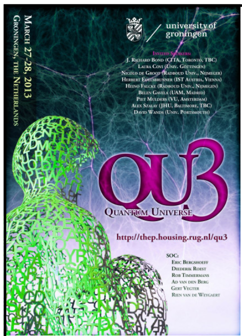

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Quantum Chromodynamics at Work

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European Research Council
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ABSTRACT

Quantum Chromodynamics at Work

Piet Mulders (Nikhef/VU University Amsterdam)

The matter in our world and the visible matter in the universe is made up of atoms consisting of electrons and nuclei built from protons and neutrons. At and above the atomic scale the interactions are primarily gravity and the electroweak forces. The latter include the interactions between charges and photons, which are beautifully described in quantum electrodynamics (QED). The atomic world also beautifully illustrates how charged particles including their spins provide a basis for enormous complexity.

In the subatomic world, the theoretical framework is Quantum Chromodynamics (QCD) that describes the strong interactions between color charges, carried by quarks and the force mediators themselves, confining them inside the nucleons, protons and neutrons. The framework of QCD certainly rivals in beauty with QED and is in fact much richer being a non-abelian gauge theory based on the symmetry group of the three color charges.

In the description of high energy scattering processes, the most important input used to encode the structure of hadrons is a set of Parton Distribution Functions (PDFs) representing probabilities for finding specific quarks and gluons (partons) carrying fractions x of the parent hadron's momentum (soft collinear part). The different functions in this set describe in essence spin-spin transfer probabilities. The interactions of the partons (hard part) are calculated using perturbation theory within the quantum field theoretical framework known as the Standard Model of particle physics.

One can go beyond this collinear approach. In that case one has to include the transverse momentum of the partons. It leads to a novel set of Transverse Momentum Dependent (TMD) distribution functions. Such functions take into account various spin-orbit correlations that may exist in a hadron, including socalled time-reversal odd functions describing spin – momentum transfer between hadrons and partons. Is this transverse momentum dependence a useful addition? Can it be measured? Can the formalism be set up and used in the same successful way as collinear PDFs, which are related to expectation values of field operators using the Operator Product Expansion in QCD? The answer is yes, but … . But after accounting for the complications, quark and gluon TMDs offer new insight into spin and orbital substructure of hadrons while they also may provide new tools to explore physics beyond the Standard Model, which is one of the ways to study the basic structure of our universe.

Content

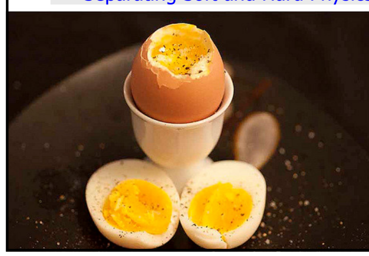
- High energies: soft x hard
  - Soft = hadron info (probabilities), hard = partonic cross section
  - Probabilities include spin-spin correlations
- Are TMD PDFs relevant and can they be measured?
  - Yes, there are besides spin-spin and also spin-orbit correlations
  - Yes, they can be measured (DY, SIDIS, …)
- But there are complications!
  - Gauge links, universality, factorization
- But also a theoretical framework to help out: QCD
  - Extension of OPE resummed into PDFs to TMDs (definite rank)
  - Distribution and fragmentation functions (time reversal)
- The reward
  - Novel hadronic info on spin and orbital structure
  - Possible use of proton as tool (playing with partons)

TMD = Transverse Momentum Dependent

PDF = Parton Distribution Function

QCD yes, but nothing on LO, NLO, ….

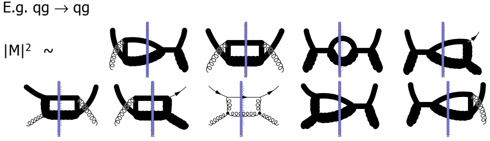
Separating Soft and Hard Physics at high energies



QCD & Standard Model

- QCD framework (including electroweak theory) provides the machinery to calculate cross sections, e.g. $\gamma^*q \to q$, $q\bar{q} \to \gamma^*$, $\gamma^* \to q\bar{q}$, $qq \to qq$, $qg \to qg$, etc.
- E.g. $qg \to qg$

$|M|^2 \sim$



- Calculations work for plane waves

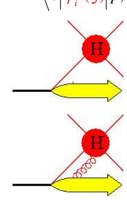
$$\langle 0|\psi_i^{(s)}(\xi)|p,s\rangle = u_i(p,s)\,e^{-ip\cdot\xi} \qquad \langle 0|A_\mu^{(\lambda)}(\xi)|p,s\rangle = \varepsilon_\mu(p,\lambda)\,e^{-ip\cdot\xi}$$

$$u(p,s)\bar{u}(p,s) = \tfrac{1}{2}(\not{p}+m)(1+\gamma_5\not{s}) \qquad \varepsilon^\mu(p,\lambda)\varepsilon^{\nu*}(p,\lambda) = -g_T^{\mu\nu}+\ldots$$

Hadron correlators

- Hadronic correlators establish the diagrammatic link between hadrons and partonic hard scattering amplitude
- Quark, quark + gluon, gluon, …

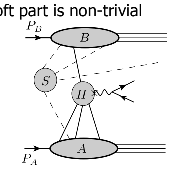
$$\langle 0|\psi_i(\xi)|p,s\rangle = u_i(p,s)\,e^{-ip\cdot\xi}$$



$$\langle X|\psi_i(\xi)|P\rangle\,e^{+ip\cdot\xi}$$

$$\langle X|\psi_i(\xi)A^\mu(\eta)|P\rangle\,e^{+i(p-p_1)\cdot\xi+ip_1\cdot\eta}$$

- Disentangling a hard process into parts involving hadrons, hard scattering amplitude and soft part is non-trivial



J.C. Collins, Foundations of Perturbative QCD, Cambridge Univ. Press 2011

Hadron correlators

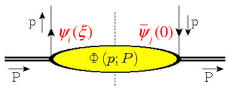
- At high energies no interference and squared amplitudes can be rewritten as correlators of forward matrix elements of parton fields
- Math: $u_i(p,s)\bar{u}_j(p,s) \Rightarrow \sum_X \langle P|\bar{\psi}_j(0)|X\rangle\langle X|\psi_i(0)|P\rangle \delta(p-P+P_X)$

$$= \sum_X \int \frac{d\xi}{2\pi} \langle P|\bar{\psi}_j(0)|X\rangle\langle X|\psi_i(0)|P\rangle e^{i(p-P+P_X).\xi}$$

$$= \sum_X \int \frac{d\xi}{2\pi} \langle P|\bar{\psi}_j(0)|X\rangle\langle X|\psi_i(\xi)|P\rangle e^{ip.\xi}$$

$$= \int \frac{d\xi}{2\pi} e^{ip.\xi} \langle P|\bar{\psi}_j(0)\,\psi_i(\xi)|P\rangle$$

- Picture:



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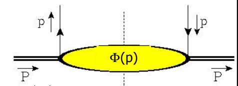
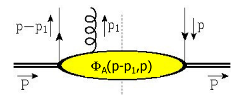
Soft part: hadron correlators

- Forward matrix elements of parton fields describe distribution (and fragmentation) parts



$$\Phi_{ij}(p;P) = \Phi_{ij}(p\,|\,p) = \int \frac{d^4\xi}{(2\pi)^4} e^{ip.\xi} \langle P|\bar{\psi}_j(0)\,\psi_i(\xi)|P\rangle$$

- Also needed are multi-parton correlators (time-ordering?)



$$\Phi^{\alpha}_{A\,ij}(p-p_1,p_1\,|\,p) = \int \frac{d^4\xi\, d^4\eta}{(2\pi)^8} e^{i(p-p_1).\xi + ip_1.\eta} \langle P|\bar{\psi}_j(0) A^{\alpha}(\eta)\,\psi_i(\xi)|P\rangle$$

- These correlators usually just will be parametrized in terms of PDFs (nonperturbative physics)

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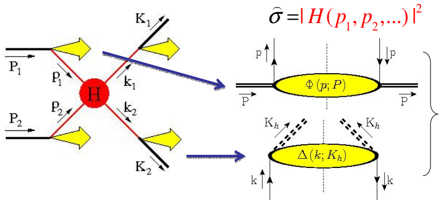
PDFs and PFFs

Basic idea of PDFs and PFFs (also for TMDs) is to obtain a full factorized description of high energy scattering processes

$$\hat{\sigma} = |\,H(p_1,p_2,\ldots)\,|^2$$

calculable



defined (!) & portable

$$\sigma(P_1,P_2,\ldots) = \iiint \ldots dp_1 \ldots \Phi_a(p_1,P_1;\mu) \otimes \Phi_b(p_2,P_2;\mu)$$

$$\otimes\, \hat{\sigma}_{ab,c\ldots}(p_1,p_2,\ldots;\mu) \otimes \Delta_c(k_1,K_1;\mu)\ldots.$$

Give a meaning to integration variables!

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Role of the hard scale

- In high-energy processes other momenta are available, such that P.P′ ~ s with a hard scale s >> M²
- Employ light-like vectors P and n, such that P.n = 1 (e.g. n = P′/P.P′) to make a Sudakov expansion of parton momentum (write s = Q²)

$$p = x P^{\mu} + p_T^{\mu} + \sigma n^{\mu} \qquad x = p^+ = p.n \quad (0 \le x \le 1)$$

~ Q   ~ M   ~ M²/Q

$$\sigma = p^- = p.P - xM^2 \sim O(M^2)$$

- Enables importance sampling (twist analysis) for integrated correlators,

$$\Phi(p) = \Phi(x, p_T, p.P) \;\Rightarrow\; \Phi(x, p_T) \;\Rightarrow\; \Phi(x) \;\Rightarrow\; \Phi$$

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(Un)integrated correlators

$$\Phi(x, p_T, p.P) = \int \frac{d^4\xi}{(2\pi)^4} e^{ip.\xi} \langle P|\bar{\psi}(0)\,\psi(\xi)|P\rangle$$

4 components

4 integrations

$$\int dp\; \Phi(p) = \int dp \int \frac{d\xi}{2\pi} e^{ip.\xi} \ldots. = \int \frac{d\xi}{2\pi} \delta(\xi) \ldots.$$

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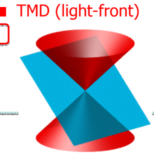
(Un)integrated correlators

$$\Phi(x, p_T, p.P) = \int \frac{d^4\xi}{(2\pi)^4} e^{ip.\xi} \langle P|\bar{\psi}(0)\,\psi(\xi)|P\rangle$$

- unintegrated

$$\Phi(x, p_T; n) = \int \frac{d(\xi.P)\, d^2\xi_T}{(2\pi)^3} e^{ip.\xi} \langle P|\bar{\psi}(0)\,\psi(\xi)|P\rangle\Big|_{\xi.n = \xi^+ = 0}$$

- TMD (light-front)
  - σ = p⁻ integration makes time-ordering automatic. The soft part is simply sliced at the light-front



$$\Phi(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{ip.\xi} \langle P|\bar{\psi}(0)\,\psi(\xi)|P\rangle\Big|_{\xi.n = \xi_T = 0} \text{ or } \Big|_{\xi^2 = 0}$$

- collinear (light-cone)
  - Is already equivalent to a point-like interaction

$$\Phi = \langle P|\bar{\psi}(0)\,\psi(\xi)|P\rangle\Big|_{\xi = 0}$$

- local
  - Local operators with calculable anomalous dimension

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Twist analysis in PDF parametrization

- Dimensional analysis to determine which correlators are important
- Maximize contractions with n (in matrix elements)

$$\dim[\bar\psi(0)\,\not n\,\psi(\xi)] = 2$$
$$\dim[F^{n\alpha}(0)F^{n\beta}(\xi)] = 2$$
$$\dim[\bar\psi(0)\,\not n\,A_T^{\alpha}(\eta)\,\psi(\xi)] = 3$$

Higher dimension compensated by 1/Q in cross section

- … or maximize # of P's in parametrization of Φ

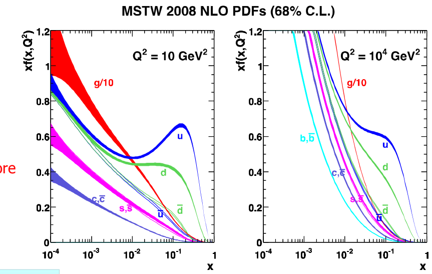
$$\Phi^q_{ij}(x) = f_1^q(x)(\not P)_{ij} \;\Leftrightarrow\; f_1^q(x) = \int \frac{d\lambda}{(2\pi)} e^{i x\lambda}\langle P|\bar\psi(0)\,\not n\,\psi(\lambda n)|P\rangle$$

- Compare
  - Free quarks $\quad u(p,s)\bar u(p,s) = \not p + m$
  - Parton model $\quad f(x)\,(\not p + m) = f(x)\,x\not P + m\,f(x)$
  - Correlators $\quad \Phi(p,P) = xf(x)\,\not P + Mx\,e(x) + ...$

Results of many years of work

PDF sets:
MSTW
CTEQ
GRV
…

But there is more
TMDs
GPDs
DPDs



This talk

relevance and measurability of TMDs

Access to transverse momenta: f(x) → f(x,p_T)

- SIDIS: $\quad$ γ*(q) + H(P) → h(K) + X
  Underlying hard process: $\quad$ γ*(q) + q(p) → q(k) $\quad p + q = k$
- Include transverse components in quark momenta leading to a non-collinearity

$$p \approx xP + p_T \qquad k \approx z^{-1}K + k_T$$
$$q_T = q + x\,P - z^{-1}K = k_T - p_T$$

- Go to sufficiently high energies to identify fractions x and z:

$$x = p.n / P.n = Q^2 / 2P.q = x_B$$
$$z = K.n / k.n = P.K / P.q = z_h$$

independent of n, up to 1/Q² corrections!

- This provides the accessible transverse momentum variable

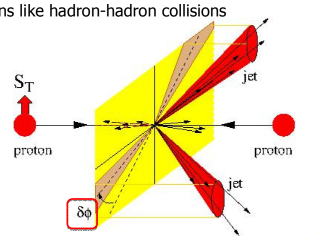
$$q_T = q + x_B P - z_h^{-1} K = k_T - p_T$$

- … which is of course also just the transverse momentum $K_{\perp(P,q)}$

Second scale!

Access to transverse momenta

- Also in more complex situations like hadron-hadron collisions



$$p_1 \approx x_1 P_1 + p_{1T}$$
$$p_2 \approx x_2 P_2 + p_{2T}$$

$$x_1 = p_1.n = \frac{p_1.P_2}{P_1.P_2} = \frac{(k_1+k_2).P_2}{P_1.P_2}$$

$$q_T = k_{jet,1} + k_{jet,2} - x_1 P_1 - x_2 P_2 = p_{1T} + p_{2T}$$

Second scale!

New information in TMD's: f(x,p_T)

- Quarks in polarized nucleon: $S = S_L\left(\frac{P}{M} + Mn\right) + S_T \qquad S_L^2 + S_T^2 = -1$

$$\Phi^q(p;P,S) \propto xf_1^q(x,p_T^2)\,\not P + S_L\,xg_{1L}^q(x,p_T^2)\,\not P\gamma_5 + xh_{1T}^q(x,p_T^2)\,\not S_T\not P\gamma_5 + \ldots$$

unpolarized quarks

T-polarized quarks in T-polarized N

chiral quarks in L-polarized N

compare
$$u(p,s)\bar u(p,s) = \tfrac{1}{2}(\not p + m)(1+\gamma_5\not s)$$

- … but also

$$\Phi^q(p;P,S) \propto \ldots + \frac{(p_T\cdot S_T)}{M}\,xg_{1T}^q(x,p_T^2)\,\not P\gamma_5 + \ldots$$

spin ←→ spin

chiral quarks in T-polarized N

New information in TMD's: $f(x,p_T)$

- … and T-odd functions

$$\Phi^q(p;P,S) \propto \ldots + i h_1^{\perp q}(x,p_T^2)\frac{p_T}{M}\not{P} + i\frac{(p_T \times S_T)}{M} x f_{1T}^{\perp q}(x,p_T^2)\not{P} + \ldots$$

T-polarized quarks in unpolarized N (Boer-Mulders)

unpolarized quarks in T-polarized N (Sivers)

spin ←→ orbit

compare $u(p,s)\bar{u}(p,s) = \frac{1}{2}(\not{p}+m)(1+\gamma_5\not{s})$

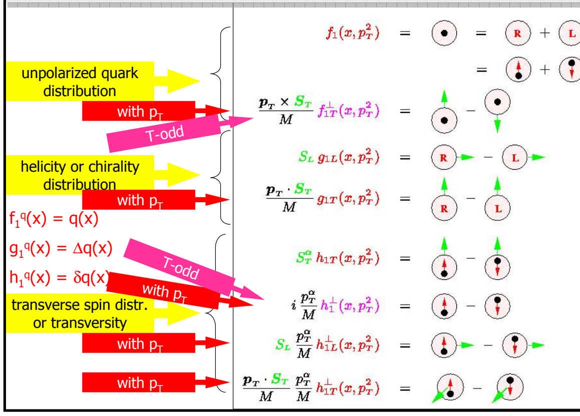
- Note that there are also parts that lack simple partonic interpretation

$$\Phi(p;P,S) \propto \ldots + M\, x e^q(x,p_T^2) + \ldots$$

Higher-twist

parton mass? But these are suppressed and linked to quark-gluon correlators via EQM

Fermionic structure of TMDs



Time reversal invariance

- TMD-correlators are not T-invariant (allowing specific spin-orbit correlations)
- QCD is T-invariant → Separate TMDs into T-even and T-odd
- T-odd observables ←→ T-odd TMDs
- Example of T-odd observable: single spin asymmetry
  E.g. left-right asymmetry in $p(P_1)\,p_\uparrow(P_2) \to \pi(K)X$
- Collinear hard T-odd contribution zero ($\sim\alpha_s^2$, $\alpha_s m_q$), p_T-contributions remain

$$p + p_\uparrow \to \pi + X \qquad \text{or} \qquad p + p_\uparrow \to \pi + X$$

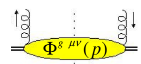
$$q + q \to q + q \qquad\qquad q + q_\uparrow \to q_\uparrow + q$$

$f_{1T}^{\perp p\to q}$ T-odd, $D_1^{q\to\pi}$ $h_1^{p\to q}$, $H_1^{\perp q\to\pi}$ T-odd

Qiu & Sterman, 1997; Boer & M; Anselmino et al.

New information in gluon TMD's: $f(x,p_T)$

- Also for gluons there are new features in TMD's



$$\Phi^{g\,\mu\nu}(p;P,S) \propto -g_T^{\mu\nu} x f_1^g(x,p_T^2) + i S_L \epsilon_T^{\mu\nu} x g_{1L}^g(x,p_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M^2} - g_T^{\mu\nu}\frac{p_T^2}{2M^2}\right) x h_1^{\perp g}(x,p_T^2) + \ldots$$

circularly polarized gluons in L-pol. N

spin ←→ spin

unpolarized gluons in unpol. N quarks

linearly polarized gluons in unpol. N (Gluon Boer-Mulders)

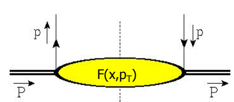
spin ←→ orbit

compare $\varepsilon^\mu(p,\lambda)\varepsilon^{\nu *}(p,\lambda) = -g_T^{\mu\nu} + \ldots$

Complications for TMDs

Large p_T

- p_T-dependence of TMDs



$$\int^\mu d^2p_T\, \Phi(x,p_T) = \Phi(x;\mu^2)$$

Fictitious measurement

Large μ^2 dependence governed by anomalous dim (i.e. splitting functions)

- $\Phi(x,p_T) \underset{p_T^2 > \mu^2}{\longrightarrow} \frac{1}{\pi p_T^2}\frac{\alpha_s(p_T^2)}{2\pi}\int_x^1 \frac{dy}{y} P\left(\frac{x}{y}\right)\Phi(y;p_T^2)$
- Consistent matching to collinear situation: CSS formalism
- Extended (Collins, Rogers, Aybat,…) and used (Boglione, …)

JC Collins, DE Soper and GF Sterman, NP B 250 (1985) 199
A Bacchetta, D Boer, M Diehl, PJM, JHEP 0808 (2008) 023

Color gauge invariance

- Gauge invariance in a non-local situation requires a gauge link U(0,ξ)

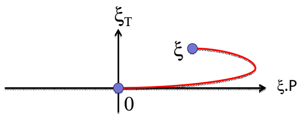
$$\bar\psi(0)\psi(\xi)=\sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\,\bar\psi(0)\partial_{\mu_1}\ldots\partial_{\mu_N}\psi(0)$$

$$U(0,\xi)=\mathscr{P}\exp\left(-ig\int_0^\xi ds^\mu A_\mu\right)$$

$$\bar\psi(0)U(0,\xi)\,\psi(\xi)=\sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\,\bar\psi(0)D_{\mu_1}\ldots D_{\mu_N}\psi(0)$$

- Introduces path dependence for $\Phi(x,p_T)$

$$\Phi^{[U]}(x,p_T)\ \Rightarrow\ \Phi(x)$$

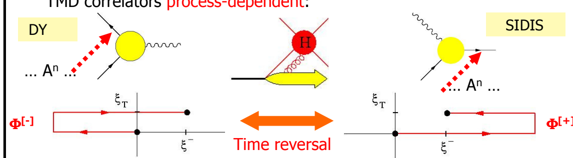


Which gauge links?

$$\Phi_{ij}^{q[C]}(x,p_T;n)=\int\frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}e^{ip.\xi}\langle P|\bar\psi_j(0)U_{[0,\xi]}^{[C]}\psi_i(\xi)|P\rangle_{\xi.n=0}$$ TMD

$$\Phi_{ij}^{q}(x;n)=\int\frac{d(\xi.P)}{(2\pi)}e^{ip.\xi}\langle P|\bar\psi_j(0)U_{[0,\xi]}^{[n]}\psi_i(\xi)|P\rangle_{\xi.n=\xi_T=0}$$ collinear

- Gauge links come from dimension zero collinear A.n gluons, but is for TMD correlators process-dependent:



AV Belitsky, X Ji and F Yuan, NP B 656 (2003) 165
D Boer, PJM and F Pijlman, NP B 667 (2003) 201

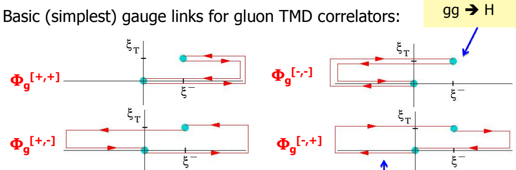
Which gauge links?

$$\Phi_g^{\alpha\beta[C,C']}(x,p_T;n)=\int\frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}e^{ip.\xi}\langle P|U_{[\xi,0]}^{[C]}F^{n\alpha}(0)U_{[0,\xi]}^{[C']}F^{n\beta}(\xi)|P\rangle_{\xi.n=0}$$

- The TMD gluon correlators contain two links, which can have different paths. Note that standard field displacement involves C = C'

$$F^{\alpha\beta}(\xi)\to U_{[\eta,\xi]}^{[C]}F^{\alpha\beta}(\xi)U_{[\xi,\eta]}^{[C]}$$

- Basic (simplest) gauge links for gluon TMD correlators:



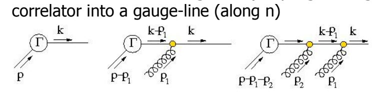
C Bomhof, PJM, F Pijlman; EPJ C 47 (2006) 147
F Dominguez, B-W Xiao, F Yuan, PRL 106 (2011) 022301

Some details on gauge links

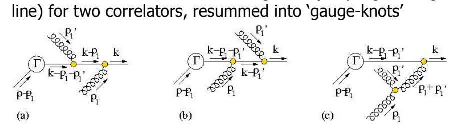
- Proper gluon fields (F rather than A and boundary terms)

$$A^\mu(p_1)=n.A(p_1)\frac{P^\mu}{n.P}+iA_T^\mu(p_1)+\ldots\ =\frac{1}{p_1.n}\left[n.A(p_1)\,p_1^\mu+iG_T^{n\mu}(p_1)+\ldots\right]$$

- Resummation of soft n.A gluons (coupling to outgoing color-line) for one correlator into a gauge-line (along n)

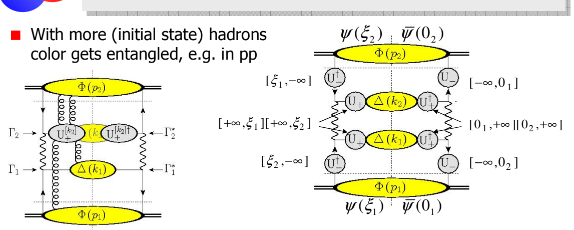


- Lowest order contributions of soft gluons (coupling to outgoing color-line) for two correlators, resummed into 'gauge-knots'



Which gauge links?

- With more (initial state) hadrons color gets entangled, e.g. in pp



- Outgoing color contributes future pointing gauge link to $\Phi(p_2)$ and future pointing part of a loop in the gauge link for $\Phi(p_1)$
- Can be color-detangled if only p_T of one correlator is relevant (using polarization, …) but must include Wilson loops in final U

T.C. Rogers, PJM, PR D81 (2010) 094006

MGA Buffing, PJM, JHEP 07 (2011) 065

Summarizing: color gauge invariant correlators

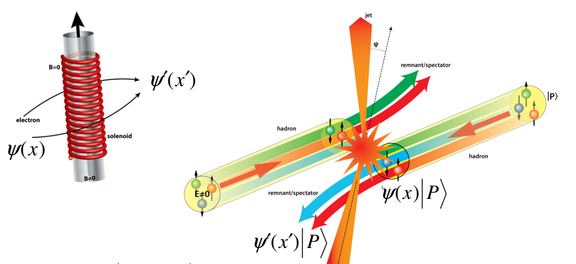
- So it looks that at best we have well-defined matrix elements for TMDs but including multiple possiblities for gauge links and each process or even each diagram its own gauge link (depending on flow of color)
- Leading quark TMDs:

$$\Phi^{[U]}(x,p_T;n)=\left\{f_1^{[U]}(x,p_T^2)-f_{1T}^{\perp[U]}(x,p_T^2)\frac{\epsilon_T^{p_TS_T}}{M}+g_{1s}^{[U]}(x,p_T)\gamma_5+h_{1T}^{[U]}(x,p_T^2)\gamma_5\,\not S_T+h_{1s}^{\perp[U]}(x,p_T)\frac{\gamma_5\not p_T}{M}+ih_1^{\perp[U]}(x,p_T^2)\frac{\not p_T}{M}\right\}\frac{\not P}{2}$$

- Leading gluon TMDs:

$$2x\,\Gamma^{\mu\nu[U]}(x,p_T)=-g_T^{\mu\nu}f_1^{g[U]}(x,p_T^2)+g_T^{\mu\nu}\frac{\epsilon_T^{p_TS_T}}{M}f_{1T}^{\perp g[U]}(x,p_T^2)+i\epsilon_T^{\mu\nu}g_{1s}^{g[U]}(x,p_T)+\left(\frac{p_T^\mu p_T^\nu}{M^2}-g_T^{\mu\nu}\frac{p_T^2}{2M^2}\right)h_1^{\perp g[U]}(x,p_T^2)-\frac{\epsilon_T^{p_T\{\mu}p_T^{\nu\}}}{2M^2}h_{1s}^{\perp g[U]}(x,p_T)-\frac{\epsilon_T^{p_T\{\mu}S_T^{\nu\}}+\epsilon_T^{S_T\{\mu}p_T^{\nu\}}}{4M}h_{1T}^{g[U]}(x,p_T^2)$$

Opportunities to see color-induced phases in QCD



$$\psi(\xi) = \boldsymbol{P}\exp\left(-ig\int_0^{\xi} ds^{\mu} A_{\mu}\right)\psi(0)$$

Figures by Kees Huyser

Next step

Basic strategy: Taylor expand

- Taylor expansion for functions around zero

$$f(z) = \sum_n \frac{f^n}{n!} z^n \qquad f^n = \left.\frac{\partial^n f}{\partial z^n}\right|_{z=0}$$

- Mellin transform for functions on [-1,1] interval

$$f(x) = -\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dn\, x^{-n} M_n \qquad M_n = \int_0^1 dx\, x^{n-1} f(x)$$

- functions in (transverse) plane

$$f(p_T) = \sum_n \sum_{\alpha_1\ldots\alpha_n} p_T^{\alpha_1}\ldots p_T^{\alpha_n} f_{\alpha_1\ldots\alpha_n} \qquad f_{\alpha_1\ldots\alpha_n} = \partial_{\alpha_1}\ldots\partial_{\alpha_n} f(p_T)\Big|_{p_T=0}$$

Operator structure in collinear case (reminder)

- Collinear functions and x-moments

$$\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P\right|\overline{\psi}(0)\,U_{[0,\xi]}^{[n]}\,\psi(\xi)\left|P\right\rangle_{\xi.n=\xi_T=0}$$

$$x^{N-1}\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P\right|\overline{\psi}(0)(\partial^n)^{N-1} U_{[0,\xi]}^{[n]}\,\psi(\xi)\left|P\right\rangle_{\xi.n=\xi_T=0}$$

x = p.n

$$= \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P\right|\overline{\psi}(0)\,U_{[0,\xi]}^{[n]}(D^n)^{N-1}\,\psi(\xi)\left|P\right\rangle_{\xi.n=\xi_T=0}$$

- Moments correspond to local matrix elements of operators that all have the same twist since dim(D^n) = 0

$$\Phi^{(N)} = \left\langle P\right|\overline{\psi}(0)(D^n)^{N-1}\,\psi(0)\left|P\right\rangle$$

- Moments are particularly useful because their anomalous dimensions can be rigorously calculated and these can be Mellin transformed into the splitting functions that govern the QCD evolution.

Operator structure in TMD case

- For TMD functions one can consider transverse moments

$$\Phi(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P\right|\overline{\psi}(0)\,U_{[0,\xi]}^{[\pm]}\,\psi(\xi)\left|P\right\rangle_{\xi.n=0}$$

$$p_T^{\alpha}\Phi^{[\pm]}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P\right|\overline{\psi}(0)U_{[0,\pm\infty]}D_T^{\alpha}U_{[\pm\infty,\xi]}\psi(\xi)\left|P\right\rangle_{\xi.n=0}$$

$$p_T^{\alpha_1}p_T^{\alpha_2}\Phi^{[\pm]}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P\right|\overline{\psi}(0)U_{[0,\pm\infty]}D_T^{\alpha_1}D_T^{\alpha_2}U_{[\pm\infty,\xi]}\psi(\xi)\left|P\right\rangle_{\xi.n=0}$$

- Upon integration, these do involve collinear twist-3 multi-parton correlators

MGA Buffing, A Mukherjee, PJM, PRD 86 (2012) 074030 , Arxiv: 1207.3221 [hep-ph]

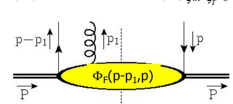
Operator structure in TMD case

- For first transverse moment one needs quark-gluon correlators

$$\Phi_D^{\alpha}(x-x_1,x_1\,|\,x) = \int \frac{d\xi.P\,d\eta.P}{(2\pi)^2} e^{i(p-p_1).\xi+ip_1.\eta} \left\langle P\right|\overline{\psi}(0)D_T^{\alpha}(\eta)\,\psi(\xi)\left|P\right\rangle_{\xi.n=\xi_T=0}$$

$$\Phi_F^{\alpha}(x-x_1,x_1\,|\,x) = \int \frac{d\xi.P\,d\eta.P}{(2\pi)^2} e^{i(p-p_1).\xi+ip_1.\eta} \left\langle P\right|\overline{\psi}(0)F^{n\alpha}(\eta)\,\psi(\xi)\left|P\right\rangle_{\xi.n=\xi_T=0}$$

- In principle multi-parton, but we need



$$\Phi_D^{\alpha}(x) = \int dx_1\,\Phi_D^{\alpha}(x-x_1,x_1\,|\,x)$$

$$\Phi_A^{\alpha}(x) = PV\int dx_1\,\frac{1}{x_1}\Phi_F^{n\alpha}(x-x_1,x_1\,|\,x)$$

$$\tilde{\Phi}_{\partial}^{\alpha}(x) = \Phi_D^{\alpha}(x) - \Phi_A^{\alpha}(x)$$ T-even (gauge-invariant derivative)

$$\Phi_G^{\alpha}(x) = \Phi_F^{n\alpha}(x,0\,|\,x)$$ T-odd (soft-gluon or gluonic pole)

Operator structure in TMD case

- Transverse moments can be expressed in these particular collinear multi-parton twist-3 correlators (which are not suppressed!)

$$\Phi_\partial^{\alpha[U]}(x) = \int d^2p_T\; p_T^\alpha \Phi^{[U]}(x,p_T;n) = \tilde{\Phi}_\partial^\alpha(x) + C_G^{[U]}\pi\Phi_G^\alpha(x)$$

T-even | T-odd

$$\Phi_{\partial\partial}^{\alpha\beta[U]}(x) = \tilde{\Phi}_{\partial\partial}^{\alpha\beta}(x) + C_{GG,c}^{[U]}\pi^2\Phi_{GG,c}^{\alpha\beta}(x) + C_G^{[U]}\pi\left(\tilde{\Phi}_{\partial G}^{\alpha\beta}(x) + \tilde{\Phi}_{G\partial}^{\alpha\beta}(x)\right)$$

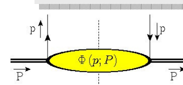
T-even | T-even | T-odd

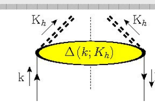
$Tr_c(GG\,\psi\bar{\psi})$ | $Tr_c(GG)\,Tr_c(\psi\bar{\psi})$

- $C_G^{[U]}$ calculable gluonic pole factors

U	$U^{[\circ]}$	$U^{[+]}U^{[\square]}$	$\frac{1}{N_c}\mathrm{Tr}_c(U^{[\square]})\,U^{[+]}$
$\Phi^{[U]}$	$\Phi^{[\circ]}$	$\Phi^{[+\square]}$	$\Phi^{[(\square)+]}$
$C_G^{[U]}$	$\circ 1$	3	1
$C_{GG,1}^{[U]}$	1	9	1
$C_{GG,2}^{[U]}$	0	0	4

Distributions versus fragmentation





- Operators:

$$\Phi^{[U]}(p\,|\,p) \sim \left\langle P\,|\,\bar{\psi}(0)U_{[0,\xi]}\psi(\xi)\,|\,P\right\rangle$$

$$\Phi_\partial^{\alpha[U]}(x) = \tilde{\Phi}_\partial^\alpha(x) + C_G^{[U]}\pi\Phi_G^\alpha(x)$$

T-even | T-odd (gluonic pole)

$$\Phi_G^\alpha(x) = \Phi_F^{n\alpha}(x,0\,|\,x) \neq 0$$

- Operators:

out state

$$\Delta(k\,|\,k) \sim \sum_X \left\langle 0\,|\,\psi(\xi)\,|\,K_h X\right\rangle\left\langle K_h X\,|\,\bar{\psi}(0)\,|\,0\right\rangle$$

$$\Delta_G^\alpha(x) = \Delta_F^{n\alpha}(\tfrac{1}{Z},0\,|\,\tfrac{1}{Z}) = 0$$

$$\Delta_\partial^{\alpha[U]}(x) = \tilde{\Delta}_\partial^\alpha(x)$$

T-even operator combination, but still T-odd functions!

Collins, Metz; Meissner, Metz; Gamberg, M, Mukherjee, PR D 83 (2011) 071503

The next step: (full) TMDs of definite rank

- Collecting right moments gives expansion into full TMDs of definite rank

$$\Phi^{[U]}(x,p_T) = \Phi(x,p_T^2) + C_G^{[U]}\pi p_{Ti}\tilde{\Phi}_G^i(x,p_T^2) + C_{GG,c}^{[U]}\pi^2 p_{Tij}\tilde{\Phi}_{GG,c}^{ij}(x,p_T^2) + \ldots$$
$$+ p_{Ti}\tilde{\Phi}_\partial^i(x,p_T^2) + C_G^{[U]}\pi p_{Tij}\tilde{\Phi}_{\{\partial G\}}^{ij}(x,p_T^2) + \ldots$$
$$+ p_{Tij}\tilde{\Phi}_{\partial\partial}^{ij}(x,p_T^2) + \ldots$$

- Depending on spin of hadron, only a finite number needed
- Example: quarks in an unpolarized target needs only 2 functions

$$\tilde{\Phi}(x,p_T^2) = \left(f_1(x,p_T^2)\right)\frac{\not{P}}{2} \qquad \pi\tilde{\Phi}_G^\alpha(x,p_T^2) = \left(ih_1^\perp(x,p_T^2)\frac{\gamma_T^\alpha}{M}\right)\frac{\not{P}}{2}$$

T-even | T-odd

Summarizing quark TMDs up to spin 1/2 targets

GLUONIC POLE RANK			
0	1	2	3
$\Phi(x,p_T^2)$	$\pi C_G^{[U]}\,\Phi_G$	$\pi^2 C_{GG,c}^{[U]}\,\Phi_{GG,c}$	$\pi^3 C_{GGG,c}^{[U]}\,\Phi_{GGG,c}$
$\tilde{\Phi}_\partial$	$\pi C_G^{[U]}\,\tilde{\Phi}_{\{\partial G\}}$	$\pi^2 C_{GG,c}^{[U]}\,\tilde{\Phi}_{\{\partial GG\}c}$	...
$\tilde{\Phi}_{\partial\partial}$	$\pi C_G^{[U]}\,\tilde{\Phi}_{\{\partial\partial G\}}$	...	...
$\tilde{\Phi}_{\partial\partial\partial}$	...	...	...

PDFs FOR SPIN 0 HADRONS		
f_1	$h_1^\perp$	

\+

PDFs FOR SPIN 1/2 HADRONS		
$g_1,\ h_1$	$f_{1T}^\perp$	$h_{1T}^{\perp(B1)},\ h_{1T}^{\perp(B2)}$
$g_{1T},\ h_{1L}^\perp$		
$h_{1T}^{\perp(A)}$		

PFFs FOR SPIN 0 HADRONS		
D_1		
$H_1^\perp$		

\+

PFFs FOR SPIN 1/2 HADRONS		
$G_1,\ H_1$		
$G_{1T},\ H_{1L}^\perp,\ D_{1T}^\perp$		
$H_{1T}^\perp$		

MGA Buffing, A Mukherjee, PJM, PRD2012 , Arxiv: 1207.3221 [hep-ph]

Where do we stand with TMDs (schematic)

- Collinear high-energy processes

$$\sigma(x_1,x_2,z) = \Phi^i(x_1)\Phi^j(x_2)\,f_C\,\hat{\sigma}_{ij\to k\ldots}^C(x_1,x_2,z)\Delta^k(z)$$

collinear PDF for parton i | color factor like $1/N_c$ | Partonic x-section | collinear PFF for parton k

Where do we stand with TMDs (schematic)

- Collinear high-energy processes:

$$\sigma(x_1,x_2,z) = \Phi^i(x_1)\Phi^j(x_2)\,f_C\,\hat{\sigma}_{ij\to k\ldots}^C(x_1,x_2,z)\Delta^k(z)$$

- Azimuthal dependences:

$$\sigma(x_1,x_2,z,q_T) = f_C^{[U_1U_2]}\,\Phi^{i[U_1(C)]}(x_2,p_{2T})\otimes\Phi^{j[U_2(C)]}(x_1,p_{1T})\,\hat{\sigma}_{ij\to k\ldots}^{[C]}(x_1,x_2,z)\Delta^k(z,k_T)$$

gauge-link process-dependent color factors

Where do we stand with TMDs (schematic)

- Collinear high-energy processes:

$$\sigma(x_1,x_2,z)=\Phi^i(x_1)\Phi^j(x_2)\,f_C\,\hat{\sigma}^C_{ij\to k\ldots}(x_1,x_2,z)\Delta^k(z)$$

- Azimuthal dependences:

$$\sigma(x_1,x_2,z,q_T)=f_C^{[U_1U_2]}\,\Phi^{i[U_1(C)]}(x_2,p_{2T})\otimes\Phi^{j[U_2(C)]}(x_1,p_{1T})\,\hat{\sigma}^{[C]}_{ij\to k\ldots}(x_1,x_2,z)\Delta^k(z,k_T)$$

$$\Phi^{i[U]}(x,p_T)=\tilde{\Phi}^i(x,p_T^2)+p_T^\alpha\tilde{\Phi}_\partial^{i\alpha}(x,p_T^2)+C_G^{[U]}p_T^\alpha\tilde{\Phi}_G^{i\alpha}(x,p_T^2)+C_{GG}^{[U]}p_T^{\alpha\beta}\Phi_{GG}^{i\alpha\beta}(x,p_T^2)+\ldots$$

gauge-link dependent TMD PDF for parton i

rank-1 TMDs (T-even)

gluonic pole factor

rank-1 ETQS TMDs (T-odd)

rank-2 ETQS TMDs (T-even)

Where do we stand with TMDs (schematic)

- Collinear high-energy processes:

$$\sigma(x_1,x_2,z)=\Phi^i(x_1)\Phi^j(x_2)\,f_C\,\hat{\sigma}^C_{ij\to k\ldots}(x_1,x_2,z)\Delta^k(z)$$

- Azimuthal dependences:

$$\sigma(x_1,x_2,z,q_T)=f_C^{[U_1U_2]}\,\Phi^{i[U_1(C)]}(x_2,p_{2T})\otimes\Phi^{j[U_2(C)]}(x_1,p_{1T})\,\hat{\sigma}^{[C]}_{ij\to k\ldots}(x_1,x_2,z)\Delta^k(z,k_T)$$

$$\Phi^{i[U]}(x,p_T)=\tilde{\Phi}^i(x,p_T^2)+p_T^\alpha\tilde{\Phi}_\partial^{i\alpha}(x,p_T^2)+C_G^{[U]}p_T^\alpha\tilde{\Phi}_G^{i\alpha}(x,p_T^2)+C_{GG}^{[U]}p_T^{\alpha\beta}\Phi_{GG}^{i\alpha\beta}(x,p_T^2)+\ldots$$

$$\Delta^{k[U]}(x,k_T)=\tilde{\Delta}^k(z,k_T^2)+k_T^\alpha\tilde{\Delta}_\partial^{k\alpha}(x,k_T^2)+k_T^{\alpha\beta}\tilde{\Delta}_{\partial\partial}^{k\alpha\beta}(z,k_T^2)+\ldots$$

universal TMD PFF for parton i

rank-1 TMDs (T-even and odd)

rank-2 TMDs (T-even and odd)

Conclusion with (potential) rewards

- (Generalized) universality via operator product expansion extends the well-known collinear distributions (including polarization, 3 for quarks and 2 for gluons) with novel TMD functions of definite rank.
- The rank m is coupled to cos(mϕ) and sin(mϕ) azimuthal asymmetries. Highest rank is $2(S_{hadron}+s_{parton})$.
- TMDs encode aspects of hadronic structure, e.g. spin-orbit correlations, such as T-odd transversely polarized quarks or T-even longitudinally polarized gluons in an unpolarized hadron (thus opening possible use for precision probing at the LHC)
- The TMDs appear in cross sections with specific calculable coefficients that depend on the flow of color in the tree-level diagrams:

  gluon + gluon → colorless (distinguish CP+ from CP- Higgs)

  gluon + gluon → quark-antiquark pair.
- Factorization studies (scaling) for TMDs are still in an early phase.

Thank you