Scaling Symmetries in Nature

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What to do with bad data

If the data values are quantities
plot it on a log:log plot.

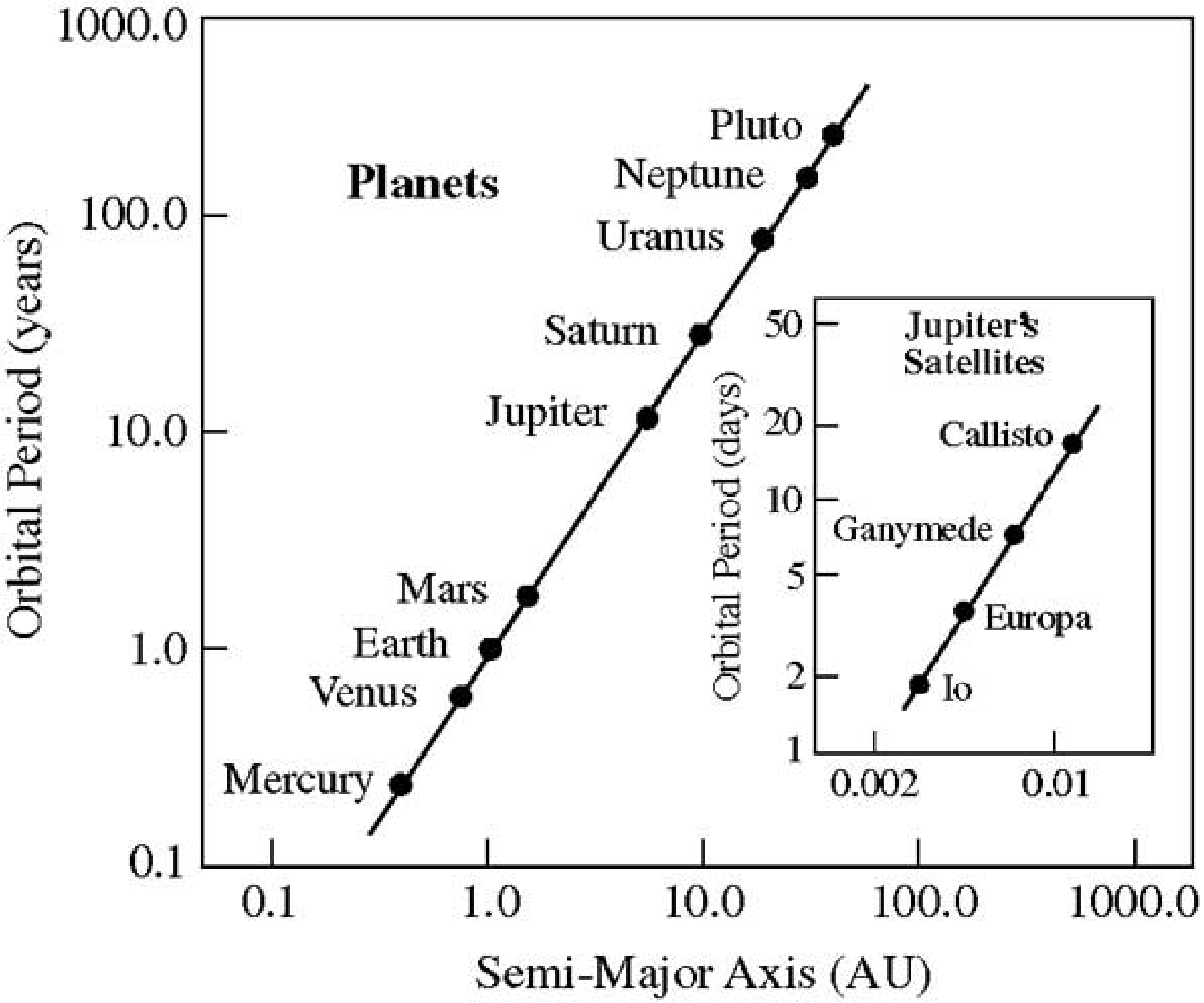
If it still looks bad
fit a straight line to help the eye.

If that doesn’t help
reject all points that are 2-sigma from the line

Make sure the slope of the line fits your prejudice

Write the paper.

Kepler’s third Law



$$T = \frac{2\pi r}{v} \Rightarrow T^2 \propto \frac{r^2}{v^2}$$

$$v^2 \propto \frac{GM}{r}$$

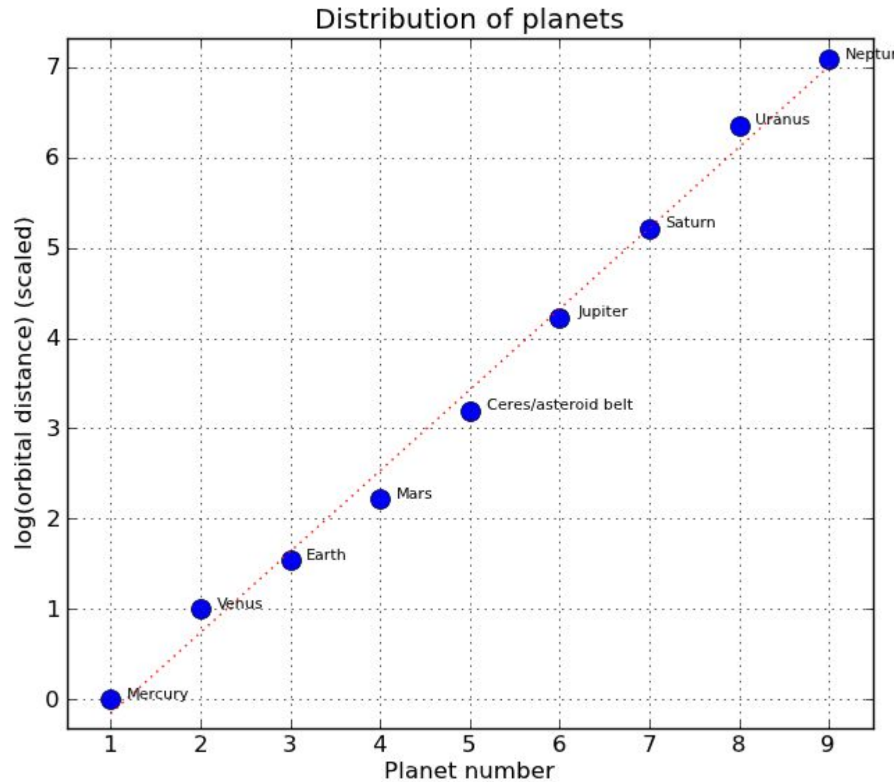
$$T^2 \propto r^3$$

Why is scaling important?

The existence of a power law relationship spanning one or more decades of scales between two quantities is suggestive of a common cause.

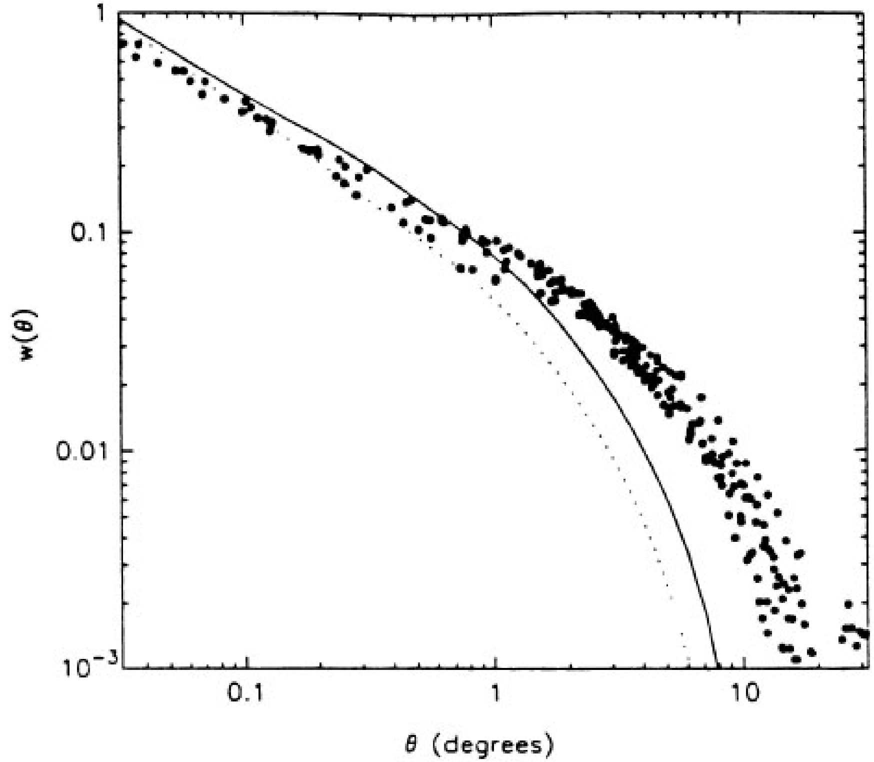
The break-down of the scaling relation is equally important since it tells us something about the driver for the association and causes us to look for a reason for the failure.

$$D\ (AU) = 0.4 + 0.3 \times 2^N$$



The Titius-Bode “Law” for the radii of the orbits of the planets

Galaxy 2-point correlation function



Efstathiou et al. Measured to two point function for galaxy clustering and concluded that the break from the smaller scale power law could **only** be explained if $\Lambda \neq 0$

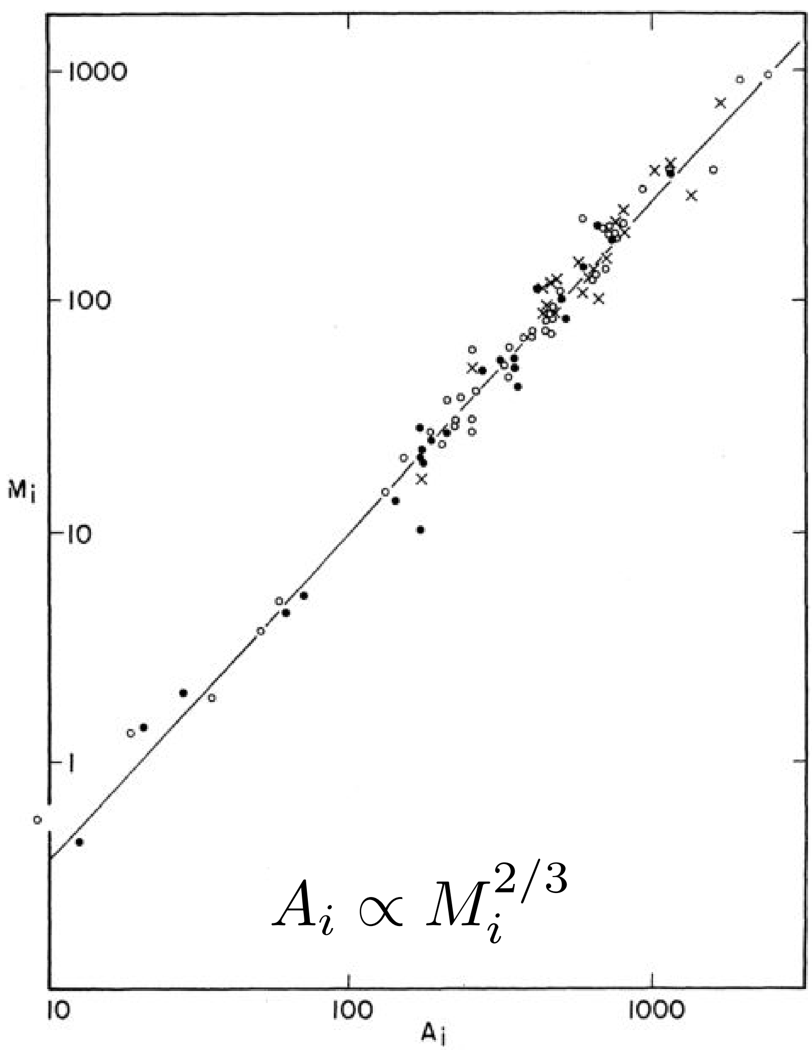
Efstathiou, G. 1991b. Phys. Scr. T36: 88-96

The discovery of Λ

It is argued here that the success of the cosmological cold dark matter (CDM) model can be retained and the new observations of very large scale cosmological structures can be accommodated in a spatially flat cosmology in which as much as 80 percent of the critical density is provided by a positive cosmological constant. In such a universe, expansion was dominated by CDM until a recent epoch, but is now governed by the cosmological constant. This constant can also account for the lack of fluctuations in the microwave background and the large number of certain kinds of objects found at high redshift.

The cosmological constant and cold dark matter
Efstathiou, G.; Sutherland, W. J.; Maddox, S. J.
Nature (ISSN 0028-0836), vol. 348, Dec. 27, 1990, p. 705-707.

Rotation of Galaxies



Heidmann, Astrophys. Lett. 1968 , **3**, 153.

Assume Newton's law of gravity.

Selection effect?
Suppose that all galaxies in the sample have the same surface brightness and same mass to light ratio.

$$v^2 \propto \frac{GM}{r} \Rightarrow M \propto rv^2$$

$$\frac{M}{r^2} = \text{const}$$

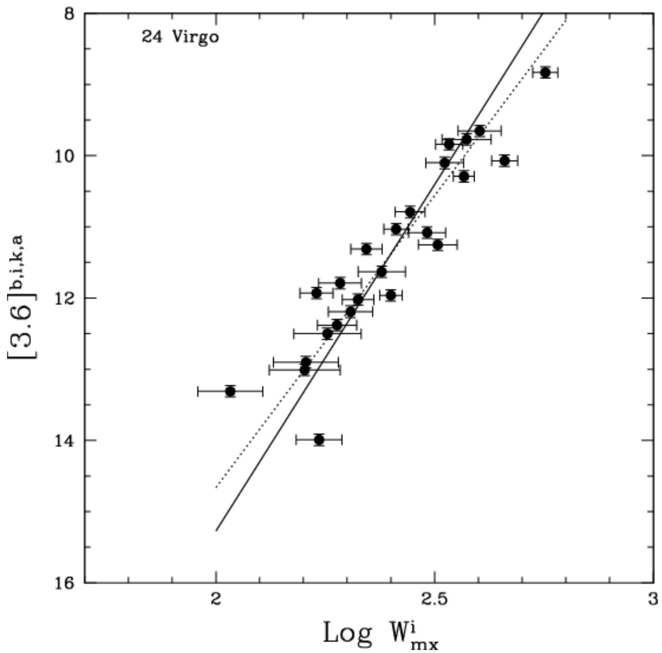
$$H \propto Mrv$$

$$\text{eliminate } r \text{ and } v$$

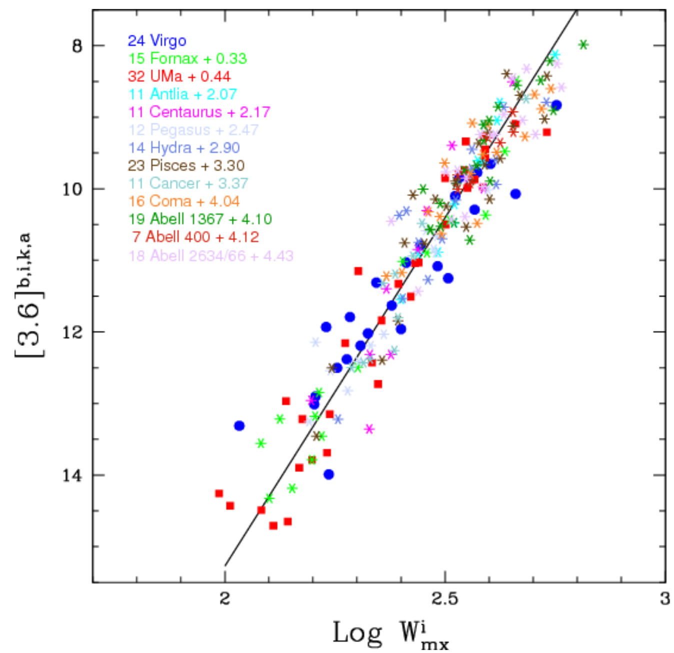
$$H \propto M^{7/4}$$

Tully-Fisher relationship

Correlation between the intrinsic brightness of a galaxy as measured in the 3.6 micron band, versus the maximum width of its rotation curve. (Sorce et al. 2015)



Calibration using Virgo Cluster



Superposition of 13 clusters to get relative distances

Slope corresponds to $\mathcal{L} \propto V^{3.6}$

Hubble constant = 74 ± 5 km s^{-1} Mpc^{-1}

G.I. Taylor’s analysis of A-bomb

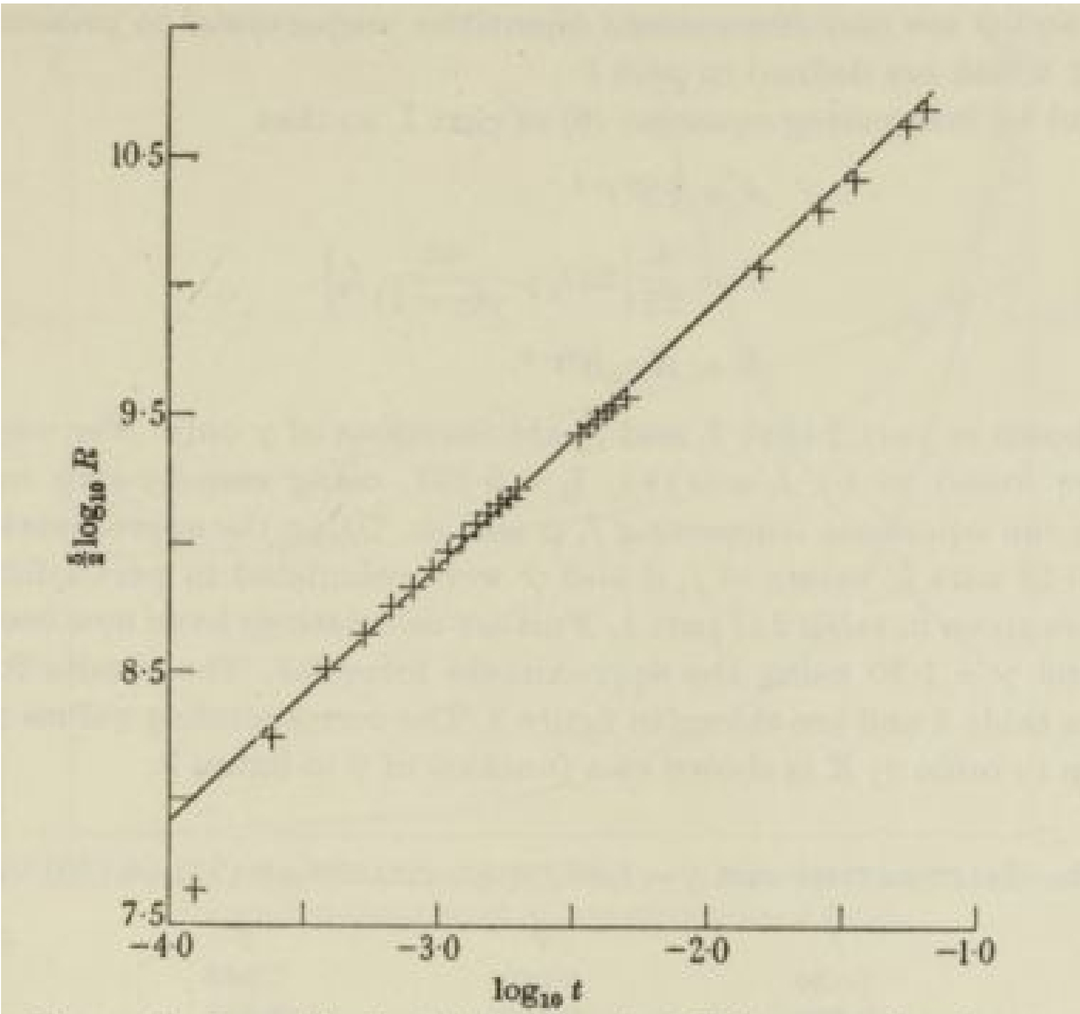
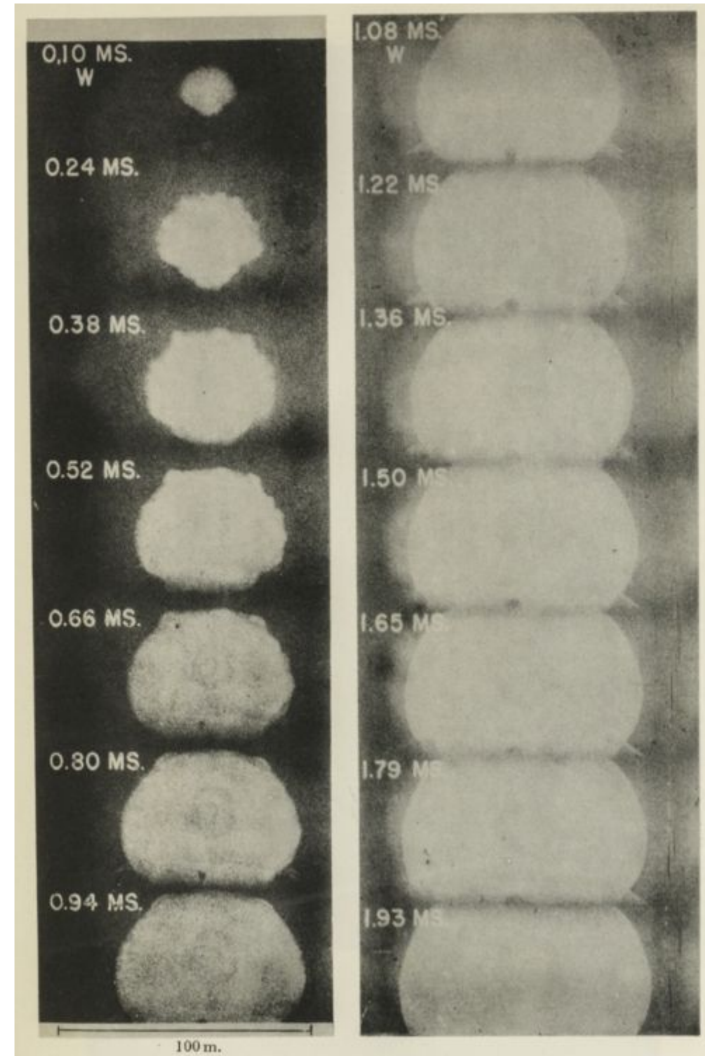


FIGURE 1. Logarithmic plot showing that $R^{\frac{5}{2}}$ is proportional to t.



The Sedov-Taylor blast wave

One of the most famous examples of scaling a physical problem is the similarity solution for a spherical blast wave created by suddenly releasing an energy E in a medium of density ρ_0.
The distance travelled by the blast is

$$r = \beta \left(\frac{E}{\rho_0} \right)^{1/5} t^{2/5}$$

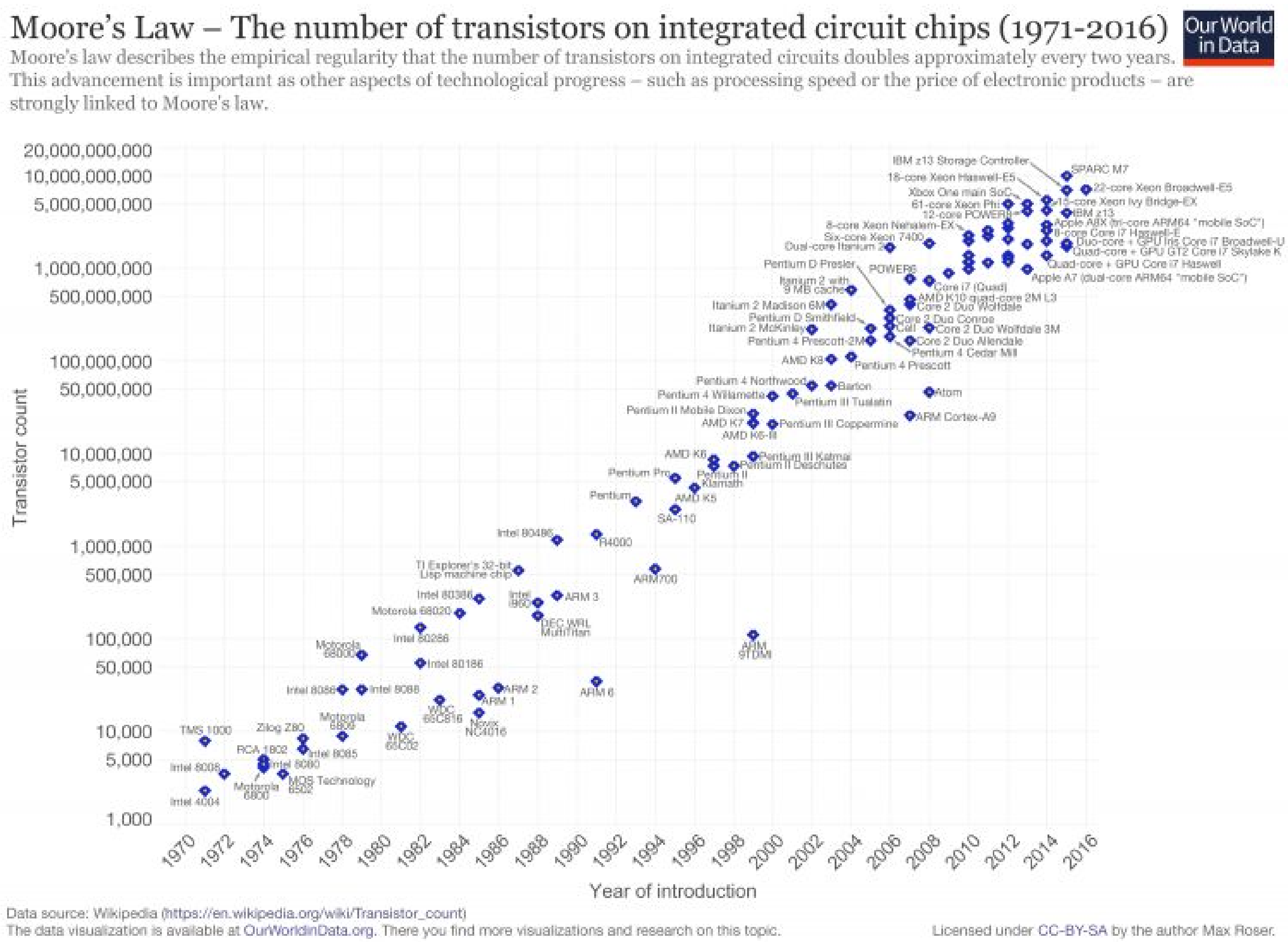
which tells us the blast energy

$$E = \left(\frac{\rho_0}{\beta^5} \right) \frac{r^5}{t^2}$$

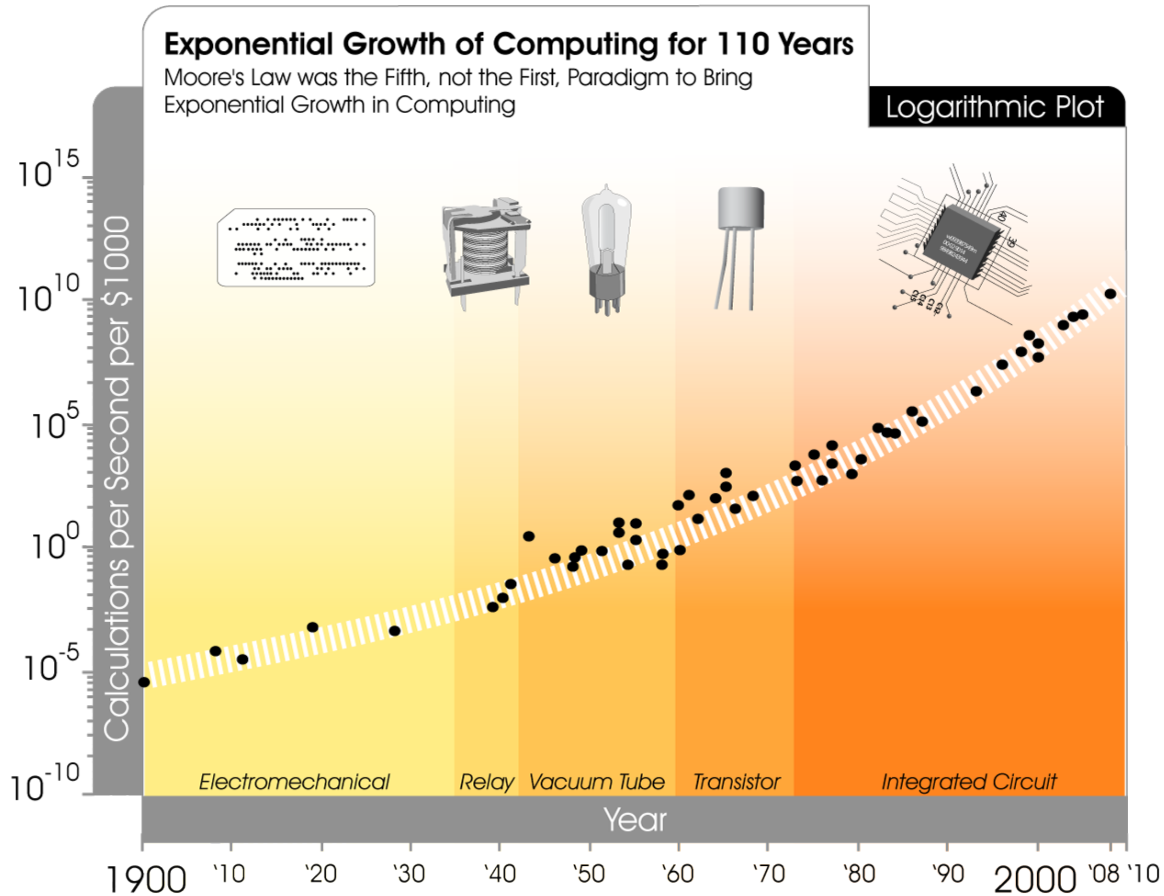
$\beta \simeq 1.0 - 1.1$ describes the physics of the gas.

In 1950 Taylor published the calculation and estimated the strength of the first 1945 atomic explosion at around 20 kilotons of TNT.
This was a highly classified number and some people were not pleased that this was published.

Moore’s law (2016)

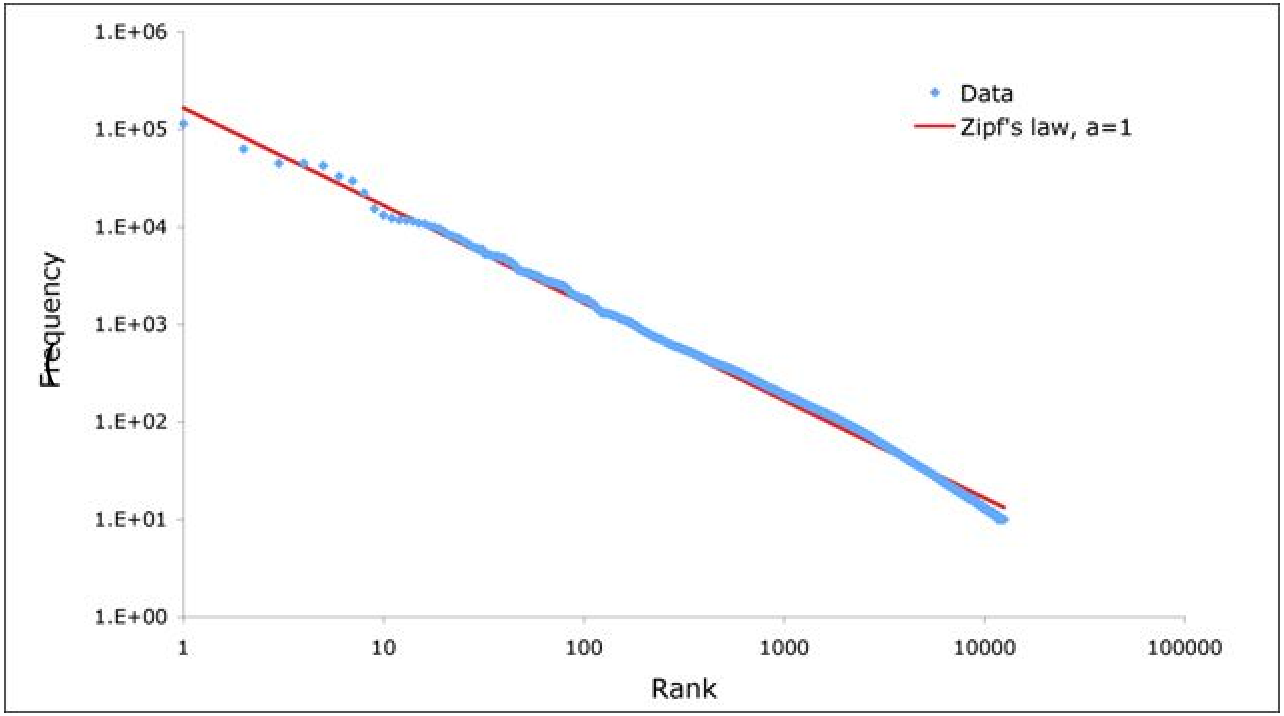


Kurzweil's extension of Moore's Law



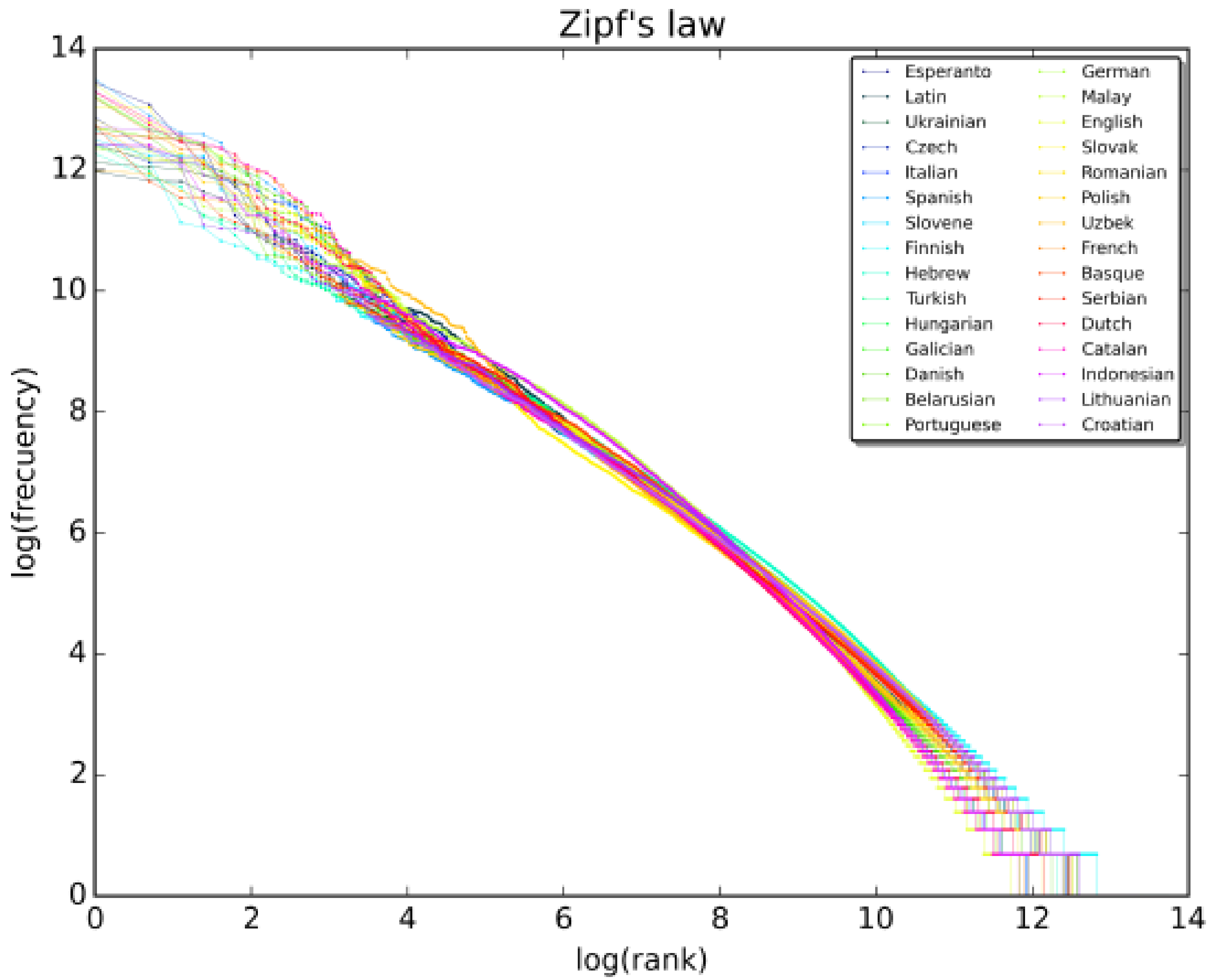
Zipf’s law for word frequency

Lexical analysis is one of the main places where we see Zipf’s law taken relatively seriously. (Zipf `discovered’ this in linguistics)



This is a survey of the “plus” webpages where “the” is ranked 1 and “of” ranked 2, with “mathematics” ranked 51.

Zipf’s Law for languages



Analysis of word frequency from Wikipedias in 15 diverse languages.
Esperanto is a made-up language so is particularly puzzling.

Zipf’s Law: size of cities

Zipf’s Law asserts that the frequencies of events are inversely proportional to their rank.

Hence with the population of cities the second ranked city (r=2) would have half the population of the first ranked, while the third-ranked (r=3) would have a third the population of the first ranked.

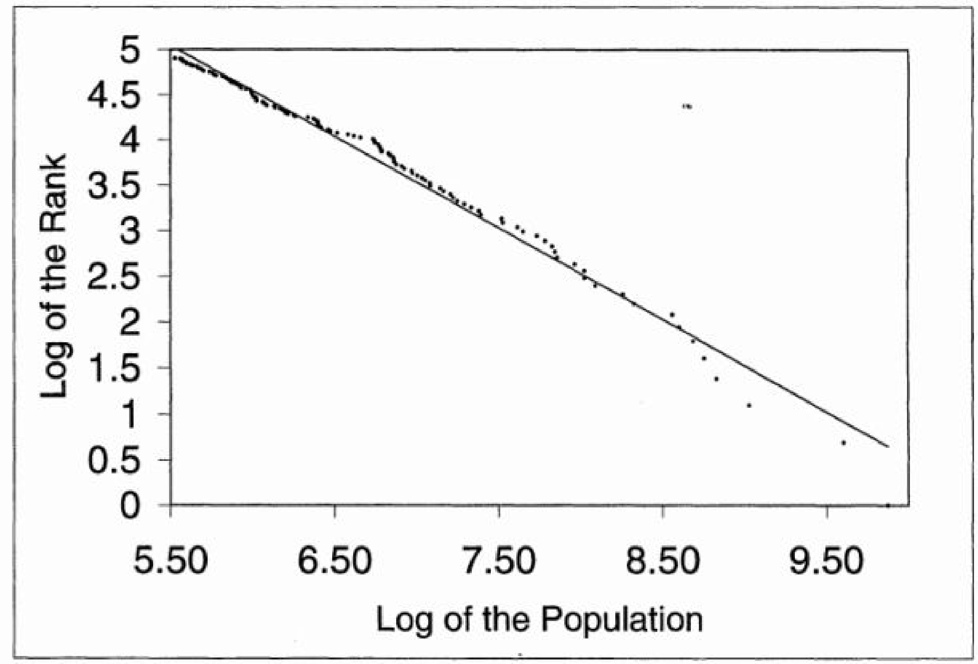
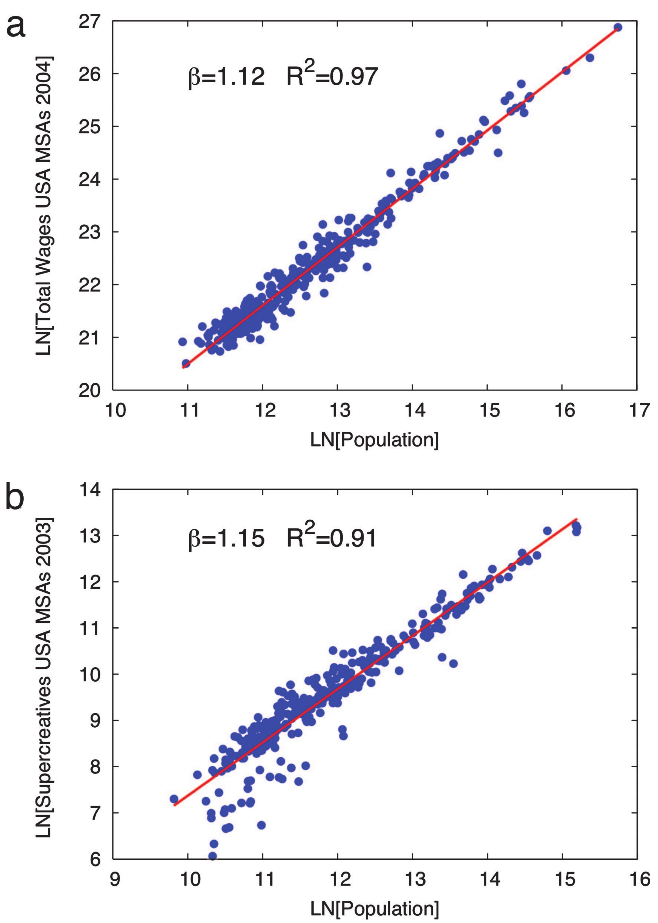


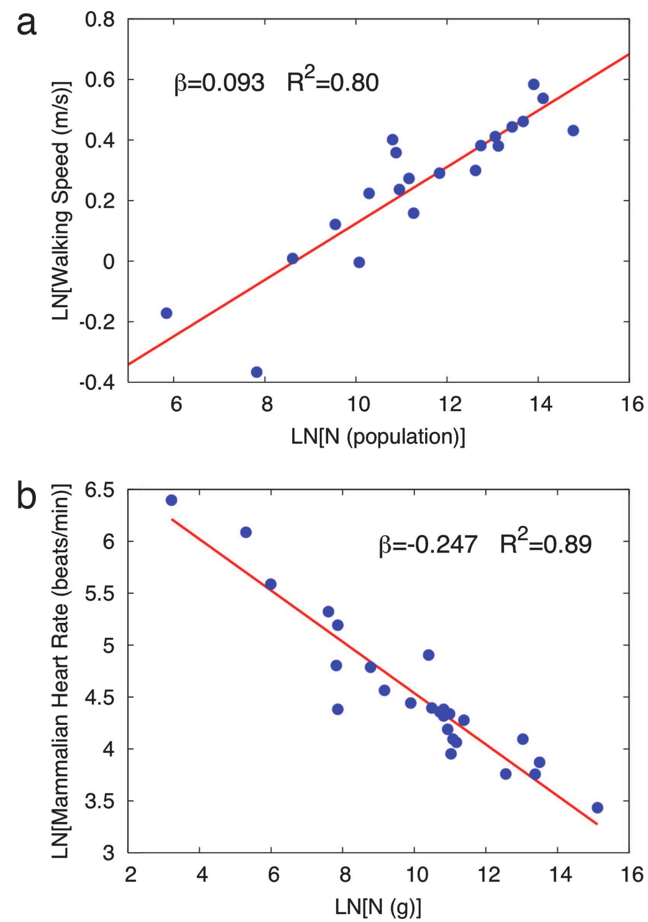
FIGURE I
Log Size versus Log Rank of the 135 largest U. S. Metropolitan Areas in 1991
Source: Statistical Abstract of the United States [1993].

With such a relationship you could predict what is the size of the 26th largest city in some arbitrary country. What could possibly be the mechanism driving that?

Growth of cities - models

Bettancourt et al. 2007, PNAS, **104**, 7301 *Growth, innovation, scaling, and the pace of life in cities*





Growth of a resource $Y(t) = Y_0 N(t)^\beta$

$N(t)$ is population growth, $Y(t)$ is wealth, pollution, ...

Tracking city resources

Y	β	95% CI	Adj-R^2	Observations	Country–year
New patents	1.27	[1.25,1.29]	0.72	331	U.S. 2001
Inventors	1.25	[1.22,1.27]	0.76	331	U.S. 2001
Private R&D employment	1.34	[1.29,1.39]	0.92	266	U.S. 2002
"Supercreative" employment	1.15	[1.11,1.18]	0.89	287	U.S. 2003
R&D establishments	1.19	[1.14,1.22]	0.77	287	U.S. 1997
R&D employment	1.26	[1.18,1.43]	0.93	295	China 2002
Total wages	1.12	[1.09,1.13]	0.96	361	U.S. 2002
Total bank deposits	1.08	[1.03,1.11]	0.91	267	U.S. 1996
GDP	1.15	[1.06,1.23]	0.96	295	China 2002
GDP	1.26	[1.09,1.46]	0.64	196	EU 1999–2003
GDP	1.13	[1.03,1.23]	0.94	37	Germany 2003
Total electrical consumption	1.07	[1.03,1.11]	0.88	392	Germany 2002
New AIDS cases	1.23	[1.18,1.29]	0.76	93	U.S. 2002–2003
Serious crimes	1.16	[1.11, 1.18]	0.89	287	U.S. 2003
Total housing	1.00	[0.99,1.01]	0.99	316	U.S. 1990
Total employment	1.01	[0.99,1.02]	0.98	331	U.S. 2001
Household electrical consumption	1.00	[0.94,1.06]	0.88	377	Germany 2002
Household electrical consumption	1.05	[0.89,1.22]	0.91	295	China 2002
Household water consumption	1.01	[0.89,1.11]	0.96	295	China 2002
Gasoline stations	0.77	[0.74,0.81]	0.93	318	U.S. 2001
Gasoline sales	0.79	[0.73,0.80]	0.94	318	U.S. 2001
Length of electrical cables	0.87	[0.82,0.92]	0.75	380	Germany 2002
Road surface	0.83	[0.74,0.92]	0.87	29	Germany 2002

Data sources are shown in *SI Text*. CI, confidence interval; Adj-R^2, adjusted R^2; GDP, gross domestic product.

The properties (resources) that are tracked by the model

The equations for city evolution

The equations:

$$dY = RN + E\,\frac{dN}{dt}$$

$$\frac{dN}{dt} = \left(\frac{Y_0}{E}\right) - \left(\frac{R}{E}\right)\frac{dN}{dt}$$

$$N(t) = \left[\frac{Y_0}{R} + \left(N(0)^{1-\beta} - \frac{Y_0}{R}\right)\exp\left[-\frac{R}{E}(1-\beta)t\right]\right]^{1/(1-\beta)}$$

Are dominated by the choices $\beta < 1, \beta = 1, \beta > 1$

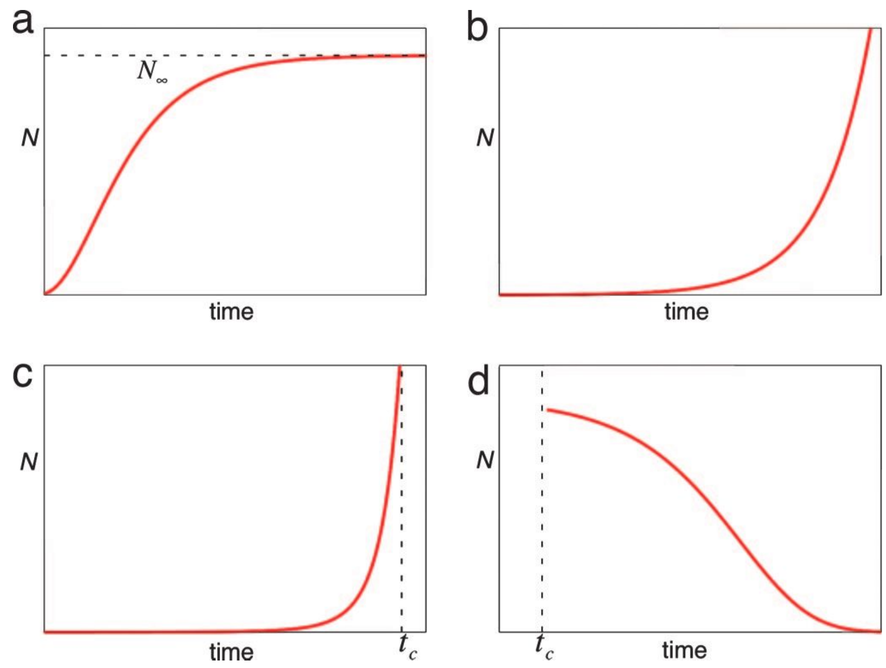
R: quantity of resource per unit time to maintain individual

E: resource it takes to add a new individual

E/R is the timescale for adding an individual to the city

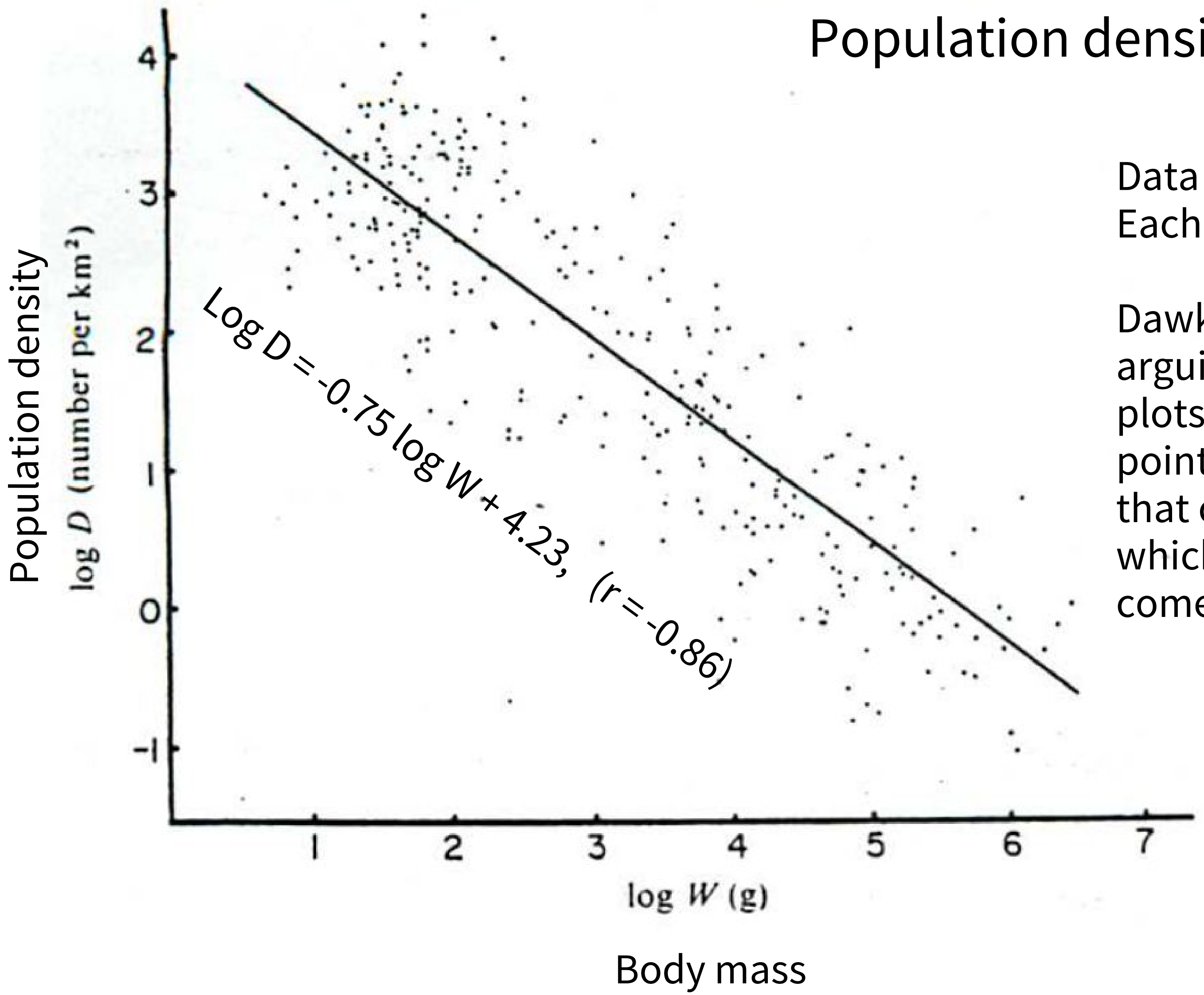
How cities may evolve

Evolution patterns



Scaling exponent	Driving force
$\beta < 1$	Optimization, efficiency
$\beta > 1$	Creation of information, wealth and resources
$\beta = 1$	Individual maintenance

Damuth relationship (2011)

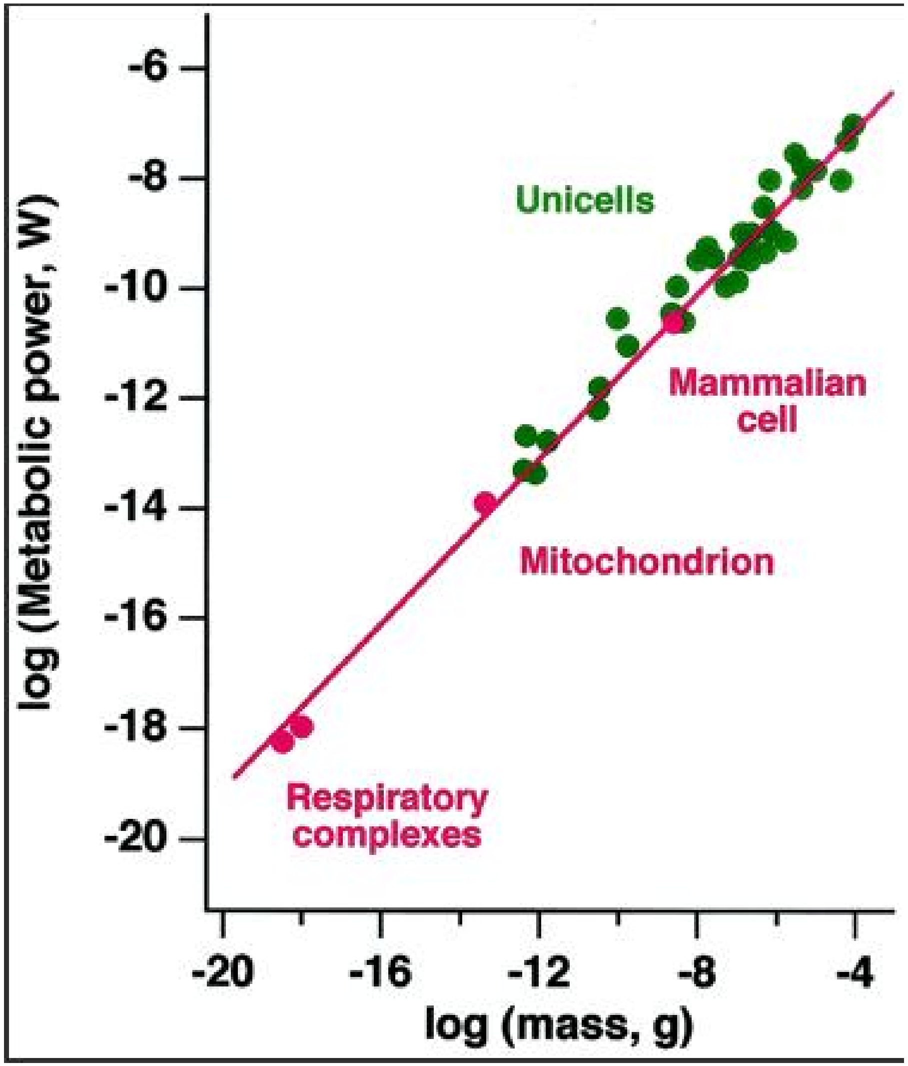


Population density vs body mass

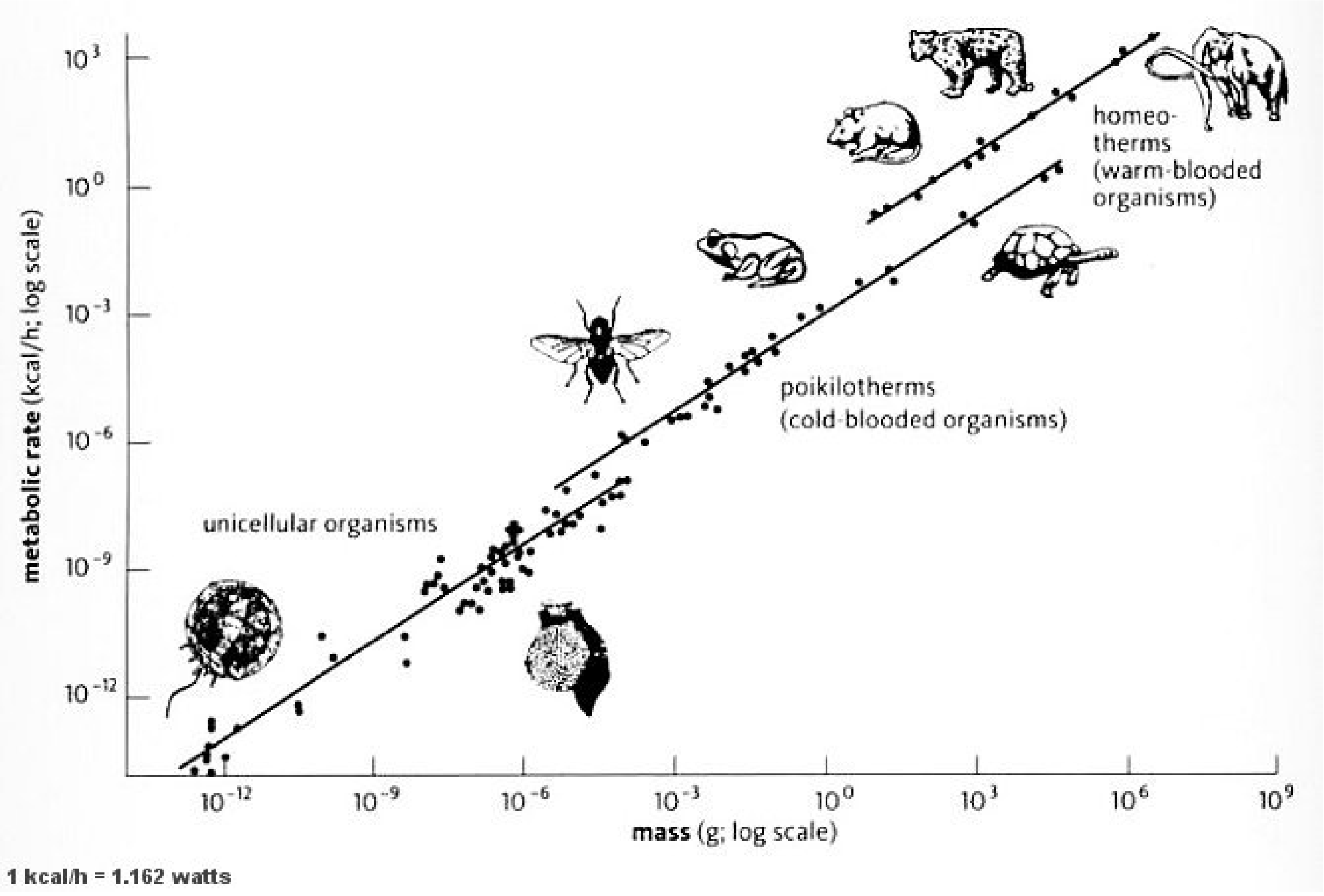
Data from 370 species.
Each point is one species.

Dawkins was highly critical of this, arguing that people are creating plots that support some particular point of contention. In this case that concerns the mechanism by which such a relationship could come about.

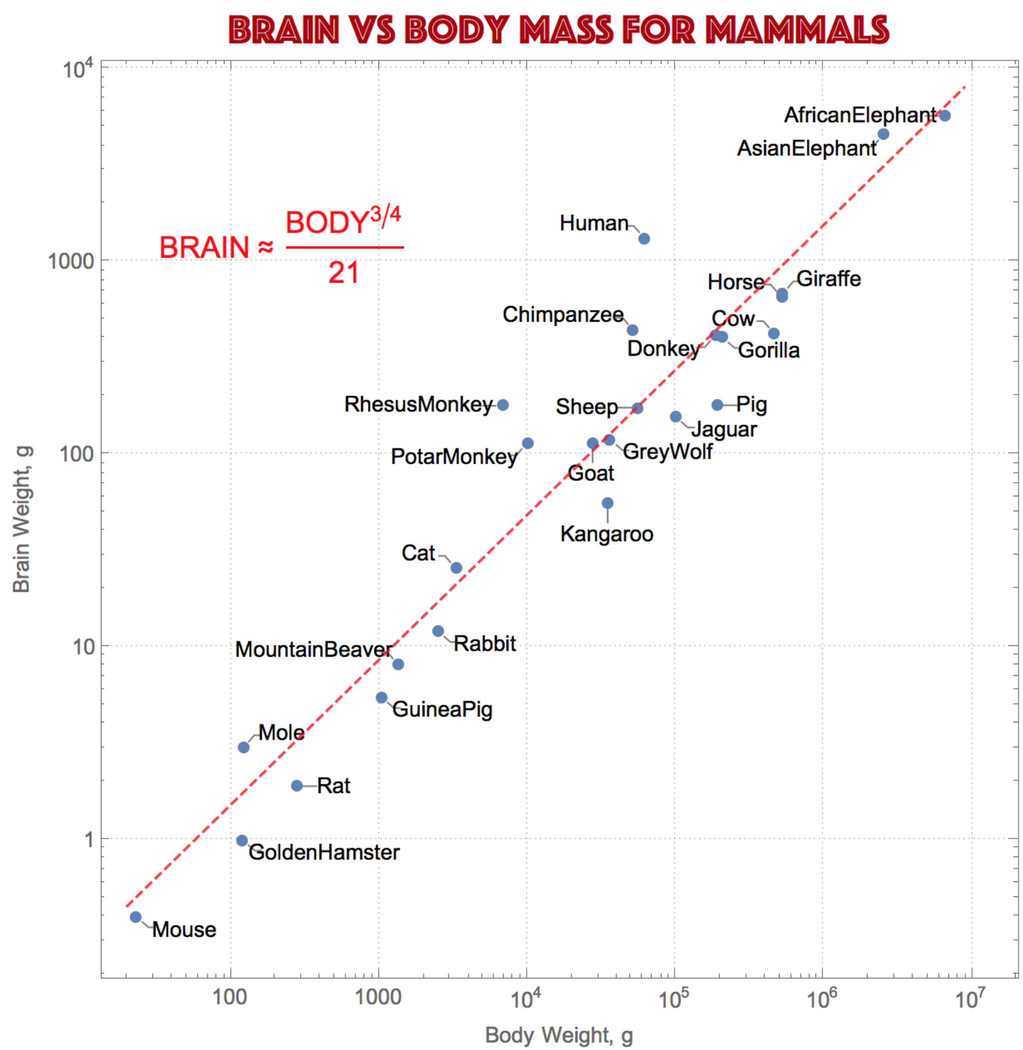
metabolic power vs mass

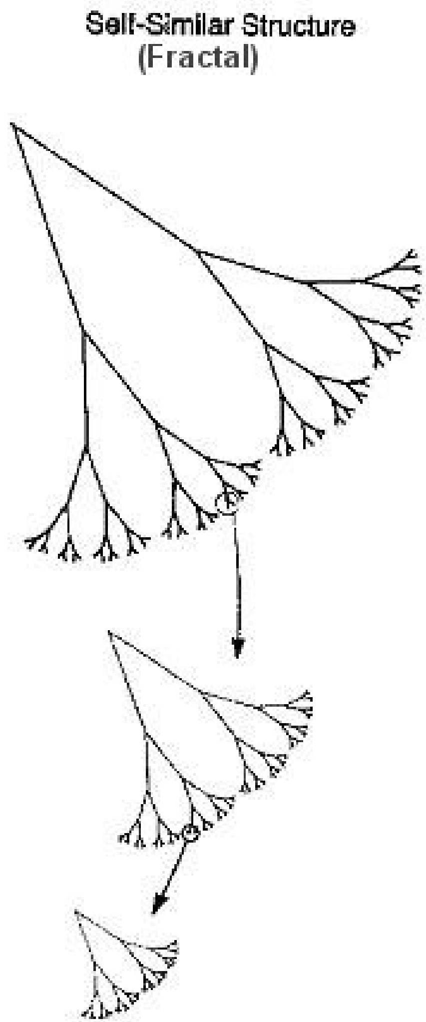


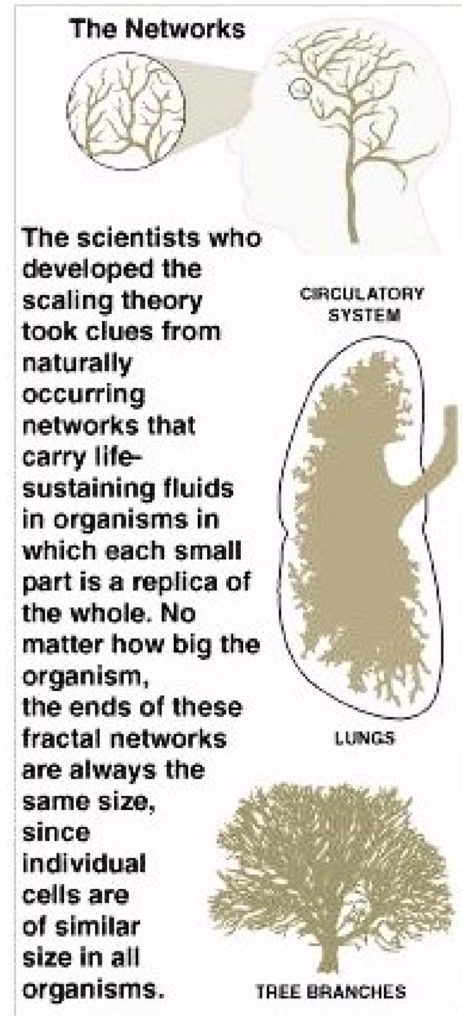
Metabolic rate vs mass



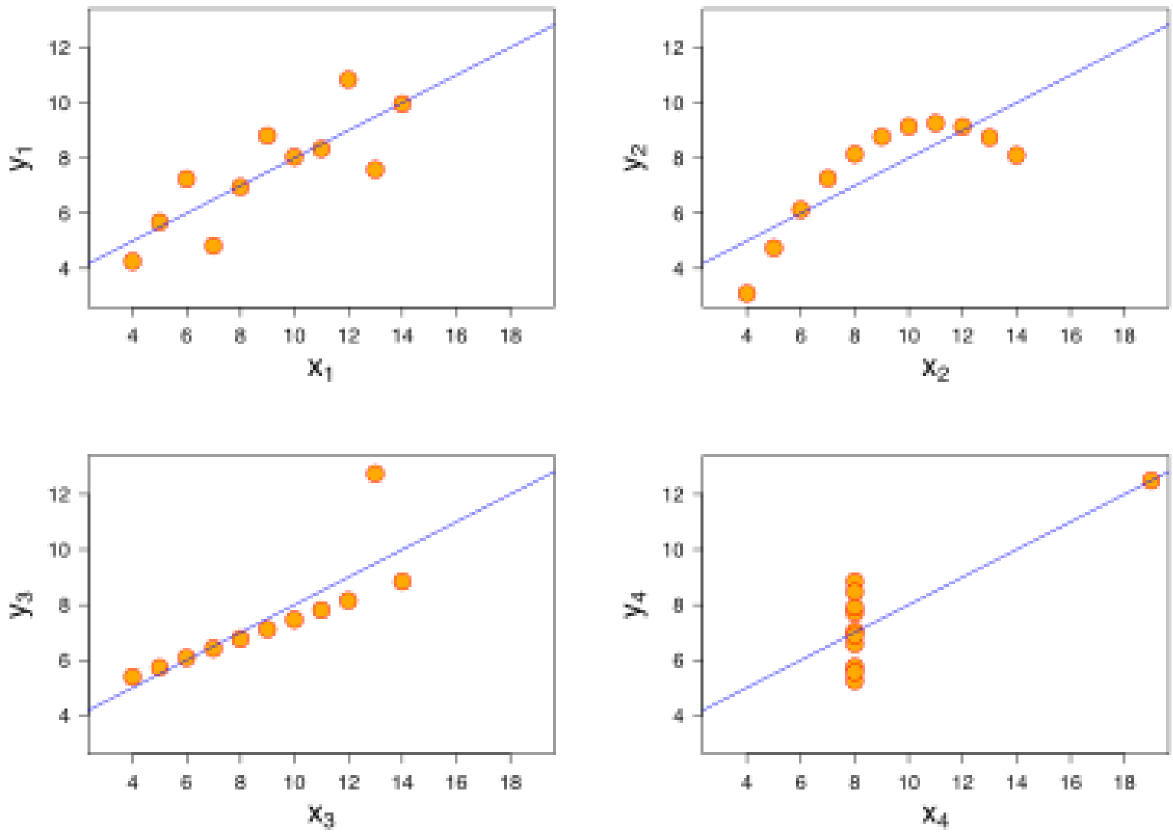
Brain mass vs body mass




Self-Similar Structure
(Fractal)


The Networks
The scientists who developed the scaling theory took clues from naturally occurring networks that carry life-sustaining fluids in organisms in which each small part is a replica of the whole. No matter how big the organism, the ends of these fractal networks are always the same size, since individual cells are of similar size in all organisms.
CIRCULATORY SYSTEM
LUNGS
TREE BRANCHES

Anscombe's quartet

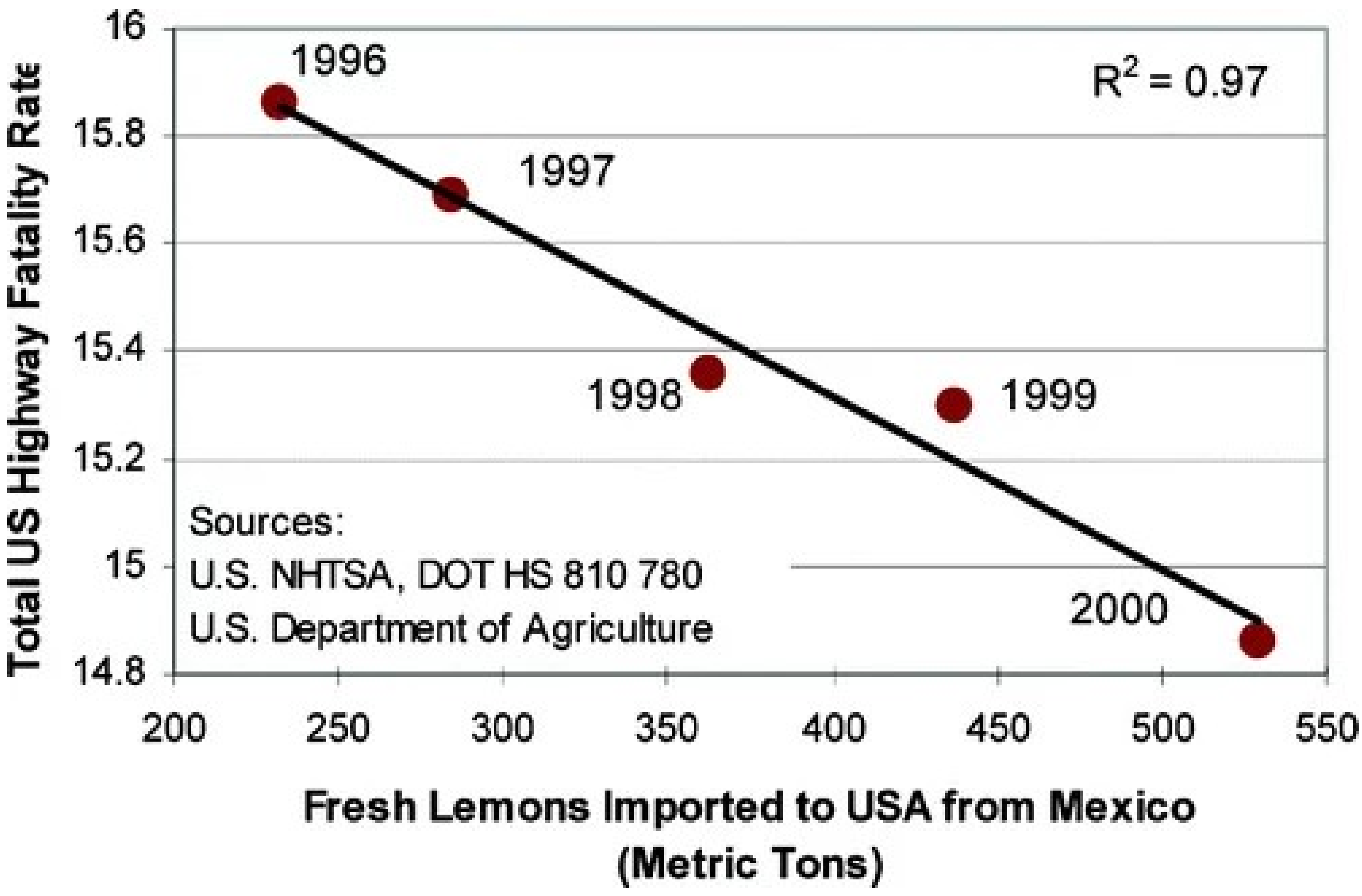


Mean x = 9
Sample x variance = 11
Mean y =7.5
Sample y-variance = 4.125
Correlation x,y = 0.816

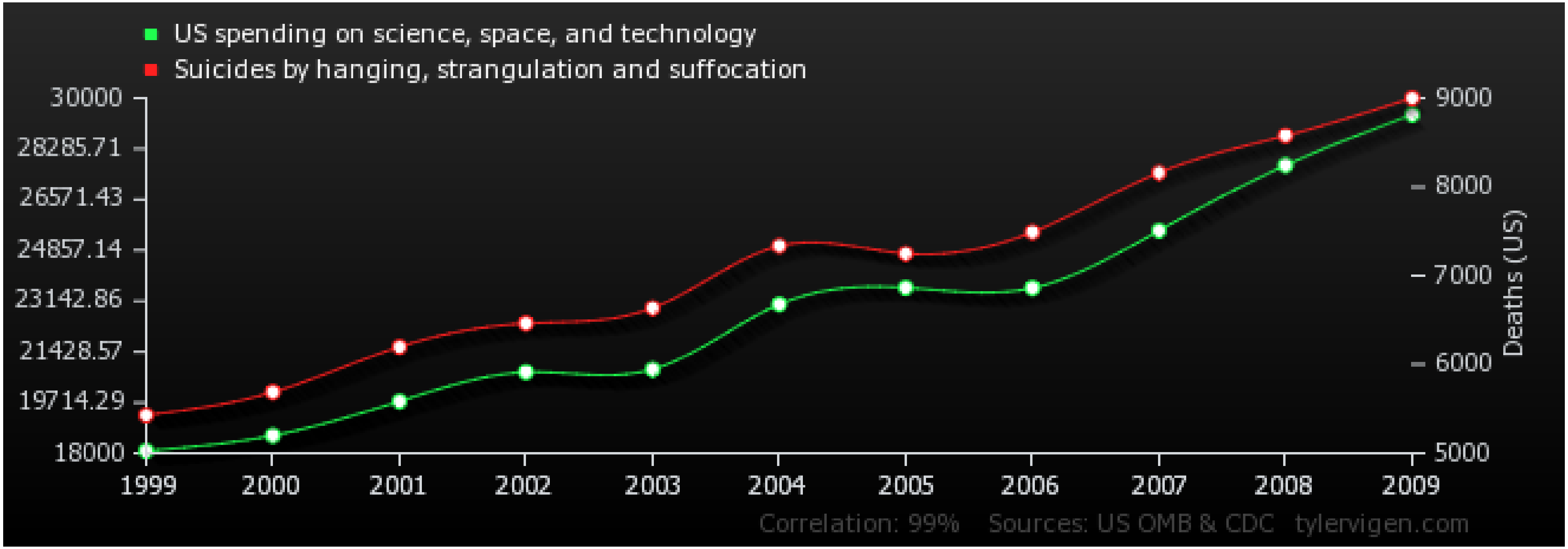
Moral:
Look at the data!

Regression line: y = 3.00 + 0.500 x

Manifestly fake (for fun)



The moral of the tale



The correlation between these two variables is 0.992

Moral of the tale: correlations does not imply causality
Nor does it indicate common cause.

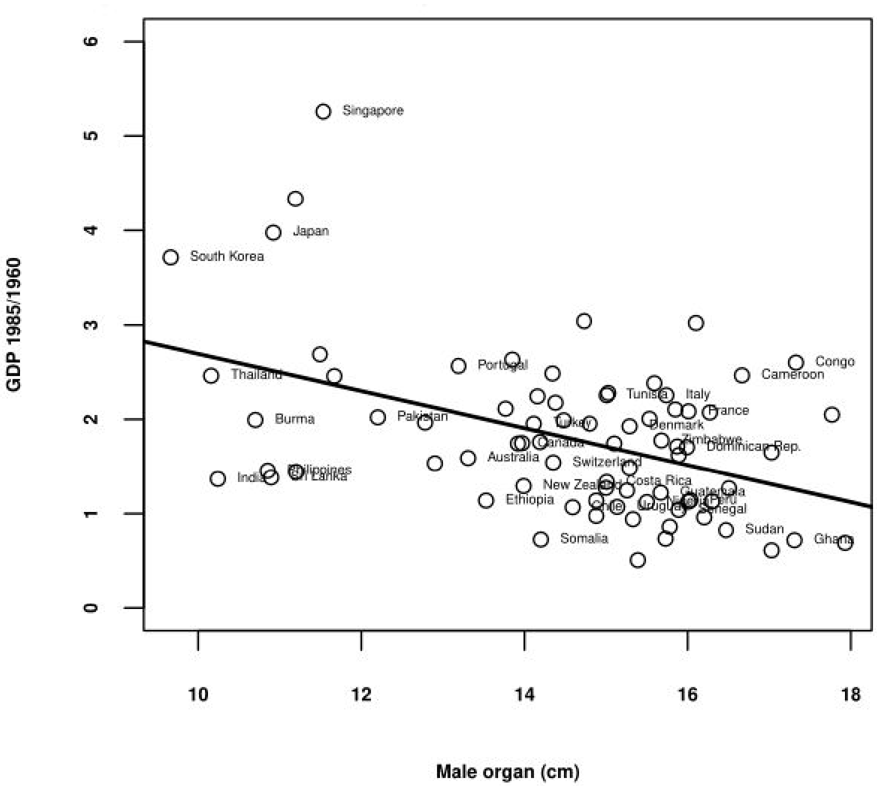
Press promotion of dubious science

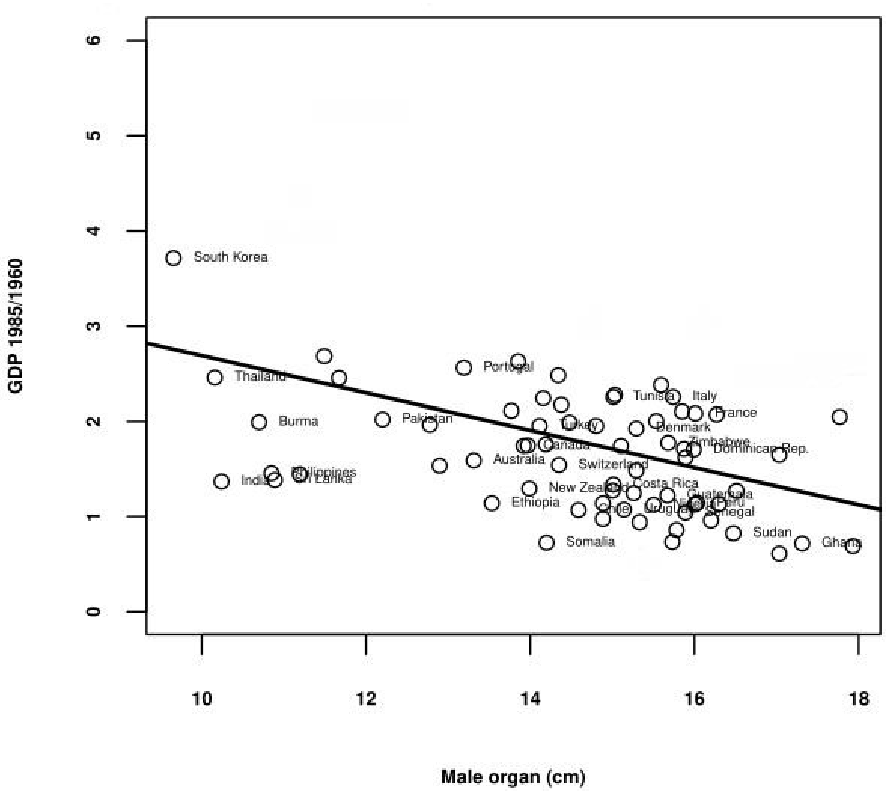
The Atlantic   Popular   Latest

Chart: This Is the Relationship Between Penile Length and Economic Growth

THE WIRE | JUL 19, 2011 | BUSINESS

Improving a correlation





2-sigma rejection helps make a correlation more convincing

T. Westling, University of Helsinki HECER: Discussion paper 335 July 2011