Understanding matter: broken and unbroken symmetries of QCD

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Quantum Universe Masterclass, Groningen, March 2018

Particle Physics: what and why

Describe the world/universe in terms of fundamental variables and interactions.

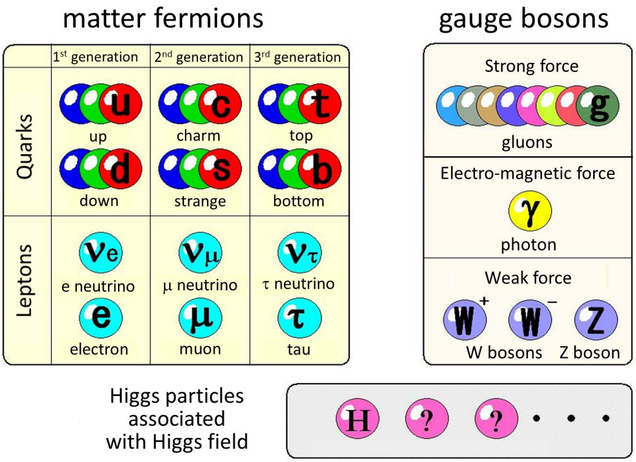
Fundamental Force Particles

Force	Particles Experiencing	Force Carrier Particle	Range	Relative Strength*
Gravity acts between objects with mass	all particles with mass	graviton (not yet observed)	infinity	much weaker
Weak Force governs particle decay	quarks and leptons	W^+, W^-, Z^0 (W and Z)	short range	↓
Electromagnetism acts between electrically charged particles	electrically charged	γ (photon)	infinity	↓
Strong Force** binds quarks together	quarks and gluons	g (gluon)	short range	much stronger

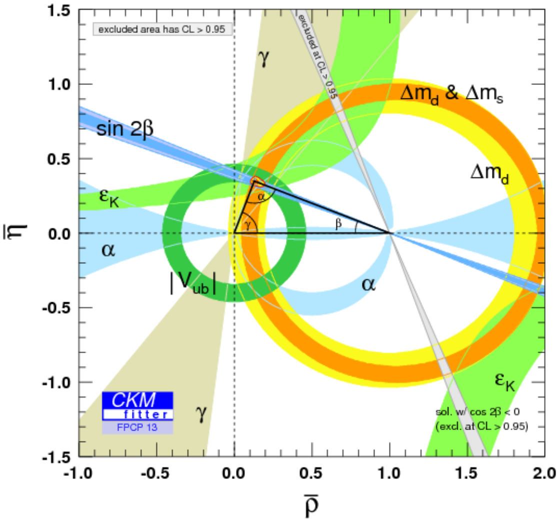
Is there one theory in which all this fits?

A unified description of 3 out of 4 forces

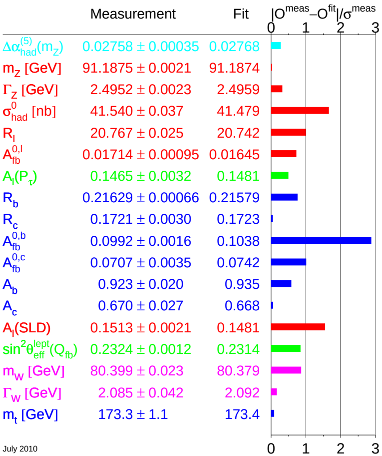
EM, Weak and Strong nuclear forces - STANDARD MODEL



Experiment and theory constrain the SM



Testing the Standard Model: theory and experiment

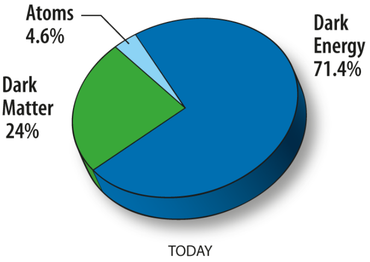


Many unanswerable questions:

- What about gravity?
- What is dark matter and dark energy?
- Why do we observe matter and almost no antimatter?
- Are quarks and leptons truly fundamental and why are there exactly 3 generations?
- …

A QCD theorist's perspective

Quantum ChromoDynamics - the theory of quarks and gluons crucial to test the Standard Model and explore New Physics!



- Precision tests of SM can reveal new physics - more important than ever!

Why focus on the 4.6%?

- **QCD is the only experimentally studied strongly-interacting quantum field theory - highlights many subtleties.**
- **A paradigm for other strongly-interacting theories in BSM physics.**
- **There are still puzzles and surprises in this well-studied arena.**

Classifying Particles

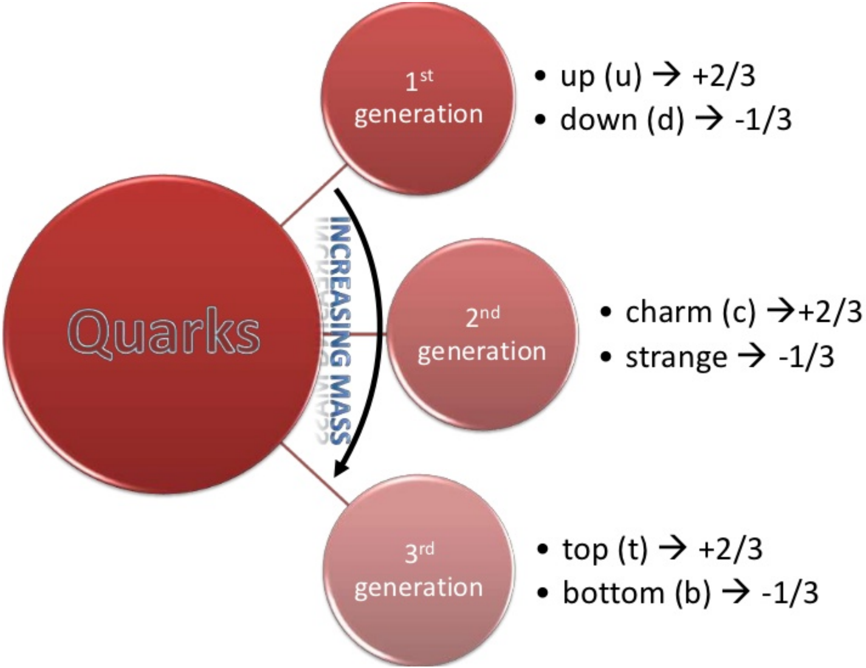
Types of particle

- **Hadrons:** built from quarks, affected by strong force
- **Quarks:** fundamental particles i.e. no internal structure (not made from smaller particles). Carry fractional charge - fractions of the charge of the e^-. Combine to form hadrons.
- **Gauge Bosons:** the force carriers - $\gamma, g, W^\pm, Z$ and h.
- **Leptons:** fundamental particles, not affected by the strong force.

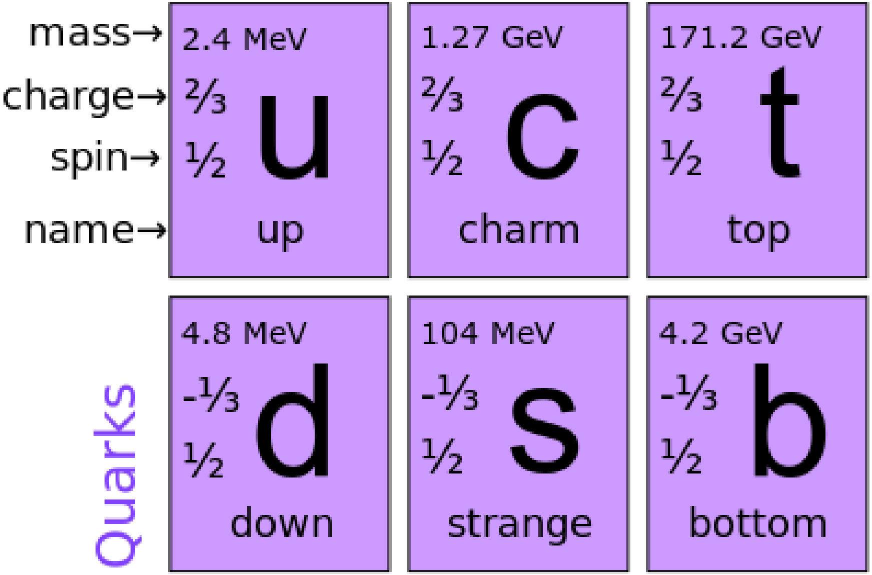
What is everything made from?

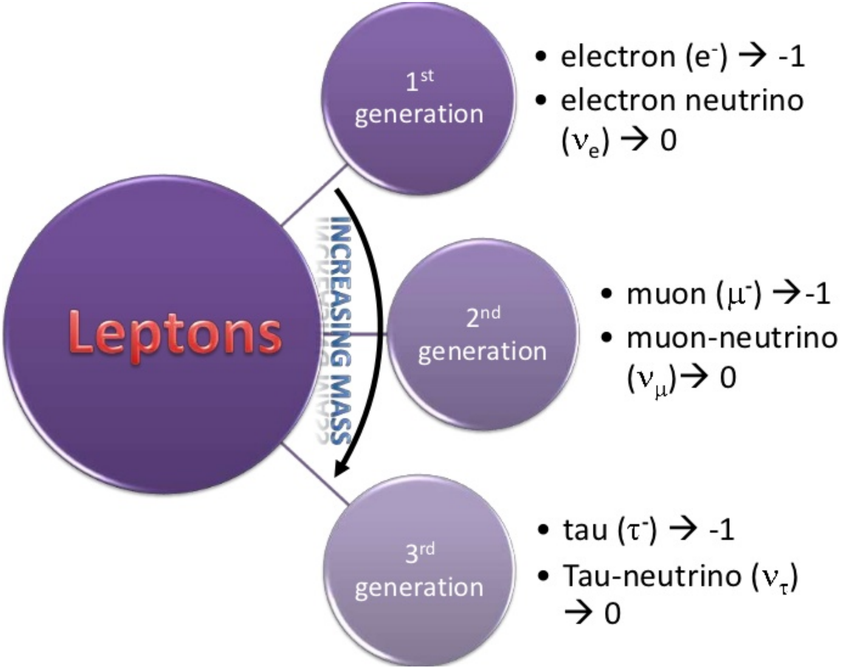
Quarks and Leptons

- to understand hadrons we need to understand the theory of their constituent fundamental particles.



from Alessio Bernardelli, SlideShare


mass→
charge→
spin→
name→
Quarks
2.4 MeV
⅔
½
u
up
1.27 GeV
⅔
½
c
charm
171.2 GeV
⅔
½
t
top
4.8 MeV
-⅓
½
d
down
104 MeV
-⅓
½
s
strange
4.2 GeV
-⅓
½
b
bottom



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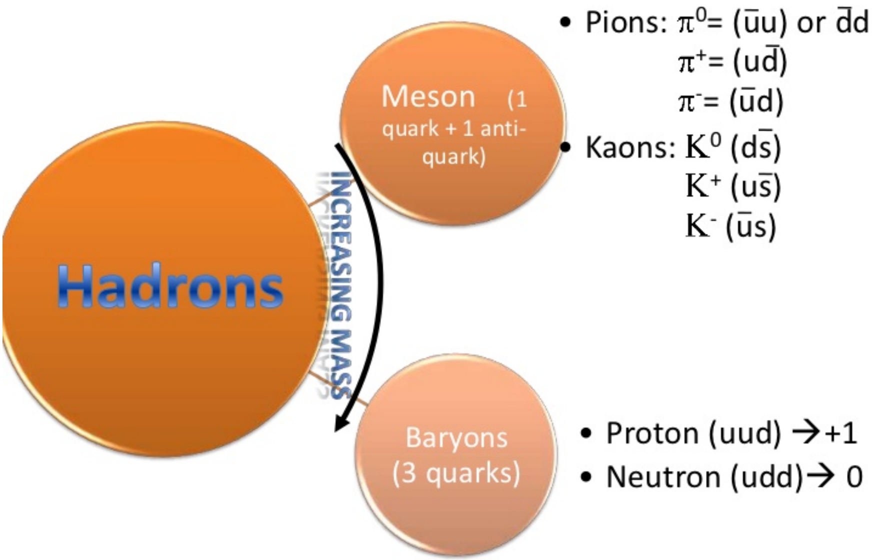


from Alessio Bernardelli, SlideShare

Colour: a new quantum number

Motivated by discovery of the $\Delta^{++}(1232)$ baryon. $|\Delta^{++}\rangle = |u^{\uparrow} u^{\uparrow} u^{\uparrow}\rangle$ and $J^P = \frac{3}{2}^+$.

Now, considering wavefunctions

$$\Psi(\Delta^{++}) = \underbrace{\Psi(r)}_{symmetric} \cdot \underbrace{\Psi_{\text{spin}}(J)}_{symmetric} \cdot \underbrace{\Psi_{\text{flavour}}}_{symmetric}$$

Contradicts the Pauli principle which demands the total wavefunction be antisymmetric.

Requires a new (and unobservable) quantum number - colour:

$$\Psi(\Delta^{++}) = \Psi(r) \cdot \Psi_{\text{spin}}(J) \cdot \Psi_{\text{flavour}} \cdot \Psi_{\text{colour}}$$

Only colourless states observed - quarks and gluons carry colour, confined in hadrons.

Color Confinement Hypothesis

Only colour singlet states can exist as free particles.

QCD and color

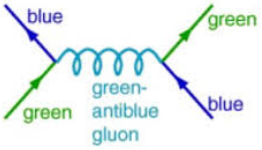
Quarks

A "new" mathematical property to explain observed particles - the spin 3/2 uuu baryon requires each quark to have a different color to not violate Pauli.

- **Quarks:** Come in three colours (r,g,b); anti-quarks have anti-colours.
- **Leptons and other Gauge Bosons:** Don't carry colour charge so don't participate in strong interaction

Gluons

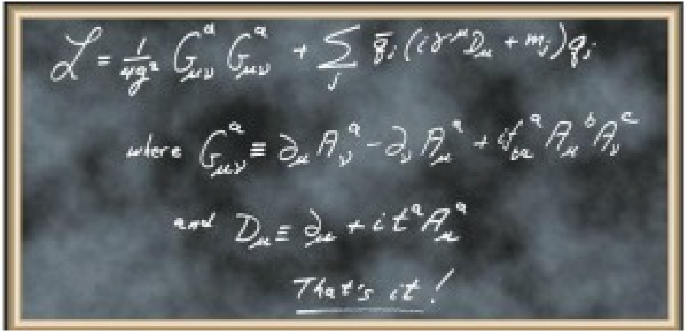
Massless spin-1 bosons. Emission or absorption of gluons by quarks changes colour of quark - **colour is conserved**.



Gluons carry colour charge e.g. gb gluon changes green quark to blue.

Quantum Chromodynamics (QCD)

The quantum field theory of the strong interaction that binds quarks and gluons to form hadrons.



from F.A. Wilczek

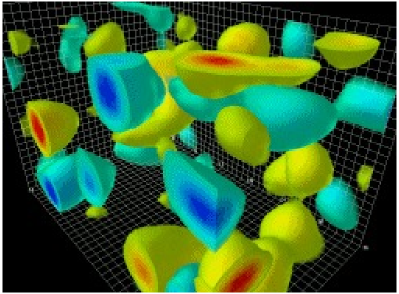
Doesn't look too bad - a bit like QED which we have a well-developed toolkit to deal with

What is a quantum field theory anyway?!

- A mathematical framework for the quantum mechanical behaviour of subatomic particle and quasi particles.
- Consider fundamental building blocks not as particles but fields - continuous throughout space e.g. EM fields
- Ripples in the field are form waves with a quantum mechanical interpretation as particles e.g. photons.
- Similar for all (fundamental) particles - quarks, gluons, Higgs etc.

Why are QFT calculations "hard"?

- QFT can describe many particles and their interactions
- The "vacuum" is not static! Even with no particles the field creates and destroys particles and antiparticles



Symmetry

- Nature exhibits many symmetries - exact and approximate.
    - **local:** act independently at each spacetime point. The basis of gauge theory eg QCD a gauge theory of SU(3) color (a local symmetry)
    - **global:** simultaneoulsy applied at all spacetime points.
- Symmetry is an invariance of a system under a set of transformations.
- Symmetries imply conservation laws, particularly conserved currents (Noether). Invariance with respect to
    - continuous transformation $\longrightarrow$ momentum conservation
    - time shift transformation $\longrightarrow$ energy conservation
    - spatial rotation $\longrightarrow$ angular momentum conservation
- Described in the mathematical language of group theory.

Breaking symmetries

Explicit breaking	symmetry is approximate; still very useful
Spontaneous breaking	equations of motion are invariant, but ground state is not
Anomalously broken	classical invariance but broken at QM level.

All relevant in QCD

QCD: AN ABUNDANCE OF SYMMETRY

- Explains nature's strong interactions in terms of fundamental variables: quarks and gluons.
- A theory rich with symmetries!

$$SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_A \times U(1)_B$$

  - Gauge "color" symmetry; Global chiral, flavour symmetry; Baryon number and axial charge (m=0) conservation, discrete C, P, T symmetries and ... .

- Quantum effects $\longrightarrow$ breaking of symmetries $\longrightarrow$ visible matter.
- Inherent properties of the theory: confinement, asymptotic freedom, anomalies, spontaneous symmetry breaking - depend on non-linear dynamics in QCD

Flavour symmetry

- From decay of particles, flavour is conserved ie the lagrangian is invariant under rotations in flavour space - for the strong interaction.

$$\Delta^{++}(uuu) \rightarrow p(uud) + \pi^{+}(\bar{d}u)$$

- Implies the conservation of flavour currents
- Flavour is conserved as a global symmetry
  - for u,d,s consider SU(3) flavour group
  - for 6 flavours consider SU(6) flavour group

Local Gauge Symmetry in QCD (& QED): colour

Local gauge invariance of $\mathcal{L} \Rightarrow$ covariant derivative $\Rightarrow$ gauge vector field A_μ

QED

$U(1)$ gauge transformation

$$\psi(x) \longrightarrow e^{i\alpha(x)}\psi(x)$$

1 gauge (vector) field

$A_\mu(x)$ - photons (massless)

QED, abelian: $[U(\alpha_1), U(\alpha_2)] = 0$

No self-interaction of A_μ since photons don't have charge

QCD

$SU(3)$ colour gauge transformation

$$\psi(x) \longrightarrow e^{i\alpha_a(x)t^a}\psi(x)$$

8 gauge (vector) fields

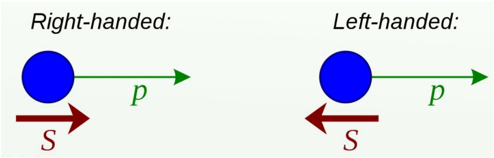
$A^a_\mu(x)$ - gluons (massless)

QCD, nonabelian: $[t_a, t_b] = if_{abc}t_c$

Self-interaction of A^a_μ since gluons have charge

Chiral symmetry

Chirality - projection of spin s on the momentum p (direction of motion)



$$\psi = \psi_R + \psi_L \,, \qquad \psi_{R,L} = \frac{1}{2}(1 \pm \gamma^5)\psi$$

QCD lagrangian is invariant under $\psi(x) \to exp\left(-i\sum_{b=1}^{8} a_b t_b \gamma^5\right)\psi(x)$ for massless quarks. ie left, right handed quarks are not mixed dynamically and conserve chirality.

- chiral invariance of QCD, $SU(3)_{\text{flavour}} \Rightarrow SU(3)_R \times SU(3)_L$
- mass term explicitly breaks chiral symmetry, although weakly for u and d quarks ...

Emergent Phenomena

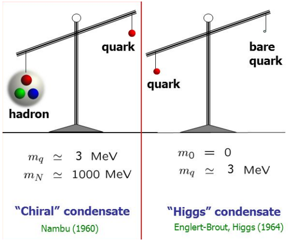
QCD demonstrates complex behaviour from seemingly simple rules

Confinement

- No quarks and gluons found free in Nature

Dynamical chiral symmetry breaking

- There is a global chiral symmetry for $m_q = 0$, approximate for light quarks
- But: opposite parity states are not close in mass - spontaneously broken symmetry
- “Higgs” mechanism in strong interactions: $q\bar{q}$ condensate fills the vacuum.



Dynamical mass generation through non-linear interactions

Very little to do with Higgs!

Massless gluons and almost massles quarks interact - generating most of the mass of nucleons

Proton: uud

- $m_u = 2.3^{+0.7}_{-0.5}$ MeV/c^2
- $m_d = 4.8^{+0.7}_{-0.3}$ MeV/c^2
- $M_P = 938.3$ MeV/c^2
- $M_\pi = 130$ MeV/c^2

- Only 1% of the proton's mass comes from the constituent quarks' intrinsic masses.
- The proton is an emergent (long-range) phenomena resulting from the collective behaviour of quarks and gluons - QCD!
- The pion is the Goldstone boson of dynamical chiral symmetry breaking.

Yang-Mills and the mass gap

A Clay Institute Millenium Prize
The prize: $1,000,000

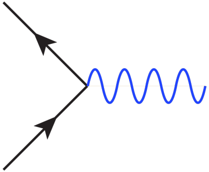
"The successful use of Yang-Mills theory to describe the strong interactions of elementary particles depends on a subtle quantum mechanical property called the 'mass gap:' the quantum particles have positive masses, even though the classical waves travel at the speed of light...
...Progress in establishing the existence of the Yang-Mills theory and a mass gap and will require the introduction of fundamental new ideas both in physics and in mathematics. "

- The mass gap explains why the strong force is short-ranged, and is intimately related to confinement in QCD.
- Confinement is a purely quantum phenomenon **and not yet understood**

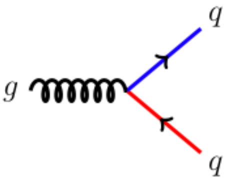
QED and QCD by Feynman diagrams

The fundamental vertices are analogous:

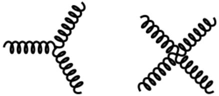
QED

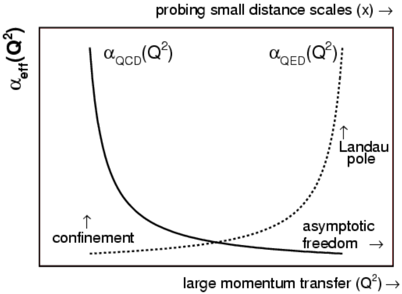


QCD



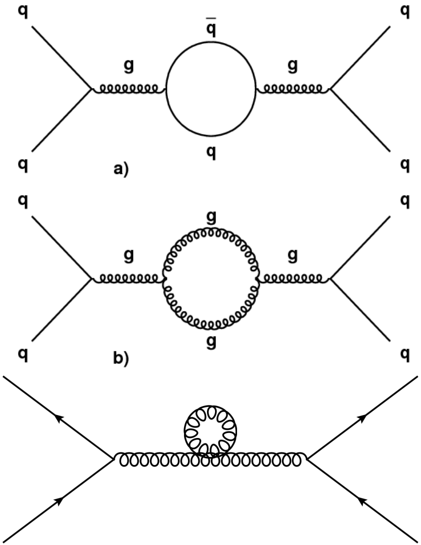
But in additon, gluons carry colour charge leading to



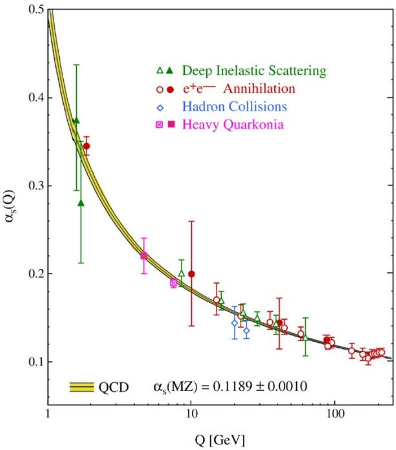


- **QED:** α varies with distance ie runs and the bare e^- is screened, reducing α
- **QCD:** screening as in QED **but** gluon loops create same-charge gluonic cloud and antiscreening dominates!
  - $\alpha_s \sim 1$ at hadronic scales, perturbation theory fails.

VACUUM POLARISATION IN QCD (AND QED)



- Vacuum polarisation diagrams in QED have QCD analogues
- In QCD there are additional vacuum polarisation diagrams arising from gluon loops
- Quark loops lead to screening - as in QED. The gluon loops lead to anti-screening.
- Net effect is the strong coupling is large at long distance, small at short distance



- In the quantum vacuum the coupling constant depends on the scale at measurement

$$\mu\frac{\partial\alpha_s}{\partial\mu}=-\frac{\beta_0}{2\pi}\alpha_s-\frac{\beta_1}{4\pi^2}\alpha_s^3-\ldots$$

- Short distance (high energy): weak coupling. **asymptotic freedom.**
- Long distance (low energy): strong coupling (IR slavery). **Confinement.**

Solution of the RGE introduces a dimensionful scale: Λ Scale invariance replaced with Dimensionsal Transmutation, relation between coupling and scale.

$$\alpha_s(\mu)=\frac{4\pi}{\beta_0\log(\mu^2/\Lambda^2_{QCD})}\left[1-\frac{2\beta_1}{\beta_0^2}\frac{\log[\log(\mu^2/\Lambda^2_{QCD})]}{\log(\mu^2/\Lambda^2_{QCD})}+\frac{4\beta_1^2}{\beta_0^4\log^2(\mu^2/\Lambda^2_{QCD})}\times\left(\left(\log[\log(\mu^2/\Lambda^2_{QCD})]-\frac{1}{2}\right)^2+\frac{\beta_2\beta_0}{8\beta_1^2}-\frac{5}{4}\right)\right]$$

QCD: making calculations

There are two regimes:

Deep inside the proton

- at short distances quarks behave as free particles
- weak coupling

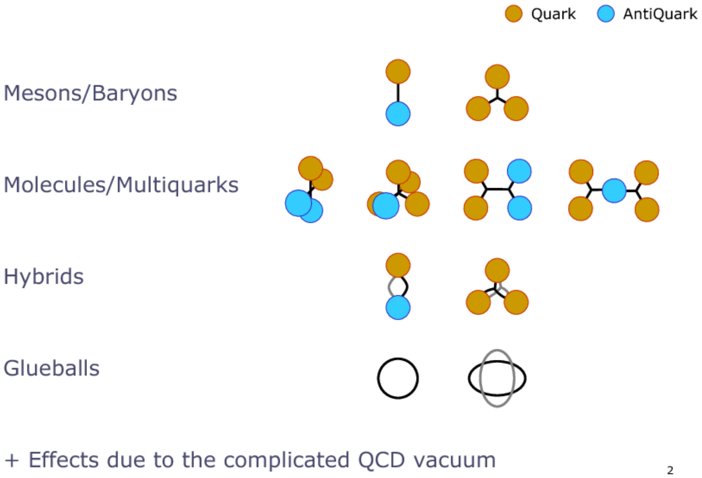
⇒ perturbation theory works

At "observable" distances

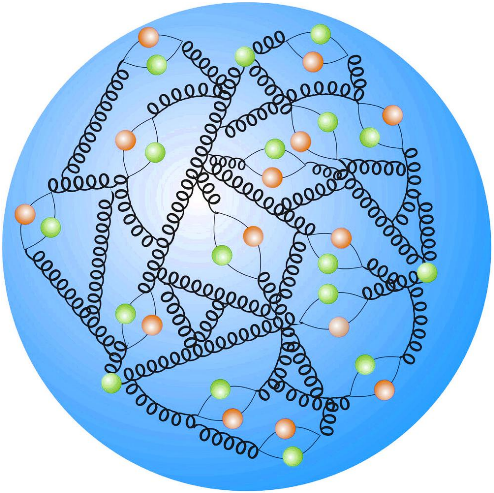
- at long distance (1fm) quarks confined
- strong coupling

⇒ perturbation theory fails: nonperturbative approach needed.

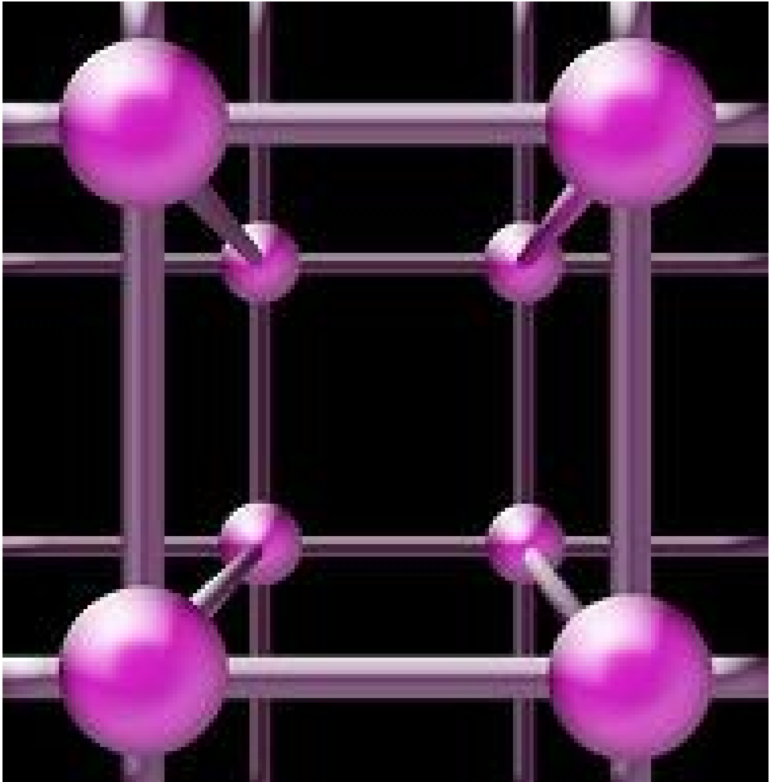
The spectrum of observable states



The secret life of hadrons



- **Can we calculate observable states from the Lagrangian in terms of fundamental variables: quarks and gluons?**




coffee break

Understanding matter: broken and unbroken symmetries in QCD

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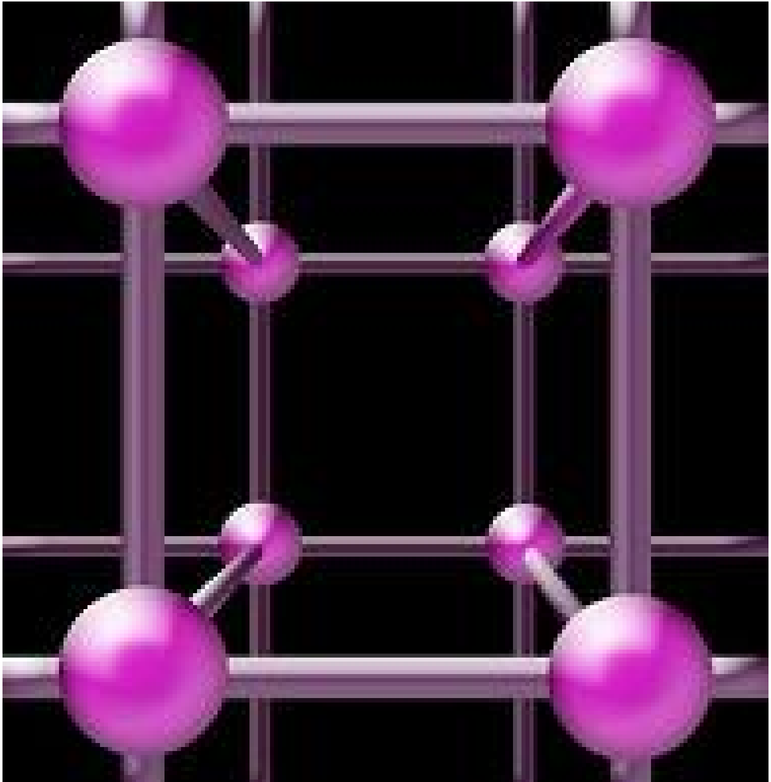
Quantum Universe Masterclass, Groningen 2018



Why Lattice QCD ?

- A systematically-improvable non-perturbative formulation of QCD
  - Well-defined theory with the lattice a UV regulator
- Arbitrary precision is in principle possible
  - of course algorithmic and field-theoretic "wrinkles" can make this challenging!
- Starts from first principles - i.e. from the QCD Lagrangian
  - inputs are quark mass(es) and the coupling - can explore mass dependence and coupling dependence but getting to physical values can be hard!

A typical road map:

- Develop methods and verify calculations through precision comparison with lattice and with experiment.
- Make predictions - subsequently verified experimentally.
- Make robust, precise calculations of quantities not accessible to experiment.

A POTTED HISTORY

- **1974** Lattice QCD formulated by K.G. Wilson
- **1980** Numerical Monte Carlo calculations by M. Creutz
- **1989** "and extraordinary increase in computing power (10^8 is I think not enough) and equally powerful algorithmic advances will be necessary before a full interaction with experiment takes place." Wilson @ Lattice Conference in Capri.
- **Now** at 100TFlops — 1PFlop
- Lattice QCD also contributing to development of computing QCDSP - QCDOC - BlueGene.

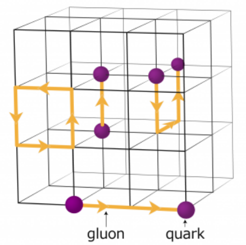
Learning from history ...
better computers help but better ideaas are crucial!

A LATTICE QCD PRIMER

Start from the QCD Lagrangian:

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi - \frac{1}{4}G^{a}_{\mu\nu}G^{\mu\nu}_{a}$$

- Gluon fields are SU(3) matrices - links of a hypercube.
  $A_{\mu}(x) \rightarrow U(x,\mu) = e^{-iagA^{b}_{\mu}(x)t^{b}}$
- Quark fields $\psi(x)$ on sites with color, flavor, Dirac indices.
  Fermion discretisation - Wilson, Staggered, Overlap.
- Derivatives $\rightarrow$ finite differences:
  $\nabla^{\text{fwd}}_{\mu}\psi(x) = \frac{1}{a}\left[U_{\mu}(x)\psi(x+a\hat{\mu}) - \psi(x)\right]$



Solve the QCD path integral on a finite lattice with spacing $a \neq 0$ estimated stochastically by Monte Carlo. Can only be done effectively in a Euclidean space-time metric (no useful importance sampling weight for the theory in Minkowski space).

Observables determined from (Euclidean) path integrals of the QCD action

$$\langle \mathcal{O} \rangle = 1/Z \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi\ \mathcal{O}[U,\bar{\psi},\psi]e^{-S[U,\bar{\psi},\psi]}$$

Correlators in Lattice Euclidean Field Theory: I

- Physical observables $\mathcal{O}$ are determined from

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\Psi \mathcal{D}\bar{\Psi} \mathcal{O} e^{-S_{QCD}}$$

- Analytically integrate Grassman fields $(\Psi, \bar{\Psi})$ $\langle \mathcal{O} \rangle \overset{N_f=2}{=} \frac{1}{Z} \int \mathcal{D}U \det M^2 \mathcal{O} e^{-S_G}$

  Calculated by importance sampling of gauge fields and averaging over ensembles.
- Simulate N_{cfg} samples of the field configuration, then

$$\langle \mathcal{O} \rangle = \lim_{N_{cfg} \to \infty} \frac{1}{N_{cfg}} \sum_{i=1}^{N_{cfg}} \mathcal{O}_i[U_i]$$

- Correlation functions have improvable(!) statistically uncertainty $\sim 1/\sqrt{N_{cfg}}$.
- Calculating $\det M$ for M a large, sparse matrix with small eigenvalues takes $> 80\%$ of compute cycles in configuration generation. $\det M = 1$ is the quenched approximation.
- $\langle \mathcal{O} \rangle$ brings M^{-1} via contractions of quark fields. Second computational overhead.
- Fermions in lagrangian: sea quarks → fermion determinant. Fermions in measurement: valence quarks → propagators

A recipe for (meson) spectroscopy

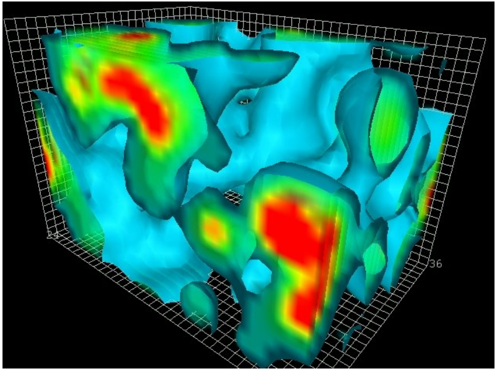
- Construct a basis of local and non-local operators $\bar{\Psi}(x)\Gamma D_i D_j \ldots \Psi(x)$
- Build a correlation matrix of two-point functions

$$C_{ij} = \langle 0|\mathcal{O}_i \mathcal{O}_j^\dagger|0\rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E_n} e^{-E_n t}$$

- Ground state mass from fits to $e^{-E_n t}$
- Beyond ground state: Solve generalised eigenvalue problem $C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$

- eigenvalues: $\lambda^{(n)}(t) \sim e^{-E_n t}\left[1 + O(e^{-\Delta E t})\right]$ - principal correlator
- eigenvectors: related to overlaps $Z_i^{(n)} = \sqrt{2E_n} e^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$

Typical size of a lattice calculation



[from D.Leinweber]

There are 2 compute intensive steps:

1. Generating Configurations - snapshots of the QCD vacuum

Volume: $32^3 \times 256$ (sites) $U_\mu(x)$ defined by $4 \times 8 \times 32^3 \times 256$ real numbers

2. Quark Propagation

Volume: $32^3 \times 256$ (sites) $\rightarrow M$ is a 100 million x 100 million sparse matrix with complex entries.

Solving QCD requires supercomputing resources worldwide.

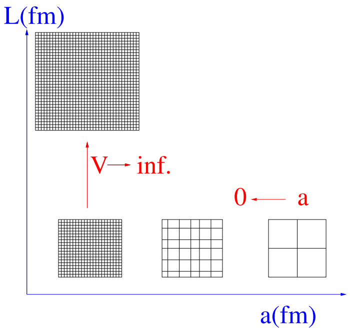
Lattice Calculations

Compromises and the Consequences

not an exhaustive list

1. Working in a finite box at finite grid spacing

- Identify a "scaling window" where physics doesn't change/changes weakly with a or V. Recover continuum **QCD** by extrapolation.
- Lattice spacing small enough to resolve structures induced by strong dynamics
- Volume large enough to contain lightest particle in spectrum: $m_\pi L \geq 2\pi$

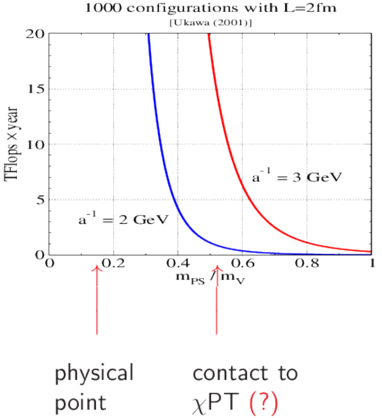


A costly procedure but a regular feature in lattice calculations now

2. Simulating at physical quark masses: light quarks

- Light quarks in gauge generation through fermion determinant M.
- Computational cost grows rapidly with decreasing quark mass $\rightarrow m_q = m_{u,d}$ costly.
- Many improvements over the years for all fermion discretisations
- The wall has come down - Physical point can be reached!
- Still costly and intricate for resonance physics.

2001 - a Berlin wall



2. Simulating at physical quark masses: heavy quarks

- Discretisation errors grow as $\mathcal{O}(am_q)$: large for reasonable a and heavy quarks
- Bottom quarks treated with Effective Field Theories - NRQCD, Fermilab etc
  - Continuum limits and EFTs can be tricky - not always possible e.g. with NRQCD
  - Controlling systematics important for precision CKM physics
- Charm quarks can be handled relativistically
  - Anisotropic lattices useful here: $a_s \neq a_t$ and $a_t m_c < 1$.

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  - Controlling systematics important for precision CKM physics
- Charm quarks can be handled relativistically
  - Anisotropic lattices useful here: $a_s \neq a_t$ and $a_t m_c < 1$.

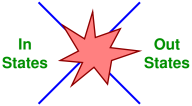
Better algorithms for physical light quarks and/or chiral extrapolation. Relativistic m_b is in reach.

Turn a weakness into a strength by using lattice simulations to study *quark mass dependence*!

3. Breaking symmetry

- Almost all symmetries of **QCD** are preserved. But Lorentz symmetry broken at $a \neq 0$ so $SO(4)$ rotation group broken to discrete rotation group of a hypercube.
- Angular momentum and parity J^P correspond to irreducible representations of the rotation group $O(3)$.
- A spatially isotropic lattice breaks $O(3) \to O_h$, the cubic point group.
- Eigenstates of the lattice $\mathcal{H}$ transform under irreps of O_h so states are classified by these irreps and not by J^P.
- Classify states by irreps of O_h and relate by subduction to J values of $O(3)$.
- 5 irreps of $O(3)$ and an infinite number for J^P so values are distributed across lattice irreps.
- Lots of degeneracies in subduction for $J \geq 2$ and physical near-degeneracies. Complicates spin identification.

4. Working in Euclidean time



- Scattering matrix elements not directly accessible from Euclidean QFT [*Maiani-Testa theorem*].
- Scattering matrix elements: asymptotic $|\mathrm{in}\rangle$, $|\mathrm{out}\rangle$ states: $\langle\mathrm{out}|e^{i\hat{H}t}|\mathrm{in}\rangle \to \langle\mathrm{out}|e^{-\hat{H}t}|\mathrm{in}\rangle$.
- Euclidean metric: project onto ground state.

- **Benefit:** can isolate lightest states in the spectrum (as we will see!). But to access radial and orbital excitations need additional ideas.
- **Problem:** direct information on scattering is lost and must be inferred indirectly.

Scattering: Lüscher method and generalisations give indirect access [later].
Excited states: use a variational method [C.Michael and I. Teasdale NPB215 (1983) 433, M. Lüscher and U. Wolff NPB339 (1990) 222]

5. Setting the scale

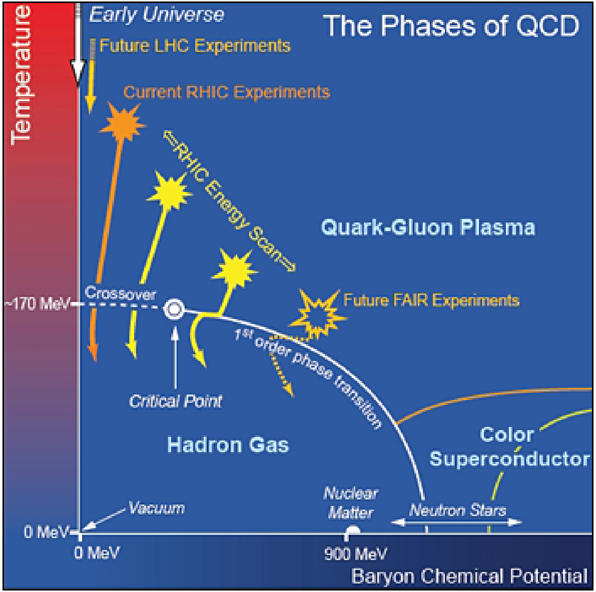
Lattice quantities are computed in lattice units e.g. am_N. and converted to physical units to compare to experiment/make predictions e.g. masses and form factors.

Choose an observable $\mathcal{O}$ that is relatively easy to calculate and insensitive to e.g. up and down quark masses (which may not be correct in the simulation) and match to its experimental value to determine a. This quantity is no longer a prediction!

Many reasonable choices and discretisation errors mean there is some uncertainty from this procedure.

Lattice QCD: exploring the properties of quarks and gluons

The QCD phase diagram

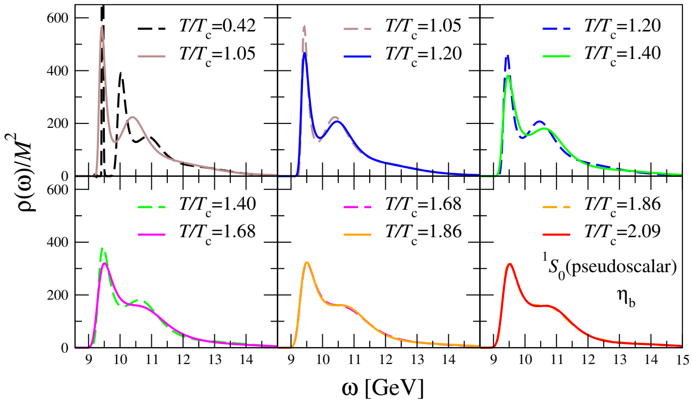


Understand from first principals

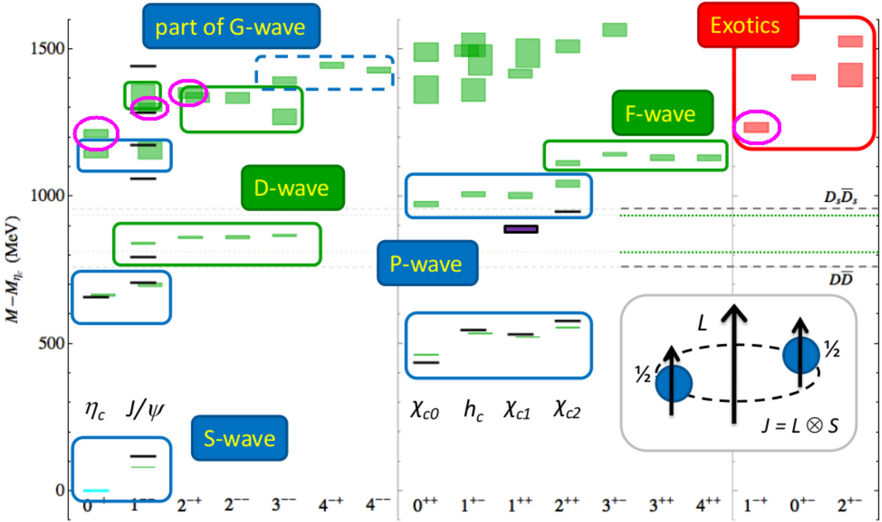
- the phase diagram
- fate of hadrons in medium
- nature and properties of the quark-gluon plasma
- ⋮

Melting and suppression of hadrons

Finite temperature
Probing the Quark-Gluon Plasma



Recent(ish) results: Lattice charmonium spectroscopy



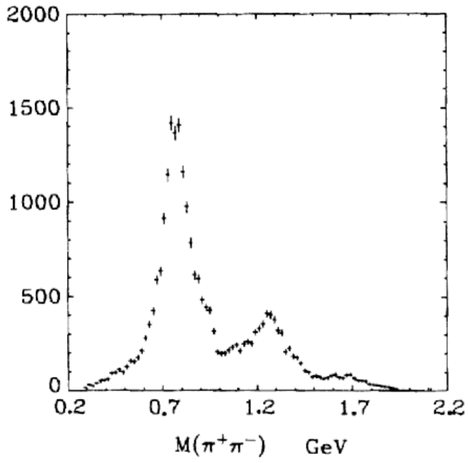
from the Hadron Spectrum Collaboration, 2012

Resonances and scattering states

- We have assumed that all states in the spectrum are stable
- Many (the majority) are not.
- A resonance is a state that forms e.g. when colliding two particles and then decays quickly to scattering states.
- These states respect conservation laws e.g. if isospin of the colliding particles is 3/2, resonance must have isospin 3/2 (Δ resonance)
- Usually indicated by a sharp peak in a cross section as a function of c.o.m. energy of the collision.

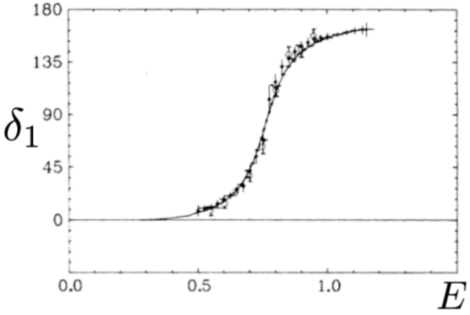
How can lattice QCD identify resonances and scattering states?

Excited hadrons are resonances

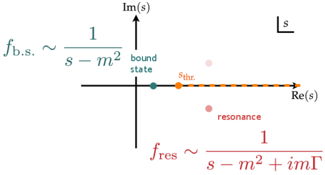


from Protopopescu et al (1972)

Resonant phase shift



Resonances are pole singularites in complex $s = E^2$



Brief overview of Lüscher's formalism

Finite Volume

Extract a discrete tower of energy levels

$$E_n(L, \vec{P})$$

depends on the volume (L) and total momentum

Infinite Volume

Decompose scattering amplitude in partial waves.

One real observable

$$\delta_l(E^*)$$

in each partial wave (l) and CM energy.

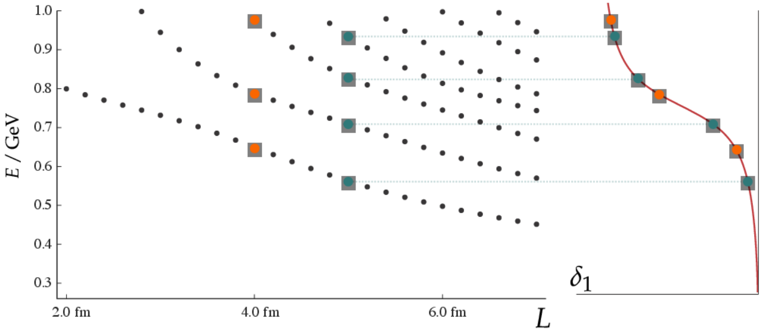
$$\det\left[\cot\delta(E_n^*) + \cot\phi(E_n, \vec{P}, L)\right] = 0$$

with $\cot\phi$ a known function (containing a generalised zeta function).

$\pi\pi$ in P-wave: $I^G J^{PC} = 1^+(1^{--})$

A relatively straightforward example

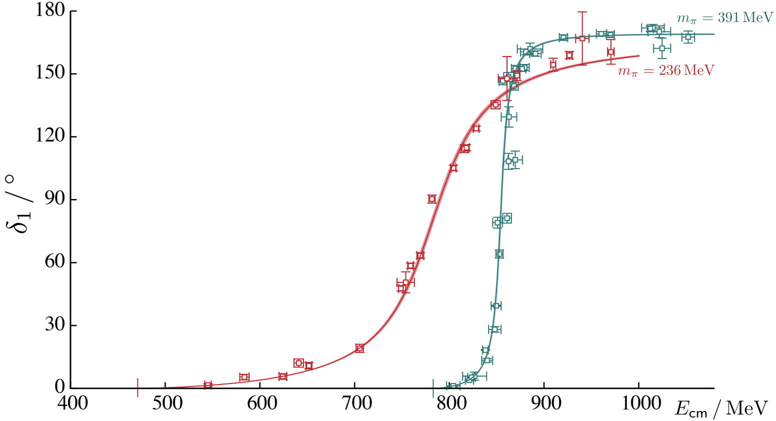
- consider the ρ on a lattices of different volumes
- at each volume extract the spectrum and use Lüscher formalism to deduce phase shift



- the more distinct spectrum points the better the phase shift picture

$\pi\pi$ in P-wave, I=1

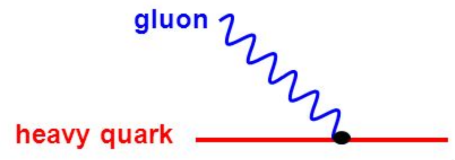
coupled $\pi\pi$, $K\bar{K}$ scattering in P-wave, PRD 92 (2015) 094592



- Includes **coupled channel** $\pi\pi$, $K\bar{K}$ at $m_\pi = 236$MeV.
- $m_R = 790(2)$ MeV; $g_R = 5.688(70)(26)$
- Reducing m_π moves mass and width in the right direction.

One last symmetry: Heavy quark symmetry

A symmetry of QCD at $m_q \to \infty$ limit



$$P_{\text{light}} \sim \Lambda_{\text{QCD}} \;;\; P_{\text{heavy}} \sim m_Q \cdot V$$

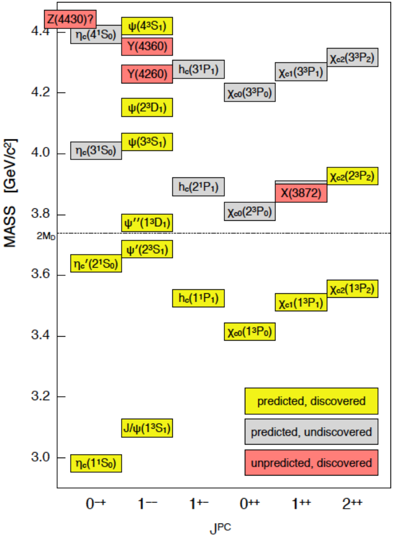
($V^\mu = (1, 0, 0, 0)$ in rest frame.

The velocity of a heavy quark is not changed by the QCD interaction: $\delta V \sim \frac{\Lambda_{\text{QCD}}}{m_Q} \to 0$

- Heavy quark number conservation
  No pair production of heavy quarks by QCD interaction
- SU(2) spin symmetry
  QCD interactions cannot flip the spin of heavy quarks

Can define an effective theory for heavy quarks and understand b and c systems together

A charming revolution!



- Charmonium the posterchild for quark models until 2003
- These days - many questions unanswered! The strong exotic matter has been around for 15 years ... and we still can't explain it ..

Heavy Quark Symmetries might yet help us.

QCD still challenging theorists!

thanks for listening!