhulst, F.C., Ormel, J., 2007. The dyadic nature of bullying and victimization: testing a dual perspective theory. Child Development 78, 1843–1854.
Wang, P., Robins, G., Pattison, P., 2009. PNet, Program for the Simulation and Estimation of Exponential Random Graph (p^*) Models (Computer Software). Department of Psychology, School of Behavioral Science, University of Melbourne, Melbourne, Australia.
Wasserman, S., Faust, K., 1994. Social Network Analysis: Methods and Applications. Cambridge University Press, Cambridge.
Witkow, M.R., Bellmore, A.D., Nishina, A., Juvonen, J., Graham, S., 2005. Mutual antipathies during early adolescence: more than just rejection. International Journal of Behavioral Development 29, 209–218.
Witvliet, M., Olthof, T., Hoeksma, J.B., Goossens, F.A., Smits, M.S.I., Koot, H.M., 2010. Peer group affiliation of children: the role of perceived popularity, likeability, and behavioral similarity in bullying. Social Development 19, 285–303.
Zijlstra, B.J.H., Veenstra, R., Van Duijn, M.A.J., 2008. A multilevel *p2* model with covariates for the analysis of binary bully–victim network data in multiple classrooms. In: Card, N.A., Selig, J.P., Little, T.D. (Eds.), Modeling Dyadic and Interdependent Data in Developmental and Behavioral Sciences. Routledge, New York, pp. 369–386.