Topology in QCD and Axion Dark Matter.

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Topological Theta Term and Strong CP Problem

- Most general gauge invariant Lagrangian of QCD:

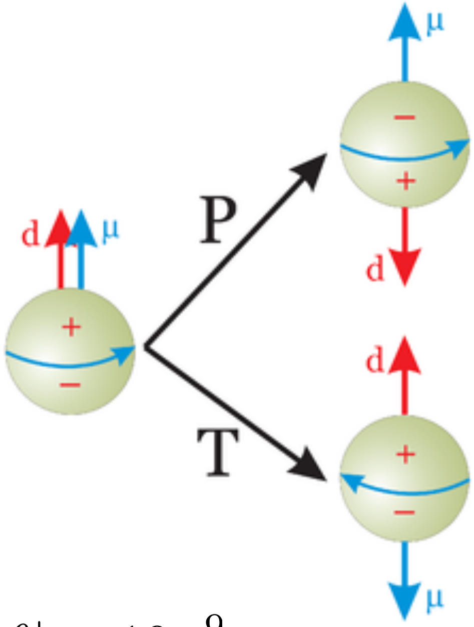
$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \overline{q}\,(i\gamma_\mu D^\mu - \mathcal{M}_q)\, q - \frac{\alpha_s}{8\pi}\,\theta\, G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

  - Parameters: strong coupling α_s, quark masses $\mathcal{M}_q = {\rm diag}(m_u, m_d, \ldots)$ and theta angle θ [Belavin et al. `75;´t Hooft 76;Callan et al. `76;Jackiw,Rebbi `76]

- Topological theta term $\propto G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} \propto \mathbf{E}^a \cdot \mathbf{B}^a$ violates P and T, and thus CP

- Most sensitive probe of P and T violation in flavor conserving interactions: electric dipole moment of neutron; experimentally

$$|d_n| < 2.9 \times 10^{-26}\ e\,{\rm cm}$$



- Strong CP problem:

$$d_n(\theta) \sim e\,\theta \frac{m_u m_d}{(m_u+m_d) m_n^2} \sim 6\times 10^{-17}\ \theta\ e\,{\rm cm} \Rightarrow |\theta| < 10^{-9}$$

Topological Theta Term and Strong CP Problem

- Theta dependence of vacuum energy density in QCD,

$$\epsilon_0(\theta) \equiv -\frac{1}{\mathcal{V}} \ln \left[\frac{Z(\theta)}{Z(0)} \right], \qquad -\pi \leq \theta \leq \pi$$

- Partition function in terms of Fourier series of Euclidean path integrals over gauge fields with fixed topological charge

$$Z(\theta) = \sum_{Q=-\infty}^{+\infty} \exp[i\theta Q] Z_Q, \qquad Q = \int d^4x \, \frac{\alpha_s}{8\pi} G^b_{\mu\nu} \tilde{G}^{b,\mu\nu} \equiv \int d^4x \, q(x)$$

$$Z_Q = \int_Q [dG][dq][d\bar{q}] \exp \left[-\int d^4x \left\{ \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + i\bar{q}\gamma_\mu D_\mu q - \bar{q}_R \mathcal{M} q_L - \bar{q}_L \mathcal{M}^\dagger q_R \right\} \right]$$

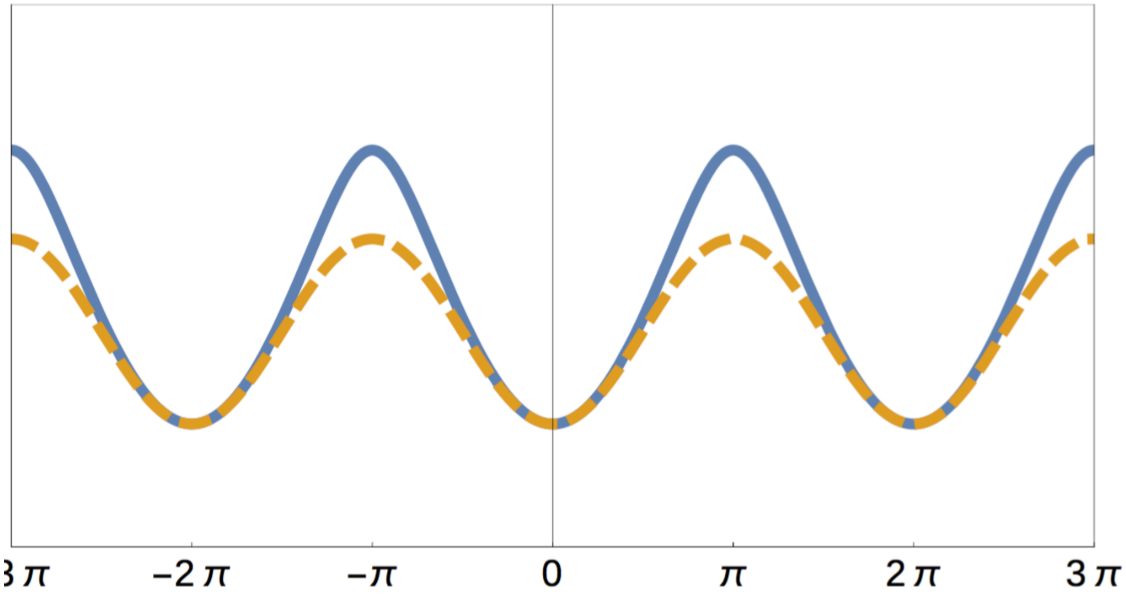
- QCD at large $\mathcal{V}$ and small m_q well described by effective field theory of Nambu-Goldstone bosons originating from spontaneous breaking of chiral symmetry of light quarks (ChPT)

- Theta dependence of vacuum energy can be inferred from ChPT by rotating theta into quark mass matrix by axial U(1) rotation

Topological Theta Term and Strong CP Problem

- In leading order SU(2) chiral perturbation theory: [Di Vecchia,Veneziano `80]

$$\epsilon_0^{(2)}(\theta) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{\theta}{2}\right)}$$



[Grilli di Cortona et al. `16]

- Minimum at vanishing theta parameter
- If theta were a dynamical field, its VEV would be zero

Axionic Solution of Strong CP Problem

- Peccei-Quinn (PQ) solution of strong CP problem: [Peccei,Quinn `77]
- Introduce field A(x) as dynamical theta parameter, respecting a non-linearly realized $U(1)_{\rm PQ}$ symmetry, i.e. a shift symmetry, $A(x)\to A(x)+{\rm const.}$ broken only by anomalous couplings to gauge fields,

$$\mathcal{L} = \frac{1}{2}\partial_\mu A \partial^\mu A - \frac{\alpha_s}{8\pi}\frac{A}{f_A} G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} - \frac{\alpha}{8\pi}\frac{E}{N}\frac{A}{f_A} F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{1}{2}\sum_f C_{Af}\frac{\partial_\mu A}{f_A}\overline{\psi}_f\gamma^\mu\gamma_5\psi_f$$

  - Can eliminate theta by shift $A(x)\to A(x)-\theta f_A$; QCD dynamics (see above) leads then to vanishing VEV, $<A>=0$, i.e. P, T, and CP conserved
  - Particle excitation of A: Nambu-Goldstone boson "axion" [Weinberg `78; Wilczek `78]
  - Potential and in particular mass can be inferred from $\epsilon_0(\theta)$:

$$V(A)\equiv\epsilon_0\left(\theta=\frac{A}{f_A}\right) \quad\Rightarrow\quad m_A^2\equiv\left.\frac{d^2V}{dA^2}\right|_{A=0} = \frac{m_\pi^2 f_\pi^2}{f_A^2}\frac{m_u m_d}{(m_u+m_d)^2}$$

  - Strength of its interactions with SM controlled by axion decay constant = PQ scale f_A
  - Must be large to avoid excessive energy losses of stars:

$$f_A \gtrsim 4\times 10^8\ {\rm GeV} \Rightarrow m_A \lesssim 14\,{\rm meV}$$

Axion Mass at Finite Temperature

> In cosmological applications, need axion potential at finite temperature,

$$V(A,T) \equiv -\frac{1}{\mathcal{V}} \ln \left[\frac{Z(\theta,T)}{Z(0,T)} \right] \Bigg|_{\theta=A/f_A}$$

> Axion mass determined by topological susceptibility (variance of topological charge distribution)

$$m_A^2(T) f_A^2 = \frac{\langle Q^2 \rangle_T \big|_{\theta=0}}{\mathcal{V}} \equiv \chi(T)$$

> At very large temperatures, above the quark hadron phase transition, expect that partition function can be described by a dilute gas of instantons and anti-instantons (minima of Euclidean action with $Q = \pm 1$)

$$A_\mu^{(I)}(x;\rho,U,x_0) = -\frac{\mathrm{i}}{g} \frac{\rho^2}{(x-x_0)^2} U \frac{\sigma_\mu \left(\overline{x} - \overline{x_0}\right) - \left(x_\mu - x_{0\mu}\right)}{(x-x_0)^2 + \rho^2} U^\dagger$$

[Belavin,Polyakov,Schwartz,Tyupkin `75]

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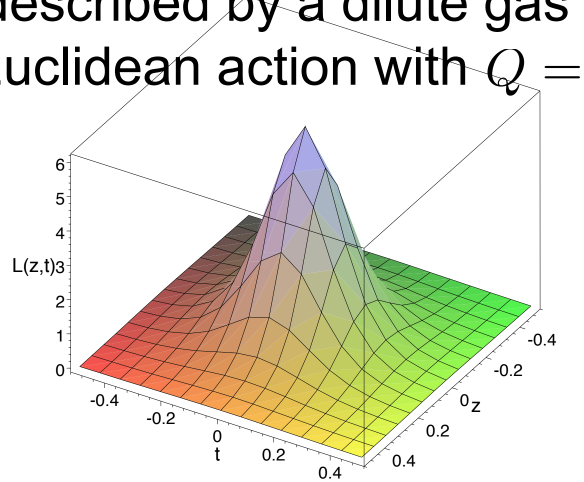
$$\mathcal{L}\left(A_\mu^{(I)}(x;\rho,U,0)\right) = \frac{12}{\pi\alpha_s}\cdot\frac{\rho^4}{(x^2+\rho^2)^4}$$

$$\Rightarrow S\left[A_\mu^{(I)}\right] = \frac{2\,\pi}{\alpha_s}$$



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$$Z(\theta,T) \simeq \sum_{n_I, n_{\bar{I}}} \frac{1}{n_I! n_{\bar{I}}!} Z_I^{n_I}(T) Z_{\bar{I}}^{n_{\bar{I}}}(T) \exp\left[i\theta (n_I - n_{\bar{I}}) \right]$$

$$m_A^2(T) f_A^2 = \chi(T) \simeq \frac{Z_I(T) + Z_{\bar{I}}(T)}{\mathcal{V}} = 2 \int_0^\infty d\rho\, D(\rho)\, G(\pi \rho T)$$

Axion Mass at Finite Temperature

- Zero temperature instanton size distribution: [´t Hooft `76]

$$D(\rho) = \frac{d}{\rho^5}\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^{2\,N_c} \exp\left(-\frac{2\pi}{\alpha_s(\mu_r)}\right)(\rho\,\mu_r)^{\beta_0}\prod_{i=1}^{n_f}(\rho\, m_i(\mu_r))$$

- At large temperature, color-electric Debye screening prohibits existence of large-scale coherent fields in plasma, leading to [Gross,Pisarski,Yaffe `76]

$$G(\pi\rho T) = \exp\left\{-\frac{6+n_f}{3}(\pi\rho T)^2 - 18\left[1-\frac{n_f}{9}\right] A(\pi\rho T)\right\}$$

$$A(\pi\rho T) = -\frac{1}{12}\ln\left[1+(\pi\rho T)^2/3\right] + \alpha\left[1+\gamma(\pi\rho T)^{-2/3}\right]^{-8}$$

- Cuts off integration at $\rho \sim 1/(\pi T)$ and ensures validity of dilute instanton gas approximation (DIGA) at large temperatures, at which $\alpha_s(\pi T) \ll 1$

- Axion mass predicted to decrease power-like with temperature in DIGA:

$$m_A^2(T) f_A^2 = \chi(T) \simeq 2\int_0^\infty d\rho\, D(\rho)\, G(\pi\rho T) \propto T^{-(7+\frac{1}{3}n_f)}$$

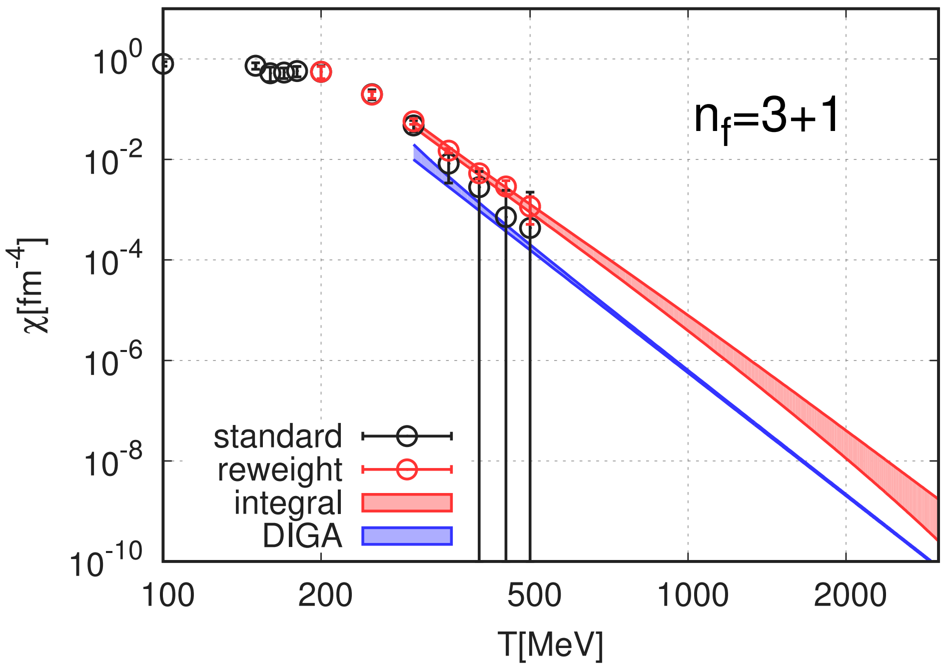
Axion Mass at Finite Temperature

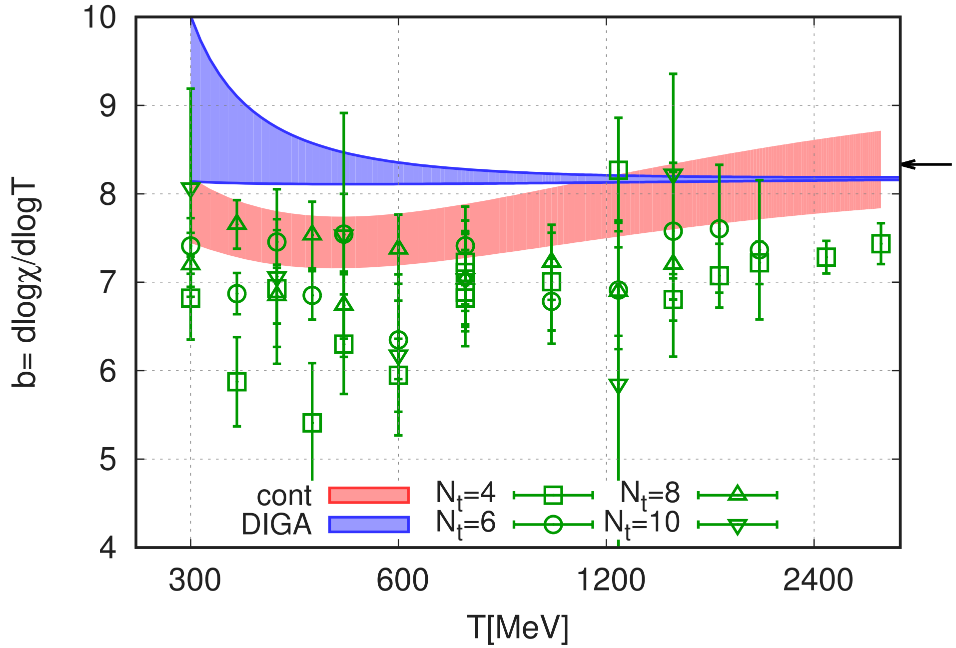
- How good is DIGA quantitatively? Check with lattice QCD!
- Topological susceptibility notoriously difficult to calculate on lattice
  1. Large cutoff effects when exploiting action with non-chiral quarks to calculate topological observables
  2. Tiny topological susceptibility needs extremely long simulation threads to observe enough changes of topological sectors
- Solutions of these problems: [Borsanyi et al. `16]
  1. Eigenvalue reweighting technique: Substitute topology related eigenvalues of non-chiral quark Dirac operator with its corresponding eigenvalues in continuum
  2. Fixed sector integral technique: Measure logarithmic differential of topological susceptibility which is related to quantities to be measured in fixed topological sectors. Then integrate.

Axion Mass at Finite Temperature

> Results:

[Borsanyi et al. `16]





> Temperature slope remarkably close to DIGA prediction

> DIGA underestimates topological susceptibility by overall normalization „K factor" of order ten (should be improved in two-loop DIGA)

Axion Mass at Finite Temperature

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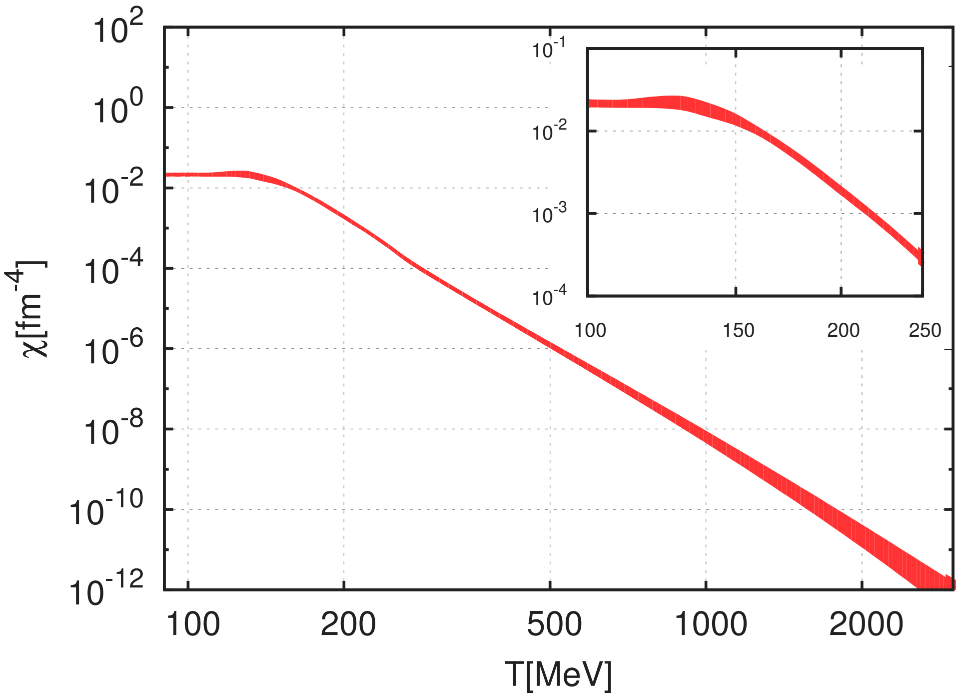
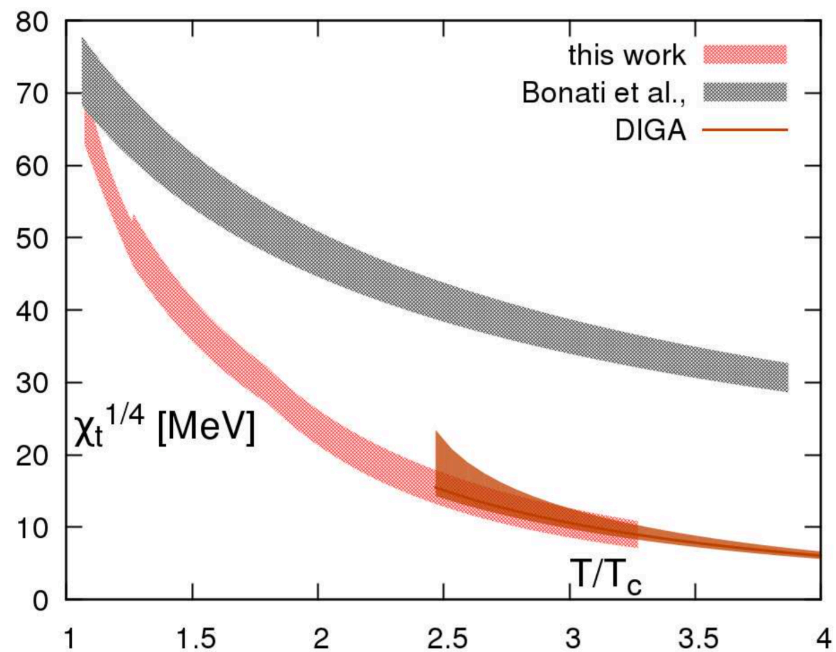


[Borsanyi et al. `16]



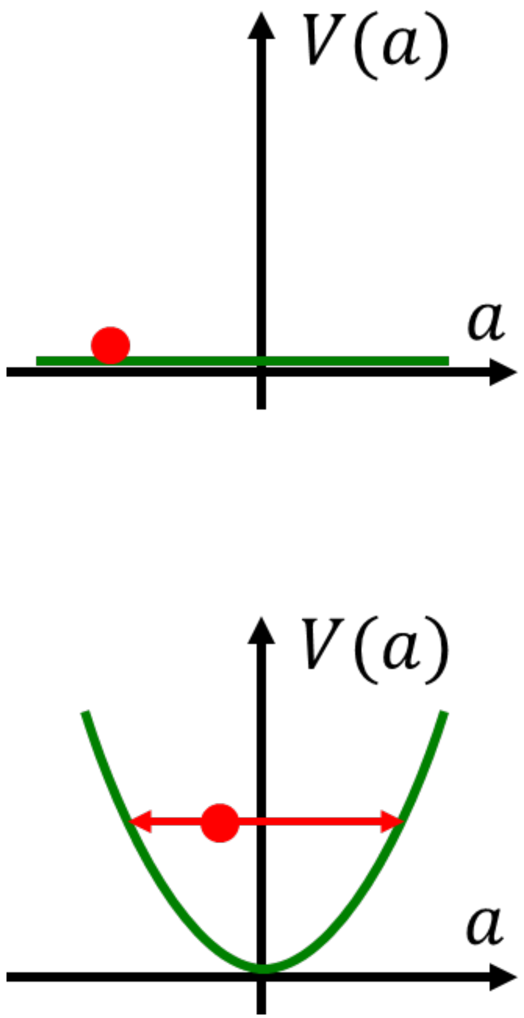
[Petreczky et al. `16]

Axion Dark Matter

> DM from vacuum realignment:

[Preskill et al 83; Abbott,Sikivie 83; Dine,Fischler 83,....]

- In early universe, axion frozen at random initial value
- Later, field feels pull of mass towards zero and oscillates around it
- Spatially uniform oscillating classical field = coherent state of many, extremely non-relativistic particles = CDM



[Raffelt]

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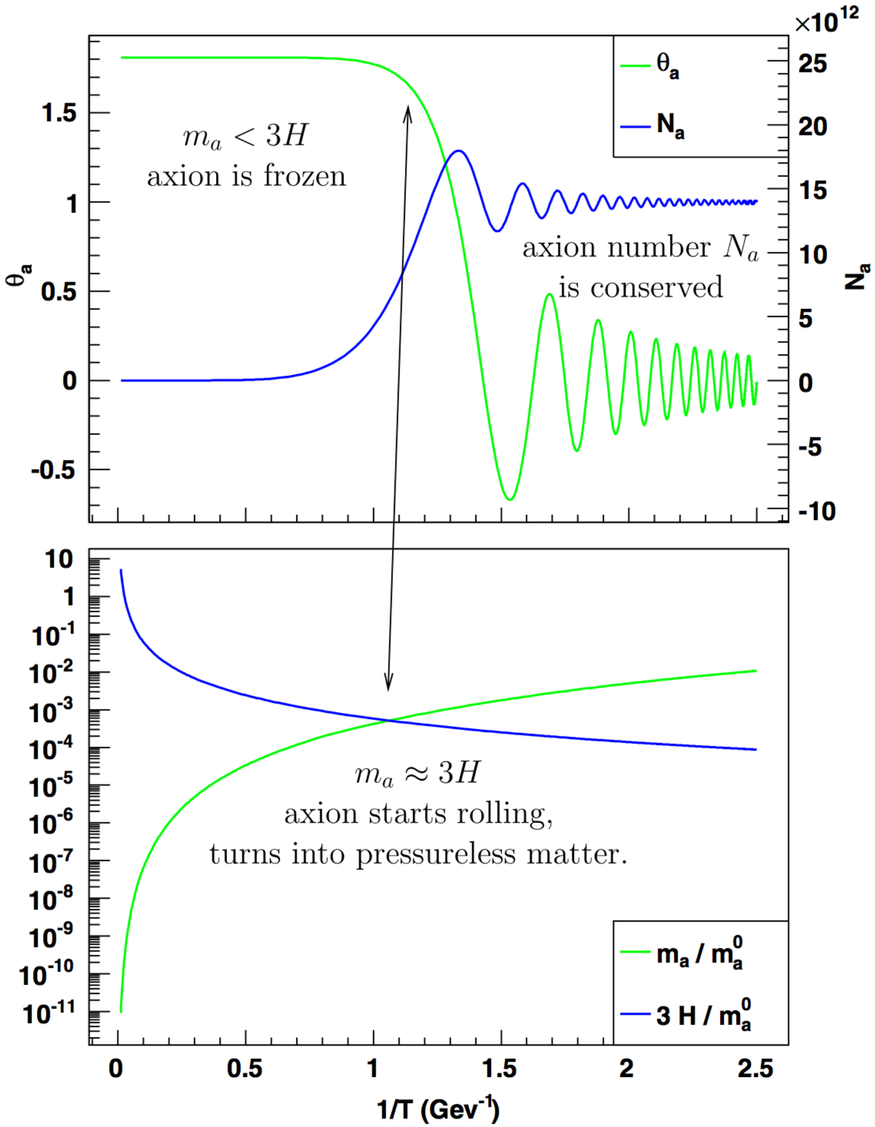
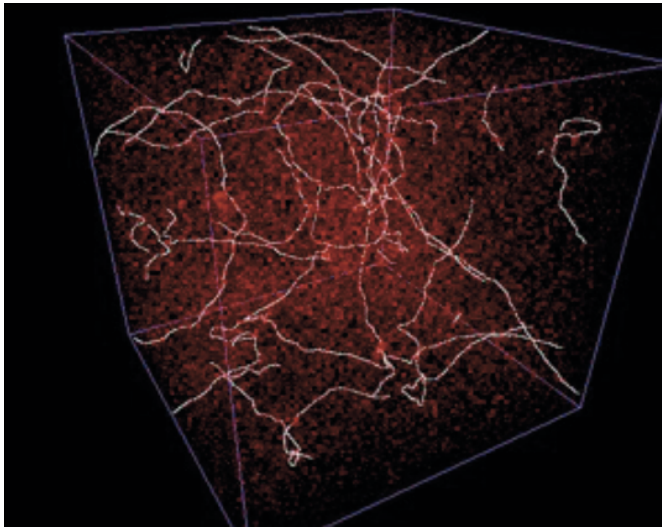


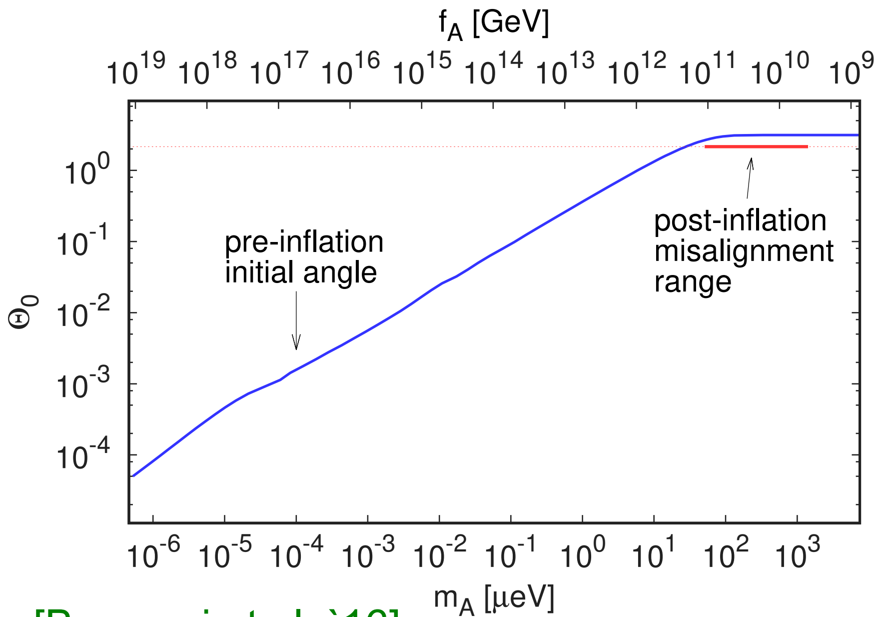
[Wantz,Shellard `09]

Axion Dark Matter

- Post-inflationary PQ restoration:
  - Initial axion angle uncorrelated at causally disconnected points
  - Average DM abundance depends only on one unknown parameter: f_A
  - Strict lower bound: $m_A > 28(2)\,\mu\text{eV}$
  - Taking into account DM production also due to axion strings and walls:

$$50\,\mu\text{eV} \lesssim m_A \lesssim 15\,\text{meV}$$





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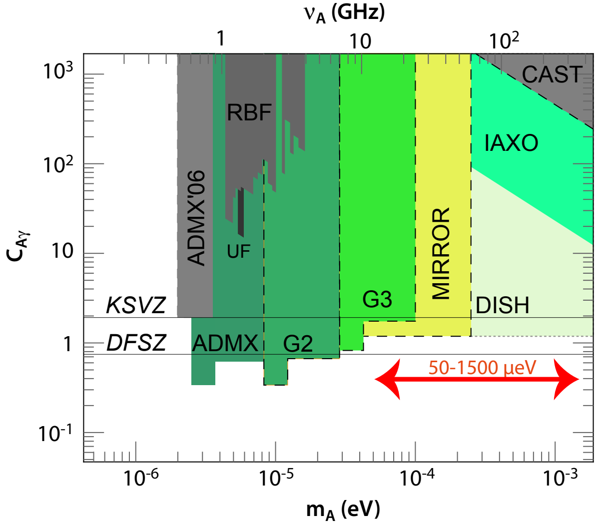

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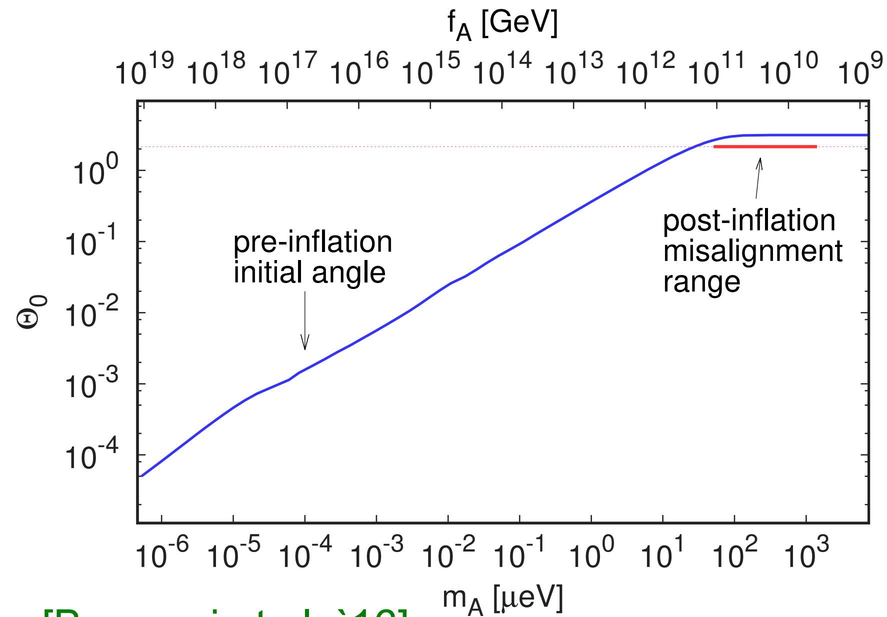
> Pre-inflationary scenario (no re-storation after inflation):

- Also dependence on universal initial axion angle
- No contribution from topological de-fects
- Strong constraints from CMB isocur-vature fluctuations



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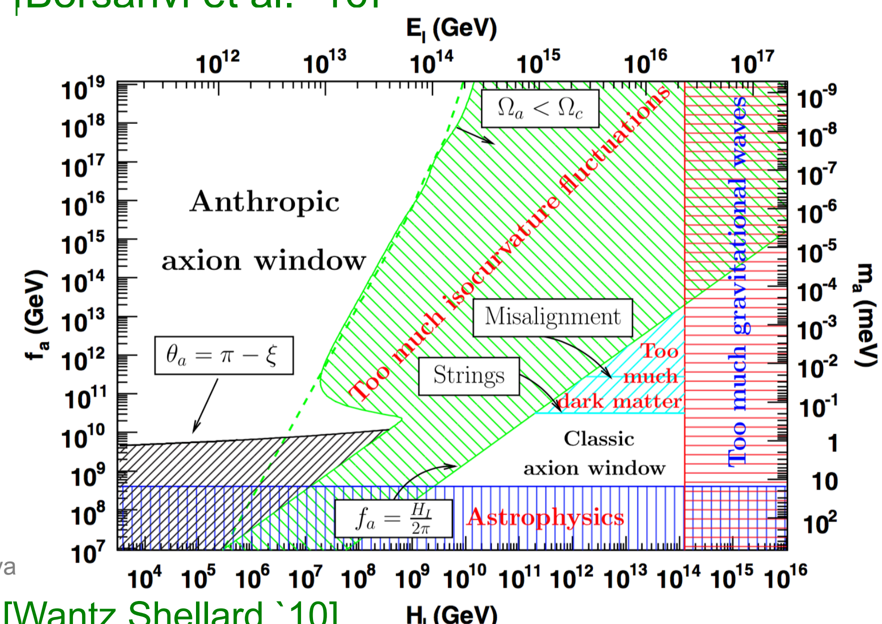
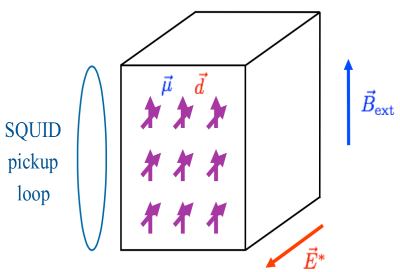


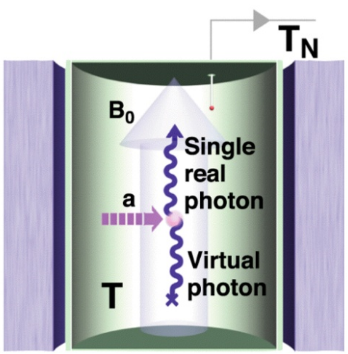
[Wantz,Shellard `10]

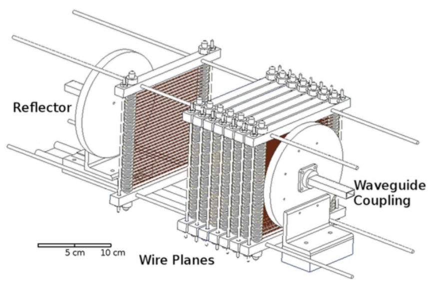
Axion Dark Matter

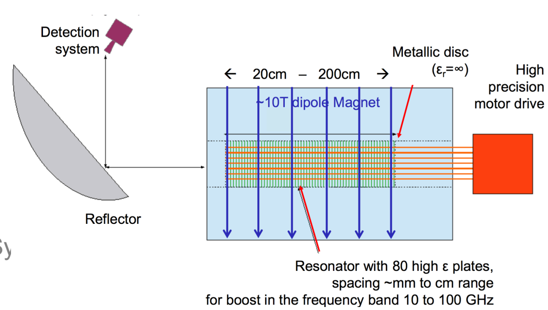
> Upcoming generation of axion dark matter experiments can probe sizeable portion of axion mass range relevant for DM:

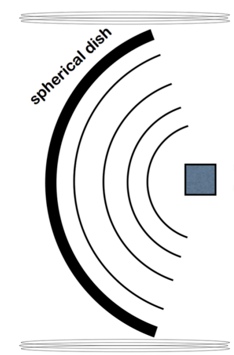
- $m_A \ll \mu\mathrm{eV}$: searches for oscillating nuclear electric dipole moments exploiting nuclear magnetic resonance techniques (CASPEr)
- $\mu\mathrm{eV} \lesssim m_A \lesssim 0.1\,\mathrm{meV}$: searches for excitations of electromagnetic resonances due to axion photon conversion in microwave cavities in superconducting solenoids (ADMX, X3, CULTASK, ...)
- $30\,\mu\mathrm{eV} \lesssim m_A \lesssim 0.3\,\mathrm{meV}$: searches for electromagnetic excitation in open dielectric/ Fabry-Perot resonator in a strong magnetic field (MADMAX/ORPHEUS, ...)
- $0.3\,\mathrm{meV} \lesssim m_A \lesssim 10\,\mathrm{meV}$: searches exploiting dish antenna or electron spin pre-cession in galactic axion wind (QUAX)

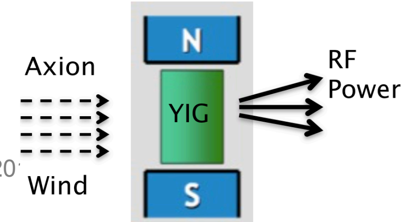












Summary

- Axions are one of the most attractive dark matter candidates
- A key quantity entering the prediction of the axion dark matter abundance is the topological susceptibility in QCD at temperatures above the quark hadron phase transition.
- The latter can be calculated semi-classically in the dilute instanton gas approximation
- Comparison with new non-perturbative continuum extrapolated lattice QCD results: Nice agreement is found for the temperature slope, but overall normalization of one-loop DIGA off by one order of magnitude
- Strict lower bound of axion mass in post-inflationary PQ symmetry restoration scenario
  - Current running experiments do not cover post-inflationary PQ symmetry restoration
  - Push to new experiments!