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Financial Fragility and the Fiscal Multiplier*

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Abstract

We show that undercapitalized banks with large holdings of government bonds subject to sovereign default risk lead to a new crowding-out channel: deficit-financed fiscal stimuli lead to higher bond yields, triggering capital losses for the banks. Banks then cut back loans, giving rise to potentially negative fiscal multipliers. Crowding out increases for longer maturity bonds and higher sovereign default risk. We estimate a DSGE model with financial frictions for Spain and find strong support for these results. The DSGE results further show strong nonlinear effects: the cumulative multiplier decreases substantially with the size of the stimulus, as well as with the amount of time between the announcement and implementation of the stimulus.

Keywords: 'Financial Intermediation; Macrofinancial Fragility; Fiscal Policy; Sovereign Default Risk'

JEL classification: E44; E62; H30

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1 Introduction

The size of the fiscal multiplier has been at the forefront of academic research ever since the financial crisis of 2007-2009 (Christiano et al., 2011; Eggertsson, 2011; Woodford, 2011; Ramey, 2019). We focus on the role of undercapitalized commercial banks that have large holdings of domestic government bonds with substantial default risk on their balance sheet in determining the size of the fiscal multiplier. We show that fiscal stimuli become much less effective when they are financed by balance-sheet-constrained commercial banks, to such an extent that the direct and even the cumulative multiplier may actually turn negative. We dissect the channels that contribute to the decreased effectiveness, and highlight the crucial role of both the size and timing of the stimulus for the size of the multiplier.

This was a major issue in Southern-Europe after the outbreak of the European sovereign debt crisis in 2011 and may well become relevant again now that central banks across the world are raising interest rates again after a long period at the Zero Lower Bound (ZLB). At the time of the European sovereign debt crisis, Southern-European banks were effectively undercapitalized (IMF, 2011; Hoshi and Kashyap, 2015), as indicated by elevated levels of non-performing loans on Southern-European banks' balance sheets, for example. In addition, Southern-European banks had domestic sovereign debt holdings amounting to at least 150% of Tier-1 capital, while CDS spreads on Southern-European sovereign debt had increased by hundreds of basis points, see Section 2. But it is not just an industrialized country issue, on the contrary: Gennaioli et al. (2018) document that 12.7% of commercial banks' assets in emerging economies consist of (predominantly) domestic government bonds, making them vulnerable to the same problems and creating the same crowding out channels we discuss in this paper.

To help intuition, we first analyse this question analytically using a two period general equilibrium model incorporating leverage constrained banks, long-term debt and endogenous sovereign default risk. To demonstrate the empirical relevance of our results we then construct a DSGE model with financial frictions and estimate the model using Bayesian techniques on Spanish data; Spain clearly fits the earlier described environment: commercial banks were undercapitalized after the burst of the real-estate bubble of the early 2000's (IMF, 2011; Hoshi and Kashyap, 2015), while the Spanish government faced substantial default risk at the height of the European sovereign debt crisis in 2011-2013.

Specifically, our quantitative model is one of a small open economy member of a monetary union, similar to Burriel et al. (2010). The policy rate is determined via a Taylor rule which features union-wide inflation and output. Union-wide inflation and output, in turn, are a weighted average of Spanish inflation and output and that in the rest of the monetary union, thereby capturing the fact that Spanish macrodevelopments have a limited impact on the Eurozone's policy rate. We introduce financial frictions as in Gertler and Karadi (2011) to capture the fact that Spanish commercial banks have been undercapitalized since the onset of the financial crisis.[1] Commercial banks have a portfolio choice between corporate loans and long-term government debt, which creates an interconnectedness between the financial system and fiscal/debt problems (Bocola, 2016; Kirchner and van Wijnbergen, 2016). A longer maturity of government bonds leads to higher potential capital losses for financial intermediaries and more pronounced adverse effects on the economy in case of a financial crisis. Therefore, we explicitly introduce long-term government bonds as in Woodford (2001) to approximate the average duration of domestic sovereign debt held by Spanish banks at the time.[2] Sovereign default risk is introduced by postulating a maximum level of taxation that is politically feasible, like in Schabert and van Wijnbergen (2006, 2014) and Corsetti et al. (2013). Uncertainty about the exact value of that limit leads to a sovereign debt discount that increases in the size of the public debt.

The first contribution of our paper is to highlight a new channel through which the effectiveness of fiscal stimuli is reduced: crowding out of private investment by government purchases is *amplified* by capital losses on existing holdings of government bonds held by balance-sheet-constrained commercial banks. To the best of our knowledge, we are the first to show the crucial role of a feedback loop between undercapitalized banks and weak sovereigns in triggering potentially negative fiscal multipliers. Other papers have investigated the link between sovereign default risk and the

[1] Throughout the paper we will interchangeably use the term 'commercial banks' and 'financial intermediaries' to denote the same group of economic agents.

[2] We do not address the issue of optimal maturity structure of the government's debt

effectiveness of fiscal stimuli, see for example Corsetti et al. (2013). Our model goes beyond this literature by pointing out that banking distress arising from capital losses on *existing* sovereign debt lead to higher interest rates on *newly issued* sovereign debt, thereby aggravating the initial sovereign debt problems. This, in turn, results in subsequent rounds of capital losses on existing government bond holdings, which lead to additional interest rate increases and so on, a negative spiral or doom loop amplification channel that has not been investigated before in connection with the effectiveness of fiscal stimuli. We find that this amplification loop is quantitatively important for the size of the multiplier when government debt is both long-term *and* subject to default risk. In the absence of one of these two ingredients, the multiplier is positive, in line with Gornicka et al. (2020). Including both ingredients, however, causes the multiplier to decrease by at least 0.60 percentage points with respect to the case where only one of these two features is included.

In a second contribution we highlight an essential non-linearity: in the presence of long-term debt and sovereign risk, we find that the size of the multiplier substantially decreases with the size of the fiscal stimulus. The intuition behind this result is that a larger stimulus increases the supply of bonds, as a result of which bond prices decrease by more with respect to smaller stimuli. As a result, commercial banks' capital losses increase, as a result of which crowding out of credit provision to the real economy increases. Specifically, we find that the difference between the multiplier of a stimulus of 0.5% of quarterly GDP and one of 4% is equal to 0.58 percentage points for a stimulus that is announced today but implemented four quarters later. Therefore, governments that find themselves in the middle of a banking-sovereign crisis should implement small fiscal stimuli, if any at all.

In a third contribution we show that the timing of the stimulus is crucial for the size of the multiplier. We show that stimuli that are announced today but implemented in the future have a lower multiplier than stimuli that are implemented immediately. The intuition behind this result is the following: a financial crisis shock causes domestic prices to decrease with respect to foreign prices in the rest of the Eurozone. A fiscal stimulus, however, increases domestic prices relative to foreign prices, everything else equal, as a result of which there is expenditure switching from domestic to foreign goods. An immediate stimulus, however, is implemented at the moment that domestic goods have become more attractive as a result of the financial crisis shock, while the delayed stimulus is implemented at the moment domestic prices have already recovered with respect to foreign prices. As a result, the relative price of domestic goods with respect to foreign goods deteriorates further for a delayed stimulus, and continues to be higher afterwards with respect to an immediate stimulus. Quantitatively, we find that the multiplier of a stimulus with an implementation lag of 4 quarters is at least 0.30 percentage points lower than that of an immediate stimulus. This is obviously a relevant point empirically: parliamentary procedures to approve a new budget can easily take up several months to a year. Ramey (2011) indeed provides empirical evidence that agents foresee most major changes in government spending. In a related analysis, Mertens and Ravn (2012) and House and Shapiro (2006) point at contractionary effects of future tax cuts in the period before they are enacted if they are anticipated.

Related Literature

In the aftermath of the recent European sovereign debt crisis, a literature has developed on the bank-sovereign nexus, which can broadly be divided into three strands: a first group of papers takes banks' sovereign exposure as given, and focuses on the impact of sovereign risk on banks' balance sheets and credit provision to the real economy (Bocola (2016) among others). A second group of papers looks at the channels through which government incentives to bail out banks are increased by excessive bank exposure to domestic sovereign debt (Acharya et al., 2014; Brunnermeier et al., 2016; Farhi and Tirole, 2018). A third group of papers focuses on the amplification of sovereign debt crises through the collateral channel: banks' ability to raise funding is seriously hampered when the value of their collateral, in the form of government bonds, deteriorates in a sovereign debt crisis and thereby induces a contraction in lending to the real economy (Engler and Große Steffen, 2016). Van der Kwaak and Van Wijnbergen (2014) also incorporate the amplification of sovereign debt crises through a balance sheet channel, but they focus on the impact of bank recapitalizations on output, not on the effectiveness of fiscal policy. We go beyond this literature by explicitly incorporating capital losses on government debt as a crowding out channel.

Empirical evidence on the effectiveness of fiscal stimuli is mixed. Blanchard and Perotti (2002), using a SVAR (Structural Vector Autoregression) approach, find a multiplier of 1 in the U.S. for government purchases. Auerbach and Gorodnichenko (2012) shows that the multiplier is moderate or even negative in expansions, while it is larger than 2 in recessions. Ramey (2019) provides an overview of ten years of renewed research on fiscal policy and finds that the wide range of reported multipliers narrows significantly to a value between 0.6 and 1 once methods for calculating multipliers are standardized. Blanchard and Leigh (2013) and Blanchard and Leigh (2014) find that multipliers in European economies around the start of the European sovereign debt crisis were substantially above one in spite of poorly capitalized banks, a finding that is challenged by Gornicka et al. (2020). Other results that are relevant for our paper are the results reported by Ilzetzki et al. (2013), who find that for countries with debt levels exceeding 60% of GDP, the impact multiplier is close to zero, and the long run multiplier -3, suggesting that debt sustainability is an important determinant of the output effects of fiscal stimuli.

Corsetti et al. (2012) specifically investigate the size of the fiscal multiplier in times of financial crises. They find that the cumulative multiplier is substantially larger than 1 during financial crises, but negative when public finances are strained. In their paper, the causality runs in one direction, namely from sovereign debt problems to higher interest rates at which banks lend to the private sector. Unlike our paper, banks in Corsetti et al. (2012) do not hold government debt, and therefore the feedback from banks charging higher interest rates on government debt is absent. The empirical evidence presented in Homar and van Wijnbergen (2017) suggests that the interaction between banks' capitalization and the effectiveness of fiscal stimuli is a very real problem: they differentiate between crises after which banks have been recapitalized and crises where they have not been recapped and show that in the latter case fiscal policy has no empirically significant impact on the speed of a recovery, while it does when banks have been recapitalized, empirical results that are in line with our theoretical predictions.

Our paper also connects to the more partial equilibrium empirical literature on the effects of banks' holdings of sovereign debt on lending to the real economy. Several papers find evidence of a crowding out of corporate lending by increased holdings of government bonds (Becker and Ivashina, 2018; Acharya and Steffen, 2015; Gennaioli et al., 2018). Other papers document a relation between capital losses on sovereign debt and corporate lending, with larger losses leading to less lending, as we predict (Popov and Horen, 2015; Acharya et al., 2018). Acharya et al. (2019) shows the converse: higher windfall gains on European banks' holdings of periphery sovereign bonds after the announcement of the Outright Monetary Transactions (OMT) program of the European Central Bank (ECB) correlated with a higher loan supply to the corporate sector. More indirectly related is the analysis of De Bruyckere et al. (2013), who show that the degree of contagion between bank and sovereign credit risk is significantly related to bank capital adequacy: weaker capitalized banks lead to more contagion. In addition, banks with a higher sovereign debt exposure experience more contagion in the form of higher banks' CDS spreads, a finding that strongly supports our predictions.

Our paper also builds on the literature in which shocks to the balance sheet of financial intermediaries affect the macroeconomy because of agency problems between deposit holders and bank owners (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011). Gertler and Karadi (2013), Bocola (2016), Kirchner and van Wijnbergen (2016), and Kollmann et al. (2013) all allow financial intermediaries to hold government bonds in addition to corporate loans, but either without sovereign risk or when it is incorporated it remains exogenous, eliminating the amplification cycle we highlight. In contrast, we endogenize the probability of sovereign default, and link it through a simple model of endogenous default like in Schabert and van Wijnbergen (2006) and Schabert and van Wijnbergen (2014) to the level of outstanding bonds and thereby introduce a new amplification cycle. Our model of endogenous default is in the language of the survey by Aguiar and Amador (2013) a non-strategic default: the government is forced into default by adverse shocks raising debt to levels requiring unsustainably high levels of taxation for debt service.

2 Stylized facts

In this section we motivate several key ingredients of our model with data from Spain, Italy and Portugal (to which we will refer as SIP from now on).[3] IMF (2011) and Hoshi and Kashyap (2015) provide evidence of the extent to which in particular Southern Europe's banks were undercapitalized

[3]For a more detailed description of the data sources see Appendix A.

during and after 2008/2009. The problem with undercapitalized banks is that they tend to engage in excessive risk shifting through zombie lending. Loans to inefficient firms are rolled over, rather than written down, which prevents productive new or expanding firms from obtaining funding. Evidence for this evergreening of bad loans is found in Peek and Rosengren (2005) and Caballero et al. (2008) in the case of Japan. Figure 1 illustrates the extent of non-performing loans (NPLs) in SIP using Germany as a benchmark.

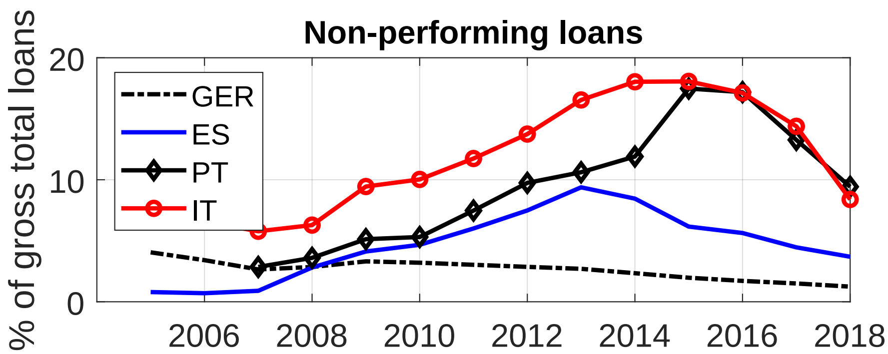


Figure 1: Non-performing loans for the aggregate commercial banking system in Germany (GER), Spain (ES), Italy (IT) and Portugal (PT). *Source*: World Bank, http://data.worldbank.org/indicator/FB.AST.NPER.ZS.

Even when there is a substantial probability that non-performing loans will not be repaid, banks were under the accounting rules at the time only allowed to provision for losses when *actual* losses materialized ("occurred losses").[4] Evergreening then allows banks to keep loans at face value on their balance sheet, despite prospective losses in the future. The fact that NPLs were at elevated levels at the time of the Eurocrisis supports the claims of IMF (2011) and Hoshi and Kashyap (2015) that Southern-European banks were effectively undercapitalized.

Another feature of the European data that is relevant for our setup is the interconnection between the commercial banking system and the sovereign. Figure 2 shows domestic government bond holdings of the aggregate commercial banking system as a percentage of aggregate Tier-1 capital at the end of 2011 across the Eurozone. Commercial banks in Southern-Europe clearly have

[4]This has changed with the IFRS9 standard, which allows provisioning for "prospective losses".

a large exposure to their domestic sovereign. Spanish banks have an exposure to domestic Spanish sovereign debt equivalent to more than 150% of Tier-1 capital, Italian banks to almost 200% of Tier-1 capital, while Greek banks have an exposure of almost 250% of Tier-1 capital to the Greek sovereign at the end of 2011. From these numbers it is clear that changes in the (market) value of sovereign debt will have substantial effects on bank capital.

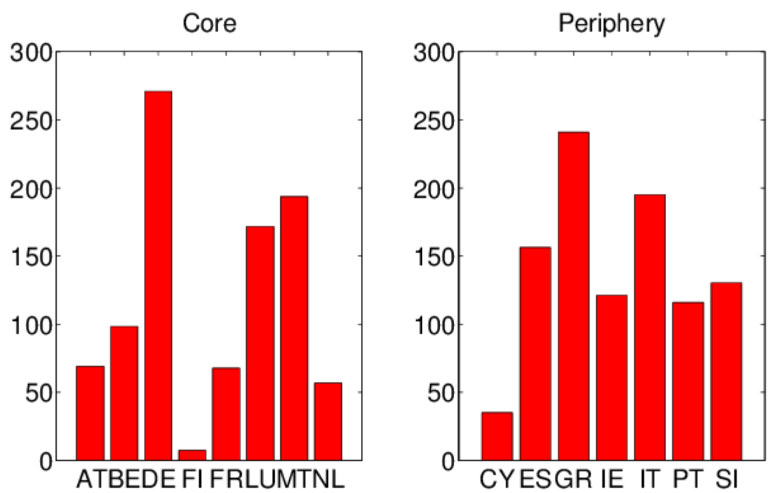


Figure 2: Banks' exposure to domestic sovereign debt (all maturities) as a percentage of their total Tier-1 capital in the core, respectively the periphery of the Eurozone. "AT" refers to Austria, "BE" to Belgium, "DE" to Germany, "FI" to Finland, "FR" to France, "LU" to Luxemburg, "MT" to Malta, "NL" to Netherlands, "CY" to Cyprus, "ES" to Spain, "GR" to Greece, "IE" to Ireland, "IT" to Italy, "PT" to Portugal, and "SI" to Slovenia. *Source*: http://www.eba.europa.eu/risk-analysis-and-data/eu-wide-stress-testing/2011/results.

Figure 3 in turn shows that Southern-European countries faced substantial sovereign default risk during the European sovereign debt crisis of 2011-2013. CDS-spreads for Italy and Spain increased from approximately 100 basis points in January 2010 to levels above 400 basis points in 2012 and 2013, reflecting a substantial increase in sovereign default risk. The likelihood of a default by the Portuguese sovereign is even larger, the CDS premium for Portuguese sovereign debt increased to more than 1000 basis points at the end of 2011. At the same time, Draghi (2018) indicated that aggregate losses for banks in Greece, Italy and Portugal amounted to 161%, 22% and 36%, respectively, of their respective end-2010 Core Tier-1 capital. These CDS data coupled with the large holdings of sovereign debt by Southern Europe's banks strongly suggest that much of their increased losses after the Great Financial Crisis (GFC) and subsequently during the Eurocrisis can be traced to their substantial holdings of sovereign debt.

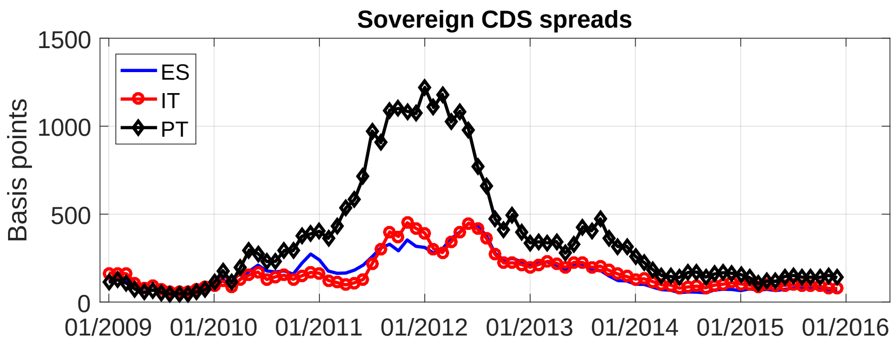
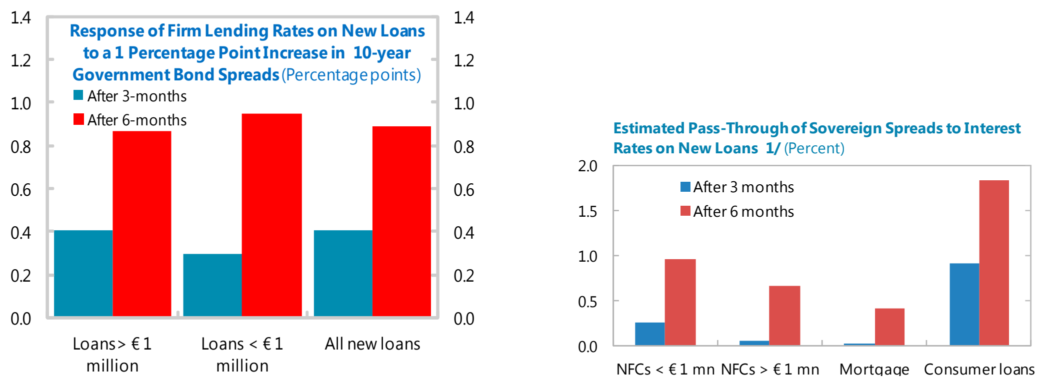


Figure 3: SNR CR 5Y Credit Default Swaps Premium in basis points (monthly) for Spain (ES), Italy (IT) and Portugal (PT). Monthly data were obtained by taking an unweighted average of daily data within a month. *Source*: Datastream, Thomson Reuters.

Figure 4 shows the estimated pass-through of sovereign spreads to interest rates on new bank loans in Italy and Spain. The sovereign spread is measured by the spread between the yield on a 10-year Italian or Spanish bond and the yield on a 10-year German Bund. the figure shows that sovereign spreads affect bank lending rates for small and large firms, and for small and large loans. For both countries, the transmission is nearly complete after six months. Therefore, Figure 4 strongly suggest that increases in sovereign default risk lead to higher interest rates on corporate loans, a key mechanism in our model.

Interest rate pass-through of sovereign spreads



(a) (b)

Figure 4: Estimated pass-through of sovereign spreads to interest rates on corporate borrowing rates in Italy (a) and Spain (b) using a monthly frequency VAR estimated from January 2006 till February 2012 (Italy) and January 2005-August 2012 (Spain). Sovereign spreads are the spread of a 10-year government bond over 10-year German Bunds. *Source*: IMF (2012), IMF (2013).

3 Analytical results in a two period model

To provide more intuition for the DSGE model results in sections 4 to 7 we first analyze a trimmed down, two period general equilibrium model analytically. Our focus in this section is on the link between government spending and banks' balance sheets, we leave an investigation of the link from intermediaries' credit provision to investment and output for the full DSGE model.

3.1 Model

Consider a small open economy, one globally traded commodity and two time periods: $t = 0$ and $t = 1$. There are households, financial intermediaries, production firms and a government. Households have a standard utility function that is concave in consumption, and receive an endowment in period $t = 0$. They divide this endowment between consumption and saving through deposits and a foreign asset. Because of perfect capital mobility, the interest rate on deposits will be equal to the foreign interest rate[5]. They consume their income net of lump sum taxes in period $t = 1$.

In period $t = 0$ production firms borrow from financial intermediaries in a perfectly competitive

[5]The monetary union eliminates exchange rate related wedges between foreign and domestic rates.

loan market. They employ these funds to purchase physical capital in the same period, which is then employed for production in period $t = 1$ using a concave production function. Financial intermediaries start period $t = 0$ with net worth and attract deposits from households to lend to production firms and purchase government bonds. These bonds are subject to default risk because the government faces a cap on tax revenues that is not perfectly known in advance, like in (Schabert and van Wijnbergen, 2006, 2014; Corsetti et al., 2013). Financial intermediaries are subject to an incentive compatibility constraint as in Gertler and Karadi (2011), which prevents them from perfectly elastically expanding their balance sheet in case opportunities for arbitrage arise.

We consider a government spending shock in period $t = 0$, after which no further spending shocks occur. Detailed descriptions and derivations can be found in Appendix B, while we focus here on the core elements and results.

3.1.1 Government

The government enters period $t = 0$ with outstanding government bonds b_{-1} that were issued at the end of period $t = -1$. These bonds are traded in period $t = 0$ in the secondary market at a price q_0^b. They do not pay coupons and their principal is to be repaid at the beginning of period $t = 1$. The government purchases goods g_0 from the production sector in period $t = 0$, which are financed by issuing new, zero-coupon bonds b_0^{new} that are to be repaid at the beginning of period $t = 1$. Since the cash flow of the new bonds is the same as that of old bonds going forward, new bonds trade at the same price q_0^b as old bonds. The government budget constraint in period $t = 0$ then equals:

$$q_0^b b_0 = g_0 + q_0^b b_{-1}, \tag{1}$$

where $b_0 \equiv b_0^{new} + b_{-1}$. Government liabilities at the beginning of period $t = 1$ are b_0 for which the government tries to raise lump sum taxes $\tau_1 = b_0$ from households. However, there is a risk that the government might not be able to do so, because of the existence of a stochastic maximum level of taxation (Schabert and van Wijnbergen, 2006, 2014; Corsetti et al., 2013). For analytical convenience we assume in this section that bond holders receive nothing in the event of a default [6]. $p(b_0)$, the probability that required taxes τ_1 are larger than the maximum level of taxation is then increasing in outstanding bonds b_0: $p'(b_0) > 0$. Aguiar and Amador (2013) refer to this type of default as a non-strategic default process.

3.1.2 Financial intermediaries

Financial intermediaries enter period $t = 0$ with net worth n_0 and raise deposits d_0 from households. On the asset side they purchase government bonds b_0 at price q_0^b and lend an amount k_0 to production firms in a perfectly competitive market for loans.[7] The balance sheet of the representative intermediary then is:

$$k_0 + q_0^b b_0 = n_0 + d_0, \tag{2}$$

Production firms use the loans to acquire physical capital with which they produce in period $t = 1$ using a concave production function $y_1 = k_0^\alpha$. The interest rate r_0^k at which production firms borrow is determined in period $t = 0$ and paid to intermediaries in period $t = 1$. Production firms take this interest rate as given when maximizing period $t = 1$ profits $y_1 - \left(1 + r_0^k\right) k_0$.[8] Therefore in equilibrium the interest rate equals the marginal productivity of capital:

$$r_0^k = \alpha k_0^{\alpha-1} - 1 \tag{3}$$

As a result, changes in lending by intermediaries in period $t = 0$ trigger changes in the equilibrium interest rate to clear the loan market. At the beginning of period $t = 1$, intermediaries' holdings of bonds b_0 are fully repaid with probability $1 - p(b_0)$, while they receive zero euros with probability $p(b_0)$. Therefore, the expected net return on bonds r_0^b is given by the relation $1 + r_0^b = [1 - p(b_0)] / q_0^b$. Financial intermediaries take the probability of default $p(b_0)$ as given. The net real return on deposits d_0 is equal to r_0^d. Expected net worth $E_0(n_1)$ at the beginning of period $t = 1$ equals the

[6] In the full model of sections 4-7 we allow for partial default like Schabert and van Wijnbergen (2006, 2014)

[7] We assume households do not deposit with bankers belonging to the same household to prevent self-financing, bypassing financial frictions (Gertler and Karadi, 2011).

[8] We assume there is full depreciation of the capital stock after production in period $t = 1$.

difference between the (gross) return on assets and on liabilities:

$$E_0(n_1) = \left(1 + r_0^k\right) k_0 + \left[1 - p(b_0)\right] b_0 - \left(1 + r_0^d\right) d_0. \tag{4}$$

After realization of n_1 in period $t = 1$, net worth is paid out to households, after which intermediaries stop operating.

Households are the ultimate owners, therefore period $t = 1$ net worth is discounted using the households' stochastic discount factor $\beta\Lambda_{0,1}$ when choosing the composition of the balance sheet in period $t = 0$. Intermediaries are subject to an incentive compatibility constraint that ensures that expected, discounted net worth $E_0(\beta\Lambda_{0,1}n_1)$ is larger than or equal to the gains from effectively running away with a fraction λ_a of asset $a = \{k_0, q_0^b b_0\}$ at the end of period $t = 0$ (Gertler and Karadi, 2011):

$$E_0(\beta\Lambda_{0,1}n_1) \geq \lambda_k k_0 + \lambda_b q_0^b b_0. \tag{5}$$

Intermediaries' optimization problem is to maximize expected, discounted net worth $E_0(\beta\Lambda_{0,1}n_1)$ subject to (2), (4) and (5). The resulting first order conditions can be found in Appendix B, which we use to rewrite intermediaries' incentive compatibility constraint in the following way:

$$(1 + \mu_0) n_0 \geq \lambda_k k_0 + \lambda_b q_0^b b_0, \tag{6}$$

where μ_0 is the Lagrangian multiplier of intermediaries' incentive compatibility constraint (5). The key insight from the above equation is that the size of intermediaries' balance sheet is limited by the amount of net worth n_0 when the constraint is binding (Gertler and Karadi, 2011). Finally, we assume that banks also purchased government bonds in period $t = -1$, and that the government does not default on outstanding liabilities in period $t = 0$. Net worth n_0 therefore equals

$$n_0 = n_0^{ex} + q_0^b b_{-1}, \tag{7}$$

where n_0^{ex} is inherited from period $t = -1$. A drop in the bond price will reduce net worth n_0, thereby tightening constraint (6) which will force intermediaries to reduce the size of the balance sheet if the constraint was or becomes binding.

3.2 A deficit-financed government spending shock

Now we turn to the key results, the analysis of the various channels through which a government spending shock crowds out private investment in the presence of undercapitalized banks, i.e. banks for which the incentive compatibility constraint (6) binds. Before doing so, we denote the effect from a government spending shock on the world interest rate by $\frac{dr_0^d}{dg_0}$. For the case of a small open economy, we get that $\frac{dr_0^d}{dg_0} = 0$, but we include the more general expression to generalize how our economy would be affected if there is a change in the world interest rate in response to our government spending shock.

We start with Proposition 1

Proposition 1. *An increase in government spending g_0 decreases the price of bonds, i.e.* $\frac{dq_0^b}{dg_0} < 0$.

Proof. We show in Appendix B that the change in the price of government bonds is given by:

$$\frac{1}{q_0^b} \cdot \frac{dq_0^b}{dg_0} = -A_1 \lambda_b - A_2 \cdot \frac{dr_0^d}{dg_0} - A_3 \cdot p'\left(b_0\right) < 0, \tag{8}$$

and we show in the Appendix that $A_i > 0$ for $i = 1, 2, 3$. Assuming $\frac{dr_0^d}{dg_0} \geq 0$, this concludes the proof. □

The first term arises because additional spending increases the supply of bonds, while intermediaries' demand for bonds is initially constrained by the binding incentive compatibility constraint (6). Therefore, the bond price must decrease to clear the market (Kirchner and van Wijnbergen, 2016). The second and third term of expression (8) are well known from the literature. The second term tells us that an increase in intermediaries' funding costs r_0^d (as a result of extra spending by the government) increases the required return on bonds (equivalently a decrease in today's bond price). This term would be zero under the assumption of a small open economy and highest if one assumes a closed economy. The third term tells us that higher debt and the ensuing increase in sovereign default risk $p'(b_0)$ lead intermediaries to demand a higher expected return, which triggers a fall in bond prices. Before discussing the intuition behind the first term, we need two propositions first.

Proposition 2. *Intermediaries' net worth decreases in response to an increase in government spending:* $\frac{dn_0}{dg_0} < 0$.

Proof. Differentiation of equation (7) with respect to g_0 shows that $\frac{dn_0}{dg_0} = b_{-1} \cdot \frac{dq_0^b}{dg_0}$. Substitution of Proposition 1 concludes the proof. □

Since intermediaries' net worth depends on the market value of their holdings of government bonds carried over from period $t = -1$, a fall in the bond price decreases the value of their existing holdings, as a result of which intermediaries' net worth directly decreases. And this result immediately leads to our main result, summarized in Proposition 3: after an increase in government spending, credit provision and private investment decrease more when government bonds are long-term than when they are short-term.

Proposition 3. *Capital losses on intermediaries' existing long-term bonds amplify the decrease in credit provision to the real economy with respect to the case where government bonds are short-term.*

Proof. First, we use the government's budget constraint (1) to replace $q_0^b b_0$ in intermediaries' incentive compatibility constraint (6), as well as expression (7) for intermediaries' net worth to get:

$$(1 + \mu_0)\left(n_0^{ex} + q_0^b b_{-1}\right) = \lambda_k k_0 + \lambda_b \left(q_0^b b_{-1} + g_0\right).$$

Next, we implicitly differentiate with respect to g_0 to get:

$$\frac{dk_0}{dg_0} = \frac{1}{\lambda_k + Cn_0}\left[\underbrace{-\lambda_b}_{\substack{\text{Direct crowding}\\\text{out by new spending}}} \underbrace{- Dn_0 \cdot \frac{dr_0^d}{dg_0}}_{\substack{\text{Change in}\\\text{funding costs}}} + \overbrace{\left(\underbrace{1+\mu_0}_{\substack{\text{Change in}\\\text{net worth}}} - \underbrace{\lambda_b}_{\substack{\text{Change in value}\\\text{of bonds that can}\\\text{be diverted}}}\right) q_0^b b_{-1} \cdot \underbrace{\frac{1}{q_0^b} \cdot \frac{dq_0^b}{dg_0}}_{<0}}^{\substack{\text{Total effect on balance sheet capacity}\\\text{due to change in value existing bonds}}}\right] < 0, \tag{9}$$

where we used that the change in the shadow value of intermediaries' incentive compatibility constraint μ_0 is given by $\frac{d\mu_0}{dg_0} = -C \cdot \frac{dk_0}{dg_0} - D \cdot \frac{dr_0^d}{dg_0}$, with $C, D > 0$, see Appendix B. □

Expression (9) shows that corporate lending and hence capital spending goes down when the government issues new debt to finance additional government consumption g_0, a drop that can be decomposed into three effects. The first term denotes crowding out of corporate lending because of limited balance sheet capacity, a point highlighted in Kirchner and van Wijnbergen (2016): corporate lending has to fall to create space on intermediaries' balance sheets to absorb the extra bonds issued by the government.

The second term comes from the fact that higher funding costs drive up the return on corporate loans (everything else equal), as a result of which the demand for loans decreases.

The third term is the key innovation in this paper: the contraction in lending that already arises when government debt is short-term is aggravated when debt held on intermediaries' balance sheets is long-term. This third term arises because of capital losses $\frac{dq_0^b}{dg_0} < 0$ on intermediaries' existing holdings of government bonds b_{-1}, which reduce net worth n_0. As a result, the incentive compatibility constraint (6) becomes more binding and corporate lending falls further. This third channel only emerges in the presence of multi-period bonds and we will see below that it is larger the longer the maturity of the bonds held by the financial intermediaries.

To sum up: we show that the contraction in lending, which already occurs when government bonds are short-term, is *amplified* because undercapitalized financial intermediaries suffer capital losses when government bonds are long-term and subject to default risk. We will investigate the extent to which this amplification is quantitatively important using the infinite horizon DSGE model presented in the next sections.

4 Extension to an infinite-horizon DSGE model

Consider now the extension of the two-period model to an infinite-horizon DSGE model to assess whether for plausible parameter values the cumulative impact on the output (multiplier) is indeed affected by long-term government debt and sovereign default risk on the balance sheet of financial intermediaries. Here we sketch the model structure, for a detailed model description see Appendix C

We employ a small open economy model of a member of a monetary union that is similar in spirit to that of Burriel et al. (2010). However, an important deviation is the way we model monetary policy. Whereas Burriel et al. (2010) implement a Taylor rule based on Spanish inflation and output, we employ a Taylor rule that responds to inflation and output of the monetary union as a whole, which in turn only partially depends on Spanish inflation and output.

Domestic production is staggered between intermediate goods producers that operate in a market with perfect competition, retail goods producers with monopoly power and subject to price-stickiness a la Calvo (1983), and final goods producers that combine the differentiated domestic retail goods using a constant elasticity of substitution (CES) technology into an aggregate commodity. These final domestic goods are used by domestic consumers, capital producers for investment, government purchases, and to retail export firms. Retail export firms are monopolists, sell to final export producers, and are subject to Calvo (1983) price-stickiness when selling to final export firms. Final export firms, in turn, combine the different retail export goods using a CES technology, after which they sell the final export good to the rest of the monetary union. The demand for exports is increasing in foreign output and in the terms of trade, which is defined as the price of imports over the price of exports. There are retail import firms that acquire goods from the rest of the monetary union, and are monopolists and subject to Calvo (1983) price-stickiness when selling to the final import firms, which in turn combine retail import goods using a CES technology. Afterwards, they sell their final import goods to domestic households for consumption and to capital producers for investment.

Financial intermediaries are again subject to an incentive compatibility constraint as in Gertler and Karadi (2011), but now deposits pay a nominal interest rate which is set by the central bank of the monetary union. Capital producing firms buy final domestic goods and final import goods, and convert these into investment goods using a CES technology. Total investment is converted into new capital, which together with surviving old capital is sold to domestic intermediate goods producers. Intermediate goods producers use capital, financed by a loan from financial intermediaries, and labor to produce intermediate goods for domestic retail firms. There is perfect competition in the intermediate goods market. After production, intermediate goods firms sell used capital to capital producers, pay wages to workers, and bring the residual to the financial intermediary.

There are two types of households: constrained households that cannot save and unconstrained households. Both household types' consumption basket is a constant elasticity of substitution function between final domestic goods and final import goods, where both households have the same degree of home bias and elasticity of substitution. Constrained households receive income from providing labor. This income is spent on consumption and lump sum taxes that can be negative, i.e. a transfer. Unconstrained households also provide labor, but can save through deposits, an internationally traded asset, and domestic government bonds, the last of which is subject to quadratic adjustment costs (Gertler and Karadi, 2013). In addition to savings, income is also used for consumption and lump sum taxes. Both household types maximize expected lifetime utility subject to their respective budget constraint. The utility function is the same for both household types, it is subject to preference shocks and separable in consumption and labor, with habit formation in consumption to capture realistic consumption dynamics (Christiano et al., 2005).

The labor market is modeled as in Erceg et al. (2000): there is a continuum of unique labor types. Each unique labor type contains members from constrained and unconstrained households (Gali et al., 2007), and is represented by a single labor union. As a result, this labor union has the power to set the nominal wage rate, after which household members provide any amount of labor demanded. Labor unions, however, are subject to Calvo (1983) type wage-stickiness, as a result of which they might not be able to choose a new wage rate, in which case there is partial indexation with previous period wage inflation. Labor agencies hire labor from all the different labor unions, and combine these labor types into final homogeneous labor using a constant elasticity of substitution production function. Final labor is then used by domestic intermediate goods producers.

The government finances itself through debt and taxes. It spends on final domestic goods and services debt liabilities. Government bonds are held by financial intermediaries and households, and can be subject to sovereign default risk following Schabert and van Wijnbergen (2006, 2014); Corsetti et al. (2013). We only discuss the non-standard blocks of the model. The standard blocks can be found in Appendix C.

Finally, the domestic economy refers to Spain, while 'foreign' refers to the rest of the monetary union, in this case the Eurozone.

4.1 The Fiscal Authority and the Central Bank

The Central Bank of the monetary union sets the nominal interest rate on deposits r_t^n according to a standard Taylor rule which minimizes deviations of union-wide output y_t^{MU} and inflation π_t^{MU} from their respective target values:

$$r_t^n = (1-\rho_r)\left[\kappa_\pi\left(\pi_t^{MU} - \bar{\pi}^{MU}\right) + \kappa_y \log\left(\frac{y_t^{MU}}{y_{t-1}^{MU}}\right)\right] + \rho_r r_{t-1}^n + \varepsilon_{r,t}, \tag{10}$$

where $\rho_r \in [0,1)$ denotes the degree of interest rate smoothing. Union-wide variables x_t^{MU} are geometric averages of the corresponding values for Spain and for the rest of the union:

$$x_t^{MU} = (x_t^*)^{1-n}(x_t)^n\,, \tag{11}$$

where x_t^* denotes the value of the variable x in the rest of the monetary union, and x_t the value of x in the domestic (Spanish) economy, with $x \in \{\pi, y\}$. The variables x_t^* are given by an exogenous AR(1) process. Finally, n denotes the relative weight of the domestic economy in the monetary union. The relation between the nominal deposit rate r_t^n and the real interest rate r_t^d is given by:

$$1 + r_t^d = \left(1 + r_{t-1}^n\right) / \pi_t, \tag{12}$$

where $\pi_t \equiv P_t / P_{t-1}$ denotes the gross inflation rate of the domestic consumer price index P_t.

The treasury of the domestic government levies lump sum taxes τ_t on households, issues bonds $q_t^b b_t$ to finance its (exogenous) expenditures g_t of final domestic goods, and services outstanding government liabilities $\left(1 + r_t^b\right) q_{t-1}^b b_{t-1}$.[9] The government budget constraint (in terms of the domestic price level P_t) when there is no sovereign default risk is given by:

$$q_t^b b_t + \tau_t \quad = \quad p_t^h g_t + \left(1 + r_t^b\right) q_{t-1}^b b_{t-1}, \tag{13}$$

where $p_t^h \equiv P_t^h / P_t$ denotes the price of final domestic goods in terms of the domestic consumer price index P_t. Government bonds have a parameterizable maturity structure as in Woodford (2001), with coupon payments x_c on outstanding bonds decaying at a rate $1 - \rho$ per period, which effectively determines the maturity of the bonds. The real return on bonds equals $1 + r_t^b = \left(x_c + (1 - \rho) q_t^b\right) / \left(\pi_t q_{t-1}^b\right)$ (cf Appendix C.2).

Lump sum taxes τ_t are levied on both constrained and unconstrained households:

$$\tau_t = \nu_r \tau_t^r + (1 - \nu_r) \tau_t^u. \tag{14}$$

where ν_r denotes the fraction of constrained households, τ_t^r the level of lump sum taxes on a constrained household, and τ_t^u the level of lump sum taxes on an unconstrained household. The level of lump sum taxes τ_t^i on a household of type $i \in \{r, u\}$ is given by:[10]

$$\tau_t^i = \bar{\tau}^i + \zeta_b \left(b_{t-1} - \bar{b}\right). \tag{15}$$

[9] We assume that the government only purchases domestic goods, and no foreign goods.

[10] We allow for the steady state level of lump sum taxes $\bar{\tau}^r$ on constrained households to be negative, in which case τ_t^r denotes a transfer to constrained households, unless the level of government debt is too far above its target value.

By responding to deviations from the steady state level of government bonds $\bar{b}$, this rule ensures intertemporal solvency of the government budget constraint in the absence of sovereign default risk (Bohn, 1998).

We introduce sovereign default risk by assuming a stochastic maximum level of taxation above which the government (partially) defaults, like in (Schabert and van Wijnbergen, 2006) (see also Schabert and van Wijnbergen (2014); Corsetti et al. (2013)). This fiscal limit will be drawn each period from a generalised beta-distribution with parameters α_b, β_b and $\bar{b}_{max}$ following Corsetti et al. (2013).[11] As a result, we can write the ex ante probability of default p_t^{def} for a given level of government debt b_t by the following cumulative distribution function:

$$p_t^{def} = F_{\beta}\left(\frac{b_t}{4\bar{y}}\frac{1}{\bar{b}_{max}}; \alpha_b, \beta_b\right). \tag{16}$$

When the level of taxes τ_t necessary to service outstanding liabilities is above the stochastic maximum level of taxation, the sovereign reduces outstanding liabilities $\left(1 + r_t^b\right) q_{t-1}^b b_{t-1}$ by a factor $1 - \vartheta_t$.[12] The haircut ϑ_t depends on whether or not the required level of taxes τ_t surpasses the draw for the fiscal limit:

$$\vartheta_t = \begin{cases} \vartheta_{def} & \text{with probability } p_t^{def}; \\ 0 & \text{with probability } 1 - p_t^{def}. \end{cases} \tag{17}$$

The gains from the (partial) default $\tau_t^{tr} = \vartheta_t \left(1 + r_t^b\right) q_{t-1}^b b_{t-1}$ are effectively transferred to unconstrained households by reducing their level of lump sum taxes from τ_t^u to $\tilde{\tau}_t^u = \tau_t^u - \tau_t^{tr} / \left(1 - \nu_r\right)$.[13] As a result of reducing their lump sum taxes, total lump sum taxes (14) under the core tax policy (15) are reduced from τ_t to $\tilde{\tau}_t = \tau_t - \tau_t^{tr}$.[14] The ex post default government budget constraint is

[11] Note that $\bar{b}_{max}$ is a parameter determining the probability of default, and does not refer to a maximum level of debt. In both Schabert and van Wijnbergen (2006, 2014); Corsetti et al. (2013) and our current setup there is only a stochastic maximum level of taxation, while there is no limit to the amount of debt that the sovereign can issue.

[12] We assume bondholders know the government's inability to raise sufficient funds, and therefore voluntarily agree to a haircut on the coupon payment and a restructuring of the outstanding government bonds.

[13] We assume that only the level of lump sum taxes of unconstrained households are reduced, as these are responsible for recapitalizing financial intermediaries, see below.

[14] As in Corsetti et al. (2013), we assume that the haircut ϑ_{def} is constant across time and independent of the draw from the fiscal limit. Implicitly, we therefore assume that the value of ϑ_{def} is always large enough to bring the ex post default level of lump sum taxes $\tilde{\tau}_t$ below the maximum level of taxation.

then given by:

$$q_t^b b_t + \tilde{\tau}_t = p_t^h g_t + (1-\vartheta_t)\left(1+r_t^b\right) q_{t-1}^b b_{t-1}, \tag{18}$$

Substitution of $\tilde{\tau}_t = \tau_t - \vartheta_t \left(1+r_t^b\right) q_{t-1}^b b_{t-1}$ into equation (18) shows that the ex post default budget constraint collapses back to the no default case (13), as all default gains are effectively transferred to (unconstrained) households in the form of lower lump sum taxes, leaving no gains for the government budget. Sovereign default risk, however, affects the government budget constraint indirectly through bond pricing q_t^b, which incorporates expectations of a sovereign default.

Government purchases $\tilde{g}_t$ are driven by a standard autoregressive process because such a process provides a clear benchmark for the fiscal multiplier. Actual government spending g_t consists of regular purchases $\tilde{g}_t$ that follow an AR(1) process, and a response to a one-off financial crisis shock $\lambda_t^k > 0$, to be specified in subsection 4.3:

$$g_t \quad = \quad \tilde{g}_t + \varsigma(\lambda_{t-l}^k - \bar{\lambda}^k), \quad \varsigma \geq 0, \quad l \geq 0, \tag{19}$$

The parameter ς determines the size of the response to a financial crisis shock, while l denotes the lags with which the government responds to the shock.

4.2 Households

Following Gali et al. (2007), there are two household types: a fraction ν_r of households is constrained in the sense that they cannot save and consume their entire after-tax period income, which consists of labor income. A fraction $1-\nu_r$ is not constrained, and saves through deposits at financial intermediaries, an internationally traded asset, and through government bonds, the first two of which contain a risk premium. In line with Schmitt-Grohe and Uribe (2003), the risk-premium is decreasing in $\left(f_t - \bar{f}\right)/y_t^h$, where f_t denotes domestic households' stock of the internationally traded asset, which is in zero net supply across the Eurozone, and where y_t^h denotes domestic output. Government bonds are subject to quadratic adjustment costs (Gertler and Karadi, 2013). In addition to income from savings, unconstrained households also receive income from labor, and from the profits of financial and non-financial firms, as unconstrained households are the ultimate owners of all firms in the domestic economy.

Both household types aim to maximize the sum of current and future expected discounted utility subject to the budget constraint. They have the same utility function, which is subject to preference shocks, and which is separate in total consumption and labor, with habit formation in total consumption to capture realistic consumption dynamics (Christiano et al., 2005). The resulting first order conditions can be found in Appendix C.1.

Final consumption c_t^q of household type $q \in \{r, u\}$ is a constant elasticity of substitution function of final domestic goods $c_t^{q,h}$, which are acquired at a nominal price P_t^h, and final import goods $c_t^{q,f}$ which are acquired at a nominal price P_t^f. The optimization problem for households of type q is to minimize expenditures $P_t^h c_t^{q,h} + P_t^f c_t^{q,f}$, subject to the following technology:

$$c_t^q = \left[(1-\upsilon_c)^{\frac{1}{\eta_c}} \left(c_t^{q,h} \right)^{\frac{\eta_c - 1}{\eta_c}} + \upsilon_c^{\frac{1}{\eta_c}} \left(c_t^{q,f} \right)^{\frac{\eta_c - 1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c - 1}}, \tag{20}$$

The resulting first order conditions for $c_t^{q,h}$ and $c_t^{q,f}$ for households of type q are standard, and can be found in Appendix C.1. Observe from the production technology (20) that we assume that the degree of home bias υ_c and elasticity of substitution η_c are the same for both household types, as a result of which we can aggregate in a straightforward way to find aggregate consumption of final domestic goods $c_t^h = \nu_r c_t^{r,h} + (1-\nu_r) c_t^{u,h}$ and final import goods $c_t^f = \nu_r c_t^{r,f} + (1-\nu_r) c_t^{u,f}$.

Finally, it suffices to mention here that the aggregate consumption expenditures are given by $P_t c_t \equiv P_t^h c_t^h + P_t^f c_t^f$. The expression for the aggregate consumer price index P_t is standard, and can be found in Appendix C.1

4.3 Financial intermediaries

The financial sector is modeled like in Gertler and Karadi (2011). Intermediary $j \in [0, 1]$ purchases government bonds $s_{j,t}^b$ at a price q_t^b and obtains claims $s_{j,t}^k$ on intermediate goods producers at a price q_t^k. Intermediaries' assets $p_{j,t}$ are funded through net worth $n_{j,t}$ and deposits $d_{j,t}$. The intermediaries' balance sheet is given by:

$$p_{j,t} \equiv q_t^k s_{j,t}^k + q_t^b s_{j,t}^b = n_{j,t} + d_{j,t}. \tag{21}$$

Claims $s_{j,t}^k$ acquired in period t pay a net real return r_{t+1}^k at the beginning of period $t+1$. Bonds $s_{j,t}^b$ pay a net real return r_{t+1}^{b*} at the beginning of period $t+1$, which includes the impact of a possible sovereign default, and deposits pay a net real return r_{t+1}^d. The law of motion for net worth of intermediary j is given by:

$$n_{j,t+1} = (1+r_{t+1}^k)q_t^k s_{j,t}^k + (1+r_{t+1}^{b*})q_t^b s_{j,t}^b - (1+r_{t+1}^d)d_{j,t}. \tag{22}$$

Intermediary j maximizes expected discounted future profits. We follow Gertler and Karadi (2011) by assuming that there is a probability $1-\theta$ that the banker has to exit the financial sector next period, in which case the intermediary will bring net worth $n_{j,t+1}$ to its (unconstrained) household. Intermediary j is allowed to continue operating with a probability θ. The banker discounts these outcomes with unconstrained households' stochastic discount factor $\beta\Lambda_{t,t+1}^u$, as unconstrained households are the ultimate owners of the intermediaries. The banker's objective is then given by the following recursive optimization problem:

$$V_{j,t} = \max E_t \left\{ \beta\Lambda_{t,t+1}^u \left[(1-\theta)n_{j,t+1} + \theta V_{j,t+1}\right] \right\}, \tag{23}$$

where $V_{j,t}$ denotes intermediary j's continuation value. Following Gertler and Kiyotaki (2010); Gertler and Karadi (2011), however, intermediaries are subject to an incentive compatibility constraint that limits the size of their balance sheet:[15]

$$V_{j,t} \geq \lambda_t^k q_t^k s_{j,t}^k + \lambda_t^b q_t^b s_{j,t}^b \tag{24}$$

Therefore, intermediaries' optimization problem is to maximize their continuation value (23), sub-

[15] We will interchangeably refer to this constraint as the 'incentive compatibility constraint' and '(endogenous) leverage constraint'.

ject to the balance sheet constraint (21), the law of motion for net worth (22), and the incentive compatibility constraint (24). We show in Appendix C.3 that intermediaries' incentive compatibility constraint (24) can be written as:

$$q_t^k s_{j,t}^k + \frac{\lambda_t^b}{\lambda_t^k} q_t^b s_{j,t}^b \leq \phi_t n_{j,t}, \quad \text{with} \quad \phi_t = \frac{\eta_t}{\lambda_t^k - \nu_t^k} \tag{25}$$

where η_t denotes the shadow value of an additional unit of net worth, and ν_t^k of an additional unit of corporate loans (Gertler and Karadi, 2011). Therefore, ϕ_t can be interpreted as an endogenous leverage ratio which limits the size of intermediaries' (weighted) assets by the amount of net worth $n_{j,t}$.

The intuition for the leverage constraint is straightforward: a higher shadow value of corporate loans ν_t^k indicates a higher value from attracting an additional unit of corporate loans, increasing expected profits everything else equal, thereby reducing the incentive for bankers to divert assets. A higher value of η_t implies higher expected profits from an additional unit of net worth, therefore allowing a higher leverage ratio. A higher fraction λ_t^a implies bankers can divert more, inducing households to provide less deposits everything else equal. The result is a tightening of the leverage constraint.

A financial crisis is modeled as an unanticipated one-off increase in λ_t^k, after which it returns to its steady state value following a standard autoregressive process (Dedola et al., 2013), while $\lambda_t^b = \left(\bar{\lambda}_b / \bar{\lambda}_k\right) \lambda_t^k$.[16]

4.3.1 Aggregation of financial variables

At the end of each period, a fraction θ of current bankers will continue operating, and retain all net worth to finance the balance sheet of their intermediary. Aggregate net worth n_t^e of continuing bankers at the beginning of period t is equal to:

$$n_t^e = \theta \left[(r_t^k - r_t^d) q_{t-1}^k s_{t-1}^k + (r_t^{b*} - r_t^d) q_{t-1}^b s_{t-1}^b + (1 + r_t^d) n_{t-1} \right],$$

[16]Specifically, the process for λ_t^k is given by $\log\left(\lambda_t^k / \bar{\lambda}^k\right) = \rho_{\lambda^k} \log\left(\lambda_{t-1}^k / \bar{\lambda}^k\right)$.

which is obtained by substituting intermediaries' balance sheet constraint (21) into the law of motion (22), after which we aggregate across continuing intermediaries. A fraction $1-\theta$ of bankers will become a worker and bring their intermediary's net worth to the household. They are replaced by another household member, who receives a starting net worth. Aggregate starting net worth is $n_t^n = \chi p_{t-1}$ (Gertler and Karadi, 2011).

The gains from default, which are handed out to unconstrained households in randomized fashion (through lower lump sum taxes) and are therefore unanticipated, are used by households to recapitalize their respective financial intermediary. Therefore, the aggregate law of motion for net worth is unaffected by the occurrence of a sovereign default:[17]

$$n_t = \theta \left[(r_t^k - r_t^d) q_{t-1}^k s_{t-1}^k + (r_t^b - r_t^d) q_{t-1}^b s_{t-1}^b + (1 + r_t^d) n_{t-1} \right] + \chi p_{t-1}. \tag{26}$$

where r_t^{b*} has been replaced by r_t^b, which is the return in case of no default, see Appendix C.4.

However, because the default gains are handed out in randomized fashion, intermediaries neither anticipate the recap nor that aggregate net worth will be unaffected by default losses. Instead, potential losses from a future default translate into a lower bond price q_t^b, and will subsequently affect the equilibrium relative to the case with no sovereign default risk (Schabert and van Wijnbergen, 2006, 2014; Corsetti et al., 2013).

4.4 Production side

There exists a continuum of domestic intermediate goods producers indexed by $i \in [0, 1]$. Each of these producers employ a standard Cobb-Douglas production technology with capital share α and lognormal productivity. Intermediate goods producers acquire physical capital $k_{i,t-1}$ at a price q_{t-1}^k at the end of period $t-1$. They borrow from financial intermediaries against future profits, which we assume they can credibly commit (Gertler and Kiyotaki, 2010). Intermediate goods producers hire labor $h_{i,t}$ in a perfectly competitive market at wage rate w_t after realization of the shocks at the beginning of period t, among which there is a capital quality shock ξ_t that causes intermediate

[17] We do so because otherwise a sovereign default would introduce a discontinuity in intermediaries' net worth, which would substantially complicate solving the model without providing additional insights.

goods producers to produce with an 'effective' capital stock $\xi_t k_{i,t-1}$ (Gertler and Karadi, 2011). They sell intermediate goods at a relative price m_t (in terms of the domestic consumer price index), pay wages, and sell the used capital stock $(1-\delta)\xi_t k_{i,t}$ for a price q_t^k to the capital producers. The remaining revenues go to financial intermediaries, who receive a net real return r_t^k:

$$1 + r_t^k = \frac{\alpha m_t y_{i,t}/k_{i,t-1} + q_t^k (1-\delta)\,\xi_t}{q_{t-1}^k}. \tag{27}$$

Capital producers purchase the physical capital stock $(1-\delta)\,\xi_t k_{t-1}$ at the end of period t at a price q_t^k, and buy investment goods i_t, which is produced using a constant elasticity of substitution (CES) function of final domestic goods i_t^h acquired at a nominal price P_t^h, and final import goods i_t^f acquired at a nominal price P_t^f. Existing capital and investment are combined into new capital k_t, which is sold at a price q_t^k to intermediate goods producers who use it for production in period $t+1$. Capital producers face convex adjustment costs that are increasing in the deviation from the level of previous period investment i_{t-1}. Hence one unit of investment i_t will produce less than one unit of capital k_t.

Domestic retail firms purchase goods $(y_{i,t})$ from intermediate goods producers at a relative price m_t, convert these one-for-one into retail goods $(y_{f,t} = y_{i,t})$, which they sell to final good producers. Retail firms produce a differentiated retail good and operate in a monopolistically competitive market, which allows them to charge a mark-up over the input price m_t. Retail firms face staggered pricing like in Calvo (1983). There is partial inflation-indexation $\pi_{t-1}^{\gamma_p}$ for retail firms that are not allowed to adjust prices.

Final domestic good producers purchase retail goods from all domestic retail firms and employ a CES-production technology. Final good producers maximize profits in a perfectly competitive market where they take prices as given and decide period by period on the amount $y_{f,t}$ to purchase from each retail firm. A more elaborate description of the production sector and the resulting first order conditions can be found in Appendix C.5.

4.5 Market clearing, trade balance, and net foreign asset position

In equilibrium, aggregate capital (k_t) is equal to the aggregate number of corporate securities owned by financial intermediaries (s_t^k), while the number of government bonds (b_t) must be equal to the sum of the bonds owned by the financial sector (s_t^b) and the household sector ($s_t^{b,h}$):

$$k_t = s_t^k, \tag{28}$$

$$b_t = s_t^b + s_t^{b,h}. \tag{29}$$

Clearing in the market for final import goods requires that demand equals supply y_t^f:

$$y_t^f = c_t^f + i_t^f, \tag{30}$$

Clearing in the market for final domestic goods requires that aggregate demand equals aggregate supply y_t^h:

$$y_t^h = c_t^h + i_t^h + g_t + x_t, \tag{31}$$

where c_t^h represents domestic demand for consumption purposes, i_t^h domestic demand for investment purposes, g_t the amount of spending by the government, and x_t the demand by the export sector.[18]

5 Calibration & estimation

We employ a mix of calibration and estimation with Bayesian methods to match the Spanish economy using data from Eurostat, the publicly available database used by Burriel et al. (2010)

[18]Note that the aggregate resource constraint does not feature the investment adjustment costs as it does in Gertler and Karadi (2011). The reason is that in our setup the investment goods are purchased first by capital goods producers, and the adjustment costs are only incurred afterwards during the conversion from investment goods into new capital. As a result, one unit of investment goods provides less than one new unit of capital. Therefore, the adjustment costs show up in the law of motion for capital rather than the aggregate resource constraint. This differs from Gertler and Karadi (2011), where one unit of investment delivers one new unit of capital. As such, the adjustment costs show up in their aggregate resource constraint.

and time series obtained from the Bank of Spain.[19] The frequency of our model is quarterly. We discuss the details of the calibration choices and especially of the Bayesian estimation setup and results in exhaustive detail in Appendix D, here we just sketch the main elements of the identification strategy.

Our strategy for identifying parameter values consists of two stages. First we partially calibrate the model by either taking parameter values that are standard in the macroeconomic literature, or by targeting first order moments such as the steady state labor supply. A key target is the (weighted) leverage ratio $\bar{\phi}$ in equation (25), which we determine by employing data from the Bank of Spain. Specifically, we construct a time series of OMFIs' total assets over capital & reserves, after which we calculate the mean over our estimation period, see Appendix D.2.[20] We follow Gertler and Karadi (2013) and divide the resulting number by 2 to obtain $\bar{\phi} = 6.48$.[21] Another calibration target is that we set $\lambda_t^b/\lambda_t^k = 0.5$ at all times following Gertler and Karadi (2013).

In the second stage we estimate the remaining deep parameters using Bayesian techniques. To do so, we perform a first order approximation around the non-stochastic steady state, and solve the model (Adjemian et al., 2011). We employ the following quarterly time series for the estimation period 2003Q1-2010Q4: real GDP per capita, real consumption per capita, real government spending per capita, real exports per capita, real imports per capita, gross inflation of the Spanish consumer price index, the real wage rate, hours worked per capita, the Spanish Non transferable three-month deposit rate, and the interest rate on loans to non-financial corporations (NFC).[22] We follow Burriel et al. (2010) and do not use investment per capita in our estimation. We estimate the model version without sovereign default risk, as sovereign risk was a relatively minor concern during the estimation period, see also Bocola (2016), who follows a similar estimation strategy in

[19] The database of Burriel et al. (2010) was downloaded in May 2018 through the following link: http://www.sepg.pap.hacienda.gob.es/sitios/sepg/en-GB/Presupuestos/Documentacion/Paginas/BasedatosmodeloREMS.aspx. An explanation of the computation of the time series can be found in Bosc et al. (2007).

[20] "OMFIs" is an abbreviation for "Other Monetary Financial Institutions", which are credit institutions and specialised lending institutions with access to the ECB balance sheet.

[21] Gertler and Karadi (2013) explain that the corporate loans in their (and our) model are more equity-like, as the ex post return is affected by productivity and capital quality shocks. Therefore, fluctuations in financial intermediaries' net worth will be overstated compared with a model where banks provide fixed principal credit. This motivates Gertler and Karadi (2013) to use a steady state leverage ratio that is half its counterpart in the data.

[22] The reason for starting the sample in 2003Q1 is that no data on the interest rate on NFC loans is available prior to 2003Q1.

this respect.[23] We calculate growth rates for the above-mentioned variables (except for interest rates and inflation) and introduce labour-augmenting technology growth into the model, which introduces a trend in the quantity variables of our model (Burriel et al., 2010). Doing so allows us to estimate the model without filtering the data beforehand, see Appendix C.13 for details.

Employing a time series for the interest rate on NFC loans allows us to identify the (steady state) diversion rate on corporate loans $\bar{\lambda}_k$ in the Bayesian estimation, for which we find the posterior mean to be equal to 0.64, see Appendix D.3. Given that we employ the posterior mean in our simulations, we end up setting $\bar{\lambda}_b = 0.32$. These values for $\bar{\lambda}_k$ and $\bar{\lambda}_b$ are higher than in the literature (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011, 2013; Karadi and Nakov, 2021), where $\bar{\lambda}_k$ and $\bar{\lambda}_b$ are typically below 0.4 and 0.2, respectively. Therefore, financial frictions seem to be relatively important for the Spanish economy.

We calibrate the remaining parameters related to sovereign default risk. To do so, we set the steady state default probability equal to 50 quarterly basis points in the model version with sovereign risk, which amounts to an annual risk of default of 2% per year (Schabert and van Wijnbergen, 2014). This is also in line with 5-year CDS spreads on Spanish government bonds at the end of 2010. We set the steady state derivative of the default probability (16) equal to 0.2. We will see below that this results in an increase in sovereign risk by 5 basis points in response to an increase in government debt of 0.5 % of quarterly steady state output, which implies a default elasticity of 0.003, see Appendix D.2. Such a number is small with respect to Schabert and van Wijnbergen (2014), for example, who work with a default elasticity of 0.01.

For brevity we relegate the specific details of this calibration/estimation procedure and an extensive discussion of the resulting values to Appendix D.

6 The fiscal multiplier, banking fragility and default risk

We first use the full model, with long-term debt to analyse the impact of a fiscal stimulus package in response to a financial crisis. To that end we simulate an unanticipated financial crisis and, with

[23]The model version without sovereign default risk is obtained by replacing equation (16) by $p_t^{def} = 0$.

one year delay, the fiscal stimulus package. The crisis is initiated by an unanticipated, one-time increase in the diversion rates for both corporate loans and sovereign debt, λ_t^k and λ_t^b respectively (Dedola et al., 2013). As there was a substantial spread between the yield on Spanish government debt and the deposit rate during the European sovereign debt crisis, we set $\lambda_t^b > 0$ and shock both rates in equal proportion (i.e. their ratio remains the same).[24] We then analyze the output response to an expansionary shift in government expenditures in response to the financial crisis shock λ_t^k to clarify the impact of a weakly capitalized banking system on the effectiveness of such a stimulus program in Section 6.1. After presenting our core results and the associated dynamic multiplier patterns, we decompose the overall weakening of the fiscal multiplier in Section 6.2 by trimming the model down step by step, so as to find out what is the most significant driver behind this weakening. We also highlight the degree to which uncertainty about the estimates of the deep parameters affect the strength of our conclusions regarding the impact of sovereign default risk.

This setup gives rise to two questions which we analyse in Sections 6.3 and 6.4, respectively. First we analyse the impact of the one year implementation delay in Section 6.3. Such a delay between announcement and implementation is rarely if ever modeled, but as we will show has a significant (and possibly surprising) impact on the results, and in particular on the size of the cumulative multiplier. Moreover there is ample evidence that such delays are substantial in practice (Beetsma et al. (2021)). The second issue is whether the size of the financial crisis matters for the impact of fiscal policy, a logical question since the 2007/8 crisis was a large one by post-war standards. We discuss this non-linearity in Section 6.4, where we also look at the impact of the size of the fiscal stimulus. To properly capture these potential nonlinear effects, we solve the model using Dynare's nonlinear perfect foresight solver (Adjemian et al., 2011). For comparability, we employ the perfect foresight solver throughout Section 6.

[24]Note that λ_t^k and λ_t^b are not legal capital requirements, in which case λ_t^b should be equal to zero according to Basel III regulations, but rather constraints imposed by depositors on financial intermediaries within a market transaction. The incentive compatibility constraint captures in reduced form financial frictions that give rise to a return difference between assets and deposit funding.

6.1 The effects of a stimulus package in the presence of financial fragility, long-term debt and sovereign default risk

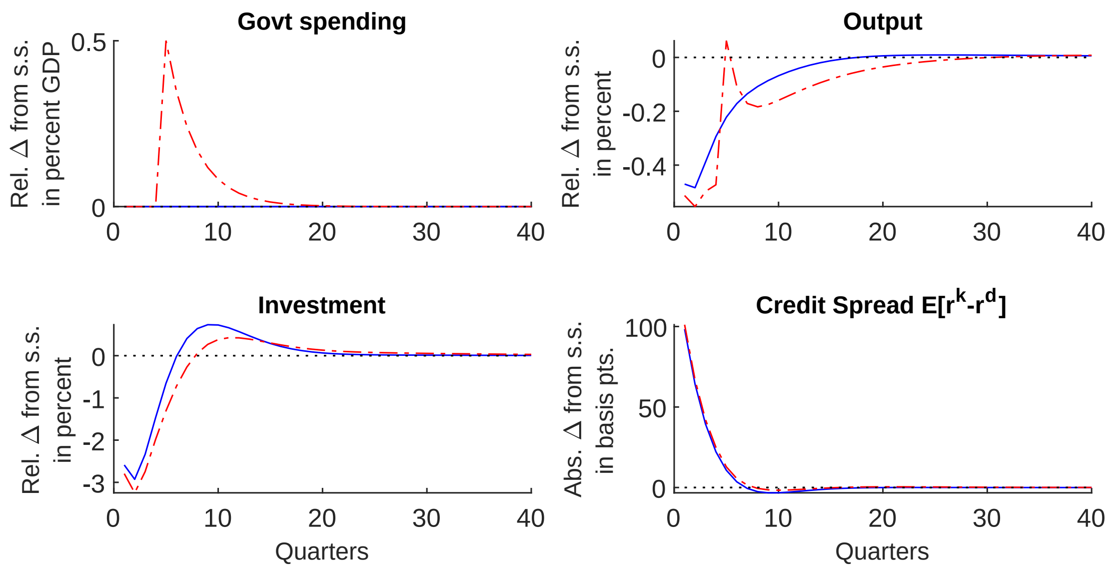
We start by discussing the impact of fiscal stimuli after a financial shock decreases banks' capitalization. Specifically, Figure 5 presents the model response to a one-time 5% increase in the diversion rate of corporate loans λ_t^k , as a result of which the diversion rate on government bonds also increases since $\lambda_t^b = \left(\bar{\lambda}_b/\bar{\lambda}_k\right) \lambda_t^k$. The financial shock comes as an unanticipated "MIT" shock, with persistence parameter $\rho_{\lambda_k} = 0.7$. This implies that output is back at the pre-crisis level of output after 20 quarters, or 5 years. We then compare a financial shock without a fiscal stimulus (blue, solid) and a fiscal stimulus that is announced on impact but implemented four quarters later (red, slotted). We introduce an implementation lag of four quarters because this represents a regular budget cycle. According to some, this is still a conservative choice: Beetsma et al. (2021) find that the crosscountry average horizon of fiscal consolidation plans ranges between 1.3 and 2.3 years. Beetsma et al. (2021), however, look at fiscal consolidation plans that aim to increase the sustainability of government finances, a process in which spending cuts are not front-loaded but typically spread out over longer horizons to mitigate the contractionary impact they might have on the economy. Our fiscal stimulus package, however, revolves around expanding government spending to stimulate the economy at a moment where it is in recession, and is therefore likely to be implemented faster. Therefore, we think an implementation lag of 2-4 quarters is a more reasonable estimate for our simulations. In Section 6.3, we investigate in more detail the extent to which the implementation lag affects the impact of the fiscal stimulus.

Consider the case without a fiscal stimulus (blue, solid line in Figure 5). In that case, the increase in the diversion rates for corporate loans and sovereign debt immediately leads to a tightening of intermediaries' leverage constraint (25). As a consequence, lending to intermediate goods producers is reduced and the credit spread increases. Consequently the demand for capital falls and the price of capital falls commensurately. And that causes a decline in intermediaries' net worth (not shown) and a further tightening of their leverage constraint and subsequent increase in the credit spread. The tightening of intermediaries' leverage constraints and a higher diversion rate for government bonds $\lambda_t^b = \left(\bar{\lambda}_b/\bar{\lambda}_k\right)\lambda_t^k$ lead to a drop in bond prices, which in turn further reduces the value of intermediaries' existing holdings of government bonds. Net worth falls further, and an additional tightening of intermediaries' leverage constraints leads to a second round of interest rate increases. The subsequent rounds of balance sheet deterioration cause the credit spread to increase by 100 basis points, investment to drop by almost 3%, and output by almost 0.5%.

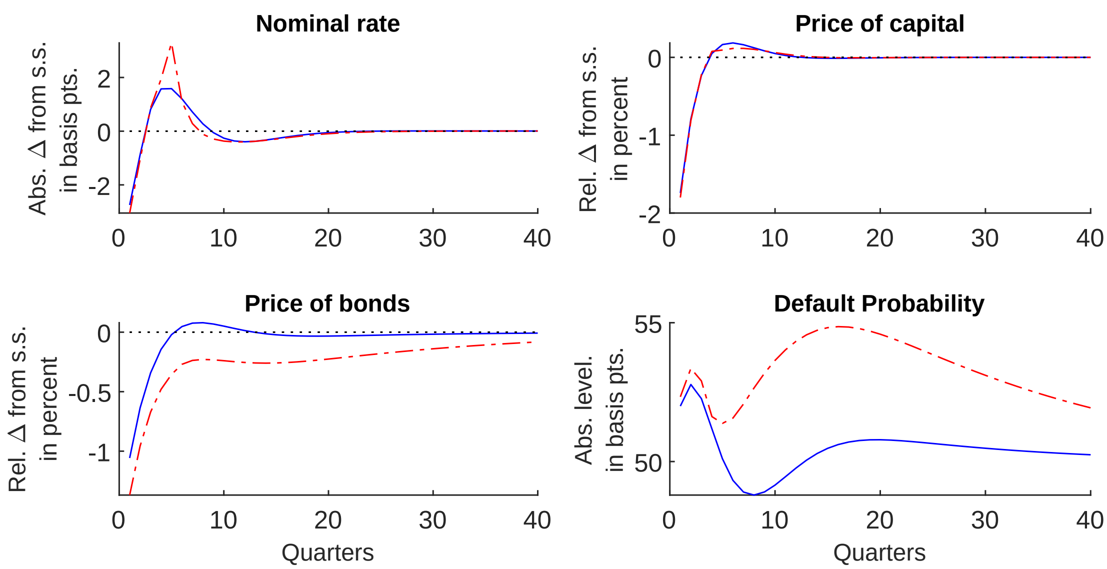
Now consider the impact of a deficit-financed stimulus package that is announced immediately at the start of the financial crisis, but implemented 4 quarters later (red, slotted line in Figure 5). Due to the forward-looking nature of financial intermediaries, they anticipate the associated future increase in government debt. Consequently, they anticipate a future bond price drop because of i) a larger supply of bonds and ii) an increase in sovereign default risk because of the higher debt levels. The effects of this anticipated future price drop are propagated through a bond price that is already lowered *before* the implementation of the fiscal stimulus package begins, which reduces the value of intermediaries' existing holdings of government bonds. The subsequent reduction in net worth today (not shown) tightens the incentive compatibility constraint (25) of financial intermediaries and makes them more balance-sheet-constrained. To sum up: the anticipation of having to finance riskier debt in the *future* makes financial intermediaries *immediately* more balance-sheet-constrained. The bond price falls by almost 0.5% further compared to the case of no additional government policy, see Figure 5.

A tightening of the incentive compatibility constraint today because of additional capital losses on government bonds also leads to higher interest rates on corporate loans and an immediate reduction in lending to the real economy. So the anticipated *future* debt issue leads to a drop in lending to the real economy *today*, with a persistent fall in investment of more than 0.5% of steady state investment with respect to the no stimulus case, and an additional decline in the capital stock (not shown) as a consequence. The anticipation by households of higher future taxes to eventually pay off the additional debt incurred to finance the future fiscal stimulus leads to a drop in consumption today (not shown), despite an increase in spending by constrained households when the stimulus is implemented. However, the effect from unconstrained households reducing their consumption level with respect to no stimulus dominates. In addition to the fall in consumption

Financial crisis, sovereign default risk, long-term bonds: no policy vs. delayed government spending



(a)



(b)

Figure 5: Plot of the impulse response functions comparing no additional policy (blue, solid) and a delayed fiscal stimulus (red, slotted) in response to a financial crisis. The delayed stimulus is announced as the crisis hits, but implemented four quarters later. The size of the stimulus equals 0.5% of quarterly steady state output and is financed through additional debt issue. Bonds have a duration of 20 quarters, and are subject to sovereign default risk. The financial crisis is initiated through a MIT-shock to the diversion rate of corporate loans of 5 percent relative to the steady state.

and investment, the fiscal stimulus increases inflation with respect to the no stimulus case, as a result of which the real exchange rate appreciates. Therefore, exports decrease with respect to the no stimulus case (not shown), and the reduction in consumption, investment, and exports cause a fall in output with respect to the case of no fiscal stimulus. Against all that is the positive direct impact of the stimulus package at the time it is actually introduced (cf the positive peak in the top two panels of Figure 5).

Figure 6 shows the net impact over time of all these (at times conflicting) effects on output. The anticipation of future tightening leads to tightening conditions today and thus a negative output effect in the months preceding the actual start of the stimulus program (cf the red slotted line in Figure 6). The net impact turns positive at the start of the implementation, but the negative channels dominate again within a year after the start of the stimulus. The subsequent period of negative impact in the end fizzles out to reach zero after 40 quarters.

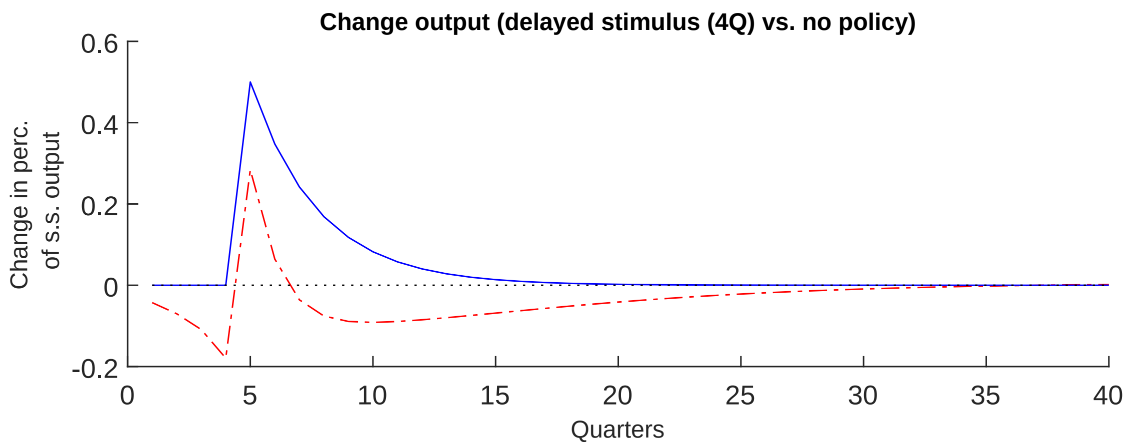


Figure 6: The red slotted line is the fiscal multiplier: the difference between output-with-stimulus and output-without-stimulus in each quarter. This represents our base case, with sovereign risk, long-term debt (duration of 20 quarters) and deficit-financed stimulus package. The solid (blue) line represents the fiscal stimulus itself, expressed as a percentage of quarterly steady state output.

The sequence of periods with negative, positive and again negative impact raises the question of whether the cumulative policy impact on output can actually turn negative. To answer that question, we calculate the *cumulative discounted multiplier* (Mountford and Uhlig, 2009). Denoting a variable from the stimulus scenario x^{st} and from the no-policy-response case x^{np}, the cumulative discounted multiplier is defined as:

$$\mu_D = \frac{\sum_j \beta^j (y^{st}_{t+j} - y^{np}_{t+j})}{\sum_j \beta^j (g^{st}_{t+j} - g^{np}_{t+j})}. \tag{32}$$

This multiplier can also be interpreted graphically from Figure 6: the numerator of μ_D is equal to the cumulative area between the red slotted line and the zero line, with areas below zero having a minus sign, while the denominator is the difference between the solid blue line (which represents additional government spending) and the horizontal zero axis. The numerical base case results, with long-term debt and sovereign risk, are summarized in row 3 of Table 1 below, which displays the cumulative discounted multiplier for several cases, of which we only consider the third in this section. The row-3 result indicates that the negative effects stemming from deteriorating balance sheets on investment and output eventually offset all the direct positive effects to such an extent that the cumulative multiplier μ_D turns negative at -0.65: the fiscal stimulus eventually becomes self-defeating and thus completely ineffective in the face of tightening balance sheet constraints in the financial intermediary sector (i.e. the banks)!

6.2 Dissecting & quantifying the various amplification mechanisms

A cumulative multiplier that is negative is surely a startling result. In this section we dissect the contributions of the different channels to the all-in overall cumulative multiplier to show how this result is built up (cf again Table 1). We do so by first calculating a base case in which (A) the stimulus is financed by one-period government bonds (so there are no capital losses on bonds) and no sovereign risk (row 1); then we introduce long-term government bonds while still assuming away sovereign default risk (row 2); and finally we add sovereign default risk back in (row 3). Row 3 corresponds to the base case analyzed in the previous section.

Without sovereign risk and with just short-term bonds, the cumulative multiplier turns positive: μ_D is 0.25 when sovereign risk is absent, and the maturity of government bonds one period (row 1). Financing the stimulus by issuing long maturity government bonds, from one quarter as assumed

Stimulus policy	Cumulative multiplier
1: Short-term debt, no sovereign risk	0.25
2: Long-term debt, no sovereign risk	0.15
3: Long-term debt, sovereign default risk	-0.65

Table 1: Table displaying the discounted cumulative dynamic multiplier for listed scenarios for a fiscal stimulus in response to a financial crisis initiated by an MIT-shock of 5% to the diversion rate on corporate loans and a fiscal stimulus of 0.5% of quarterly output. All stimuli are debt-financed.

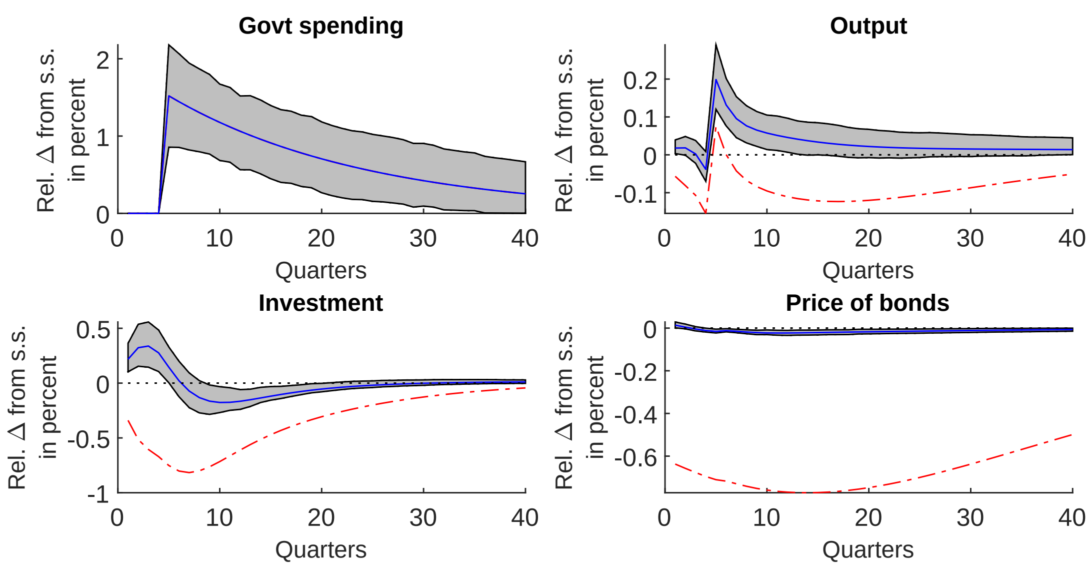
in row 1 to an average duration of 20 quarters in row 2, decreases the cumulative multiplier: μ_D falls from 0.25 to 0.15, which explains 11% of the total decline of 90 basis points (from 0.25 to -0.65). An increase in the supply of bonds leads to intermediaries demanding a higher (expected) return on bonds, which is achieved through a drop in the bond price. The resulting capital losses on intermediaries' existing bond holdings further tighten leverage constraints. As a result there is an additional drop in lending, aggregate investment and output compared with the case where government bonds have a maturity of one quarter. This effect corresponds to the third term in expression (9), which in turn is driven by the first two terms of expression (8).

Finally we add back in sovereign default risk: compare row 2 with row 3 in Table 1. The additional drop in μ_D from 0.15 to -0.65 (or about 89% of the total decline of 90 basis points) is caused by larger capital losses on existing government bond holdings: a deficit-financed stimulus not only increases the supply of bonds, but also leads to higher sovereign default risk. This effect is captured by the third term of expression (8).

But are the results with sovereign risk *significantly* different from those without sovereign risk given the precision with which we have estimated the model parameters? To answer that question we use the posterior distributions generated by our Bayesian estimation to assess whether the impulse response functions from the model version with sovereign default risk lie within the uncertainty bands around the comparable IRFs generated from the model version without sovereign risk (cf Figure 7). Note that we show the response to an isolated government spending shock without prior financial crisis, as λ_t^k and λ_t^b were constant in the Bayesian estimation.

The figure clearly shows that the impulse response functions for bond prices, investment, and output of the model version with sovereign default risk lie outside the uncertainty bands around

Gov't spending shock, long-term bonds: no default vs. default



(a)

Figure 7: Plot of the impulse response functions for a government spending shock of one standard deviation in the absence of sovereign default risk (blue, solid), and with sovereign default risk (red, slotted). The grey areas denote the 90% HPD interval from the Bayesian estimation of the model version without sovereign risk. In contrast to the rest of Section 6, the impulse response functions are generated using a first order perturbation around the steady state.

the IRFs generated with the model version without sovereign risk. These results confirm that the combination of long-term government bonds and sovereign default risk is key to the deterioration of the fiscal multiplier and that this effect is statistically significant.

We delve deeper into the impact of debt maturity on the effectiveness of fiscal stimuli in Figure 8 below: it shows the decline in the cumulative dynamic multiplier μ_D as a function of average debt maturity of existing and new debt (we recalculate ρ into the more intuitive but equivalent metric of average duration, measured in quarters, see Appendix C.2). The blue, solid line depicts the case with no sovereign default risk, whereas the red, slotted line depicts the case with sovereign default risk.

Discounted cumulative multiplier as a function of duration

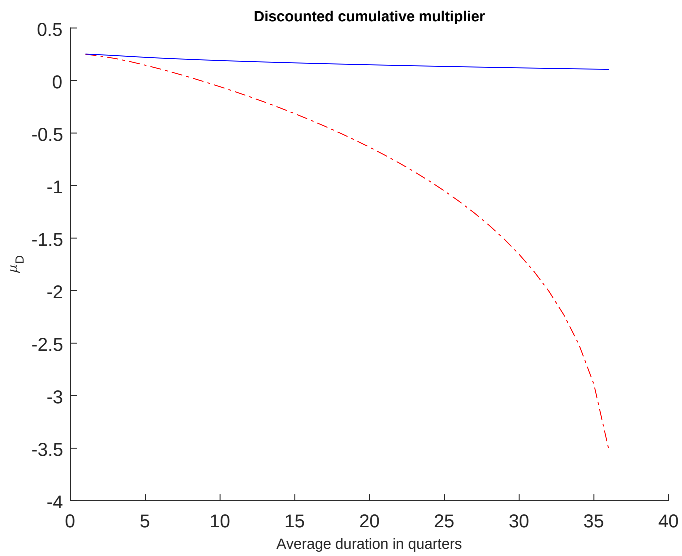


Figure 8: Average duration of government debt (horizontal axis) vs. discounted cumulative multiplier. The blue, solid line depicts the case with no sovereign default risk, whereas the red, slotted line depicts the case with sovereign default risk.

The figure clearly shows that the longer the average duration and thus the larger the capital losses that arise from higher interest rates and higher sovereign default risk, the smaller the cumulative multiplier. Quantitatively sovereign risk is the main cause for the appearance of cumulative negative multipliers, but the figure also shows that it only does so to a significant extent in the presence of longer maturity debt, at low maturity the effect of sovereign default risk is negligible for short-term bonds, where both multipliers (with and without sovereign risk) are equal to 0.25.

6.3 Timing matters: Implementation delays and the multiplier

In the previous sections we studied a fiscal stimulus that was announced upon arrival of the financial crisis shock, but implemented four quarters later. Modeling a fiscal stimulus in this way deviates from most of the literature, where announcement and implementation typically coincide but Beetsma et al. (2015) show that in reality there are substantial delays in implementation of fiscal policy measures. Given the discrepancy between most of the academic literature and actual practice, the question arises to what extent such an implementation delay affects our results.

In Figure 9 below we compare the impact of an immediate stimulus for which announcement and implementation coincide (blue, solid line), and the delayed stimulus (implementation 4 quarters after it is announced) from the previous section (red, slotted line). The figure shows starkly different patterns of in particular the output response to the two stimuli, although the stimuli are equal in magnitude and only differ in their timing.

First, note that output under a delayed stimulus is always below that under an immediate stimulus, except for the periods in which the delayed stimulus is actually implemented. To better understand the difference between the two stimuli, we plot the individual components of the demand for domestic final output from the aggregate resource constraint (31). In addition, we display a measure of competitiveness, which we define as the price of imports over the price of exports ("Terms of trade").

Figure 9 shows that the financial crisis shock causes domestic demand and hence total output to decrease initially. As a result domestic prices decrease with respect to foreign prices, and competitiveness improves. As a consequence there is a switch from foreign goods to domestic goods for both consumption and investment, and exports increase. An immediate stimulus, however, increases domestic demand on impact with respect to a delayed stimulus, as a result of which competitiveness goes down, everything else equal, and demand for domestic final goods for consumption, investment, and exports decrease with respect to a delayed stimulus in the first four quarters of the crisis

Financial crises, sovereign default risk, long-term bonds: immediate vs. delayed government spending

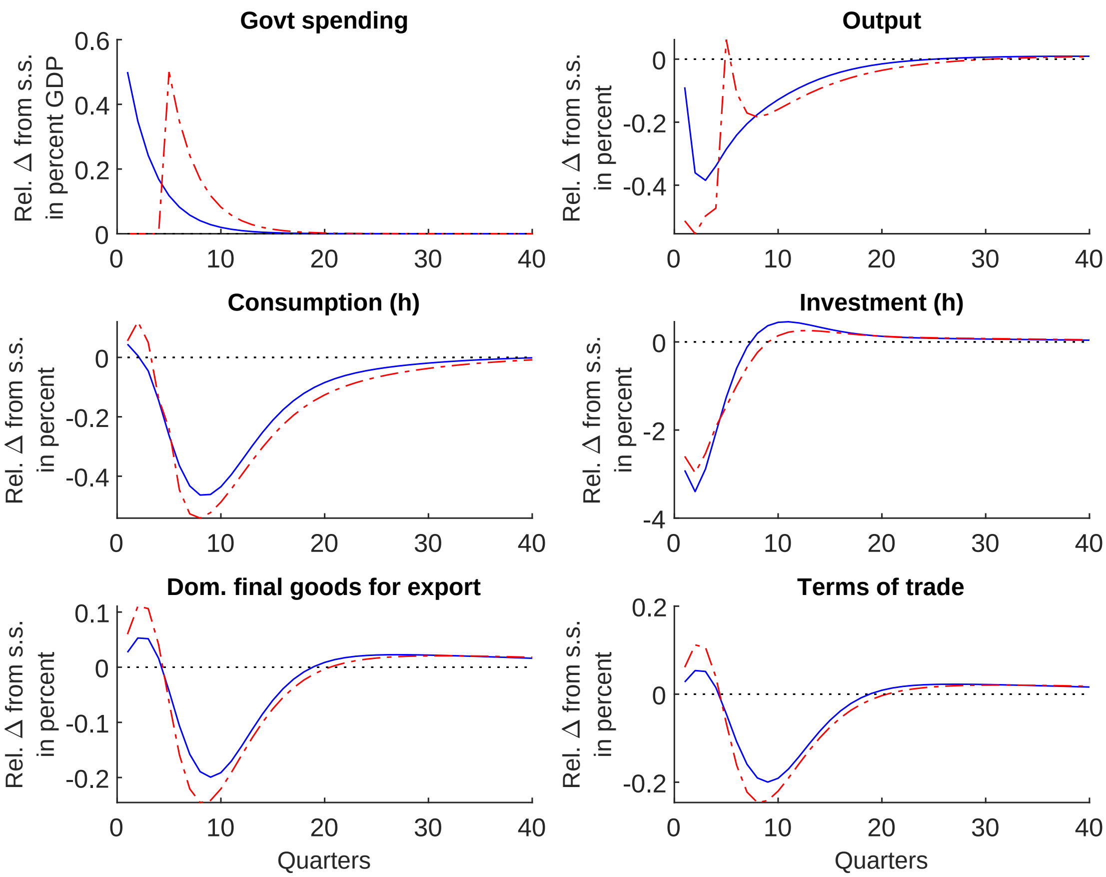


Figure 9: Plot of the impulse response functions comparing an immediate fiscal stimulus (blue, solid) and a delayed fiscal stimulus (red, slotted) in response to a financial crisis. The delayed stimulus is announced as the crisis hits, but implemented four quarters later. The size of the stimulus equals 0.5% of quarterly steady state output and is financed through additional debt issue. Bonds have a duration of 20 quarters, and are subject to sovereign default risk. The financial crisis is initiated through an MIT-shock to the diversion rate of corporate loans of 5 percent relative to the steady state.

(before the delayed stimulus is implemented). But the direct effect from additional government purchases offsets these negative impact effects and causes output under an immediate stimulus to be above that under a delayed stimulus in the first four quarters. For the same reason the reverse happens in the second set of four quarters, when the immediate stimulus is over but the delayed stimulus is implemented.

Once the delayed stimulus is implemented however, the terms of trade under a delayed stimulus deteriorate with respect to those under the immediate stimulus, as competitiveness goes down and demand for domestic final goods for consumption, investment, and exports falls. Furthermore, it takes a long time before the difference in competitiveness between the two stimuli disappears because the delayed stimulus is implemented when the terms of trade are already deteriorating. This contrasts with the immediate stimulus, which is implemented at the moment the terms of trade are improving as a result of the financial crisis. The net outcome is that output under an immediate stimulus is always above output under a delayed stimulus, except in the quarters where the delayed stimulus is implemented.

Figure 10 shows the (cumulative) multiplier μ_D explicitly as a function of the length of the implementation delay. Clearly the larger the delay between announcement and implementation, the lower the cumulative multiplier, which decreases from -0.14 for an immediate stimulus to -0.65 for a stimulus that is implemented four quarters after announcement, which marks a decrease in the multiplier of no less than 0.51 percentage points. So the delay with which the stimulus is implemented has a substantial effect on the eventual (and the cumulative) size of the multiplier. The result that the implementation lag can be important is in line with the empirical analysis of Mertens and Ravn (2012), who show that pre-announced tax cuts that have not yet been implemented have a contractionary effect on output, investment, and hours worked. House and Shapiro (2006) find similar results within a dynamic general equilibrium model that investigates the US tax rate changes of 2001.

In Table 2 we repeat the deconstructionist exercise of Section 6.2. Several conclusions emerge from the table. First, comparison of the first and the second column clearly shows the differential impact of a stimulus measure that is immediately executed and one whose implementation is de-

Discounted cumulative multiplier vs. implementation lag

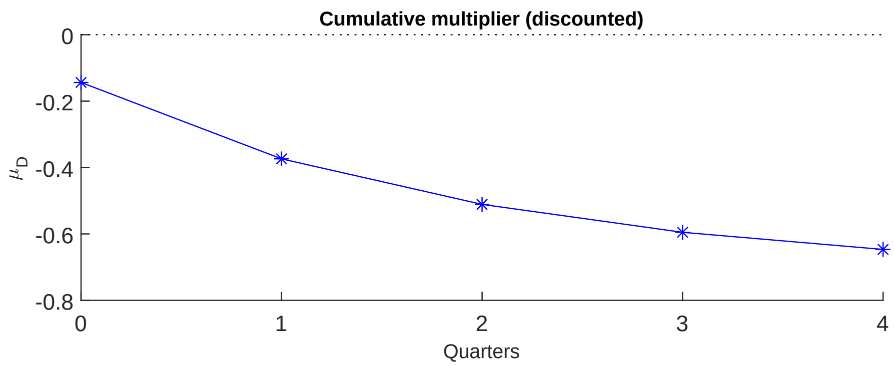


Figure 10: Figure displaying the cumulative discounted multiplier μ_D versus the lag between the announcement of the deficit-financed fiscal stimulus and its implementation. The size of the stimulus is 0.5% of quarterly output. Bonds have an average duration of 20 quarters, and are subject to sovereign default risk. The financial crisis is initiated through an MIT-shock to the diversion rate of corporate loans of 5 percent relative to the steady state.

layed by 4 quarters after the announcement. For all external debt configurations the cumulative multipliers of the delayed stimulus are between thirty and fifty basis points lower than the multipliers following an immediately executed stimulus package. The negative multiplier also emerges for the immediately implemented stimulus (at -0.14 instead of -0.65, but still negative). The second conclusion confirms earlier results also: extending debt maturity has a significantly negative impact on the multiplier, and adding in sovereign risk does that also but substantially more so.

Stimulus policy	Immediate multiplier	Delayed multiplier
1: Short-term debt, no sovereign risk	0.64	0.25
2: Long-term debt, no sovereign risk	0.47	0.15
3: Long-term debt, sovereign default risk	-0.14	-0.65

Table 2: Table displaying the discounted cumulative dynamic multiplier for listed scenarios for a fiscal stimulus of 0.5% of quarterly output in response to a financial crisis. All stimuli are debt-financed. Short-term debt consists of one-period bonds (scenario 1), while long-term debt are bonds with an average duration of 20 quarters (scenario 2 and 3).

6.4 The nonlinear impact of the degree of undercapitalization & the size of the fiscal stimulus

We now turn to the next question, does it matter for the impact of fiscal stimuli whether they are in response to a small or a large financial crisis? The idea here is that the impact of a binding incentive compatibility constraint (25) is likely to be nonlinear, as a result of which larger decreases in financial intermediaries' net worth lead to a disproportionally larger credit contraction. Similarly, larger fiscal stimuli might lead to disproportionally larger capital losses and crowding out of credit provision, which adds an additional non-linearity because the size of the multiplier depends on these capital losses. Are larger fiscal stimuli less effective than smaller stimuli (keep in mind that μ_D is scale independent by design). To investigate to what extent this is the case, we first calculate in Table 3 the cumulative discounted multiplier (32) for financial crisis shocks λ_t^k that range from 2% to 15% of steady state on impact, while keeping the size of the fiscal stimulus fixed at 0.5% of quarterly output. Second, we calculate in Table 4 the cumulative discounted multiplier for fiscal stimuli ranging from 0.5% to 4% of quarterly output on impact, while keeping the financial crisis shock equal to 5% on impact.

We see from Table 3 that the size of the multiplier decreases with the size of the financial crisis shock λ_t^k, both for immediate and delayed stimuli. However, the quantitative difference is relatively small: the size of the multiplier decreases by 0.06 percentage points for an immediate stimulus (from -0.13 to -0.19), and by 0.07 percentage points for a delayed stimulus (from -0.63 to -0.70). Therefore, the size of the financial crisis shock does not seem to be quantitatively important for the size of the multiplier. Also observe that the size of the multiplier continues to be smaller for delayed stimuli than for immediate stimuli when the size of the financial crisis shock increases. Therefore, if a government decides to implement a fiscal stimulus in the middle of a combined financial and sovereign debt crisis, the government has to act fast, and leave as little time as possible between the announcement of a stimulus and its actual implementation, which to our knowledge is a new result. Existing DSGE models with frictions would most likely produce the same result, but the widely accepted practice of solving a first order approximation of the model makes this sort of non-linearities disappear.

Cumulative multipliers for different size of the financial crisis shock λ_t^k

Impact change in λ_t^k	Immediate multiplier	Delayed multiplier
2%	-0.13	-0.63
5% (base case)	-0.14	-0.65
8%	-0.16	-0.66
10%	-0.16	-0.67
15%	-0.19	-0.70

Table 3: Discounted cumulative dynamic multipliers for different size of impact shock to λ_t^k for a fiscal stimulus that is equal to 0.5% of quarterly output. The reported multipliers correspond to the model version with long-term government debt and sovereign default risk. The delayed stimulus features an implementation lag of 4 quarters.

Things are substantially different when we investigate how the size of the fiscal stimulus affects the multiplier in Table 4. Here, we see a substantial decrease when increasing the impact size of the fiscal stimulus from 0.5% to 4% of quarterly output. The multiplier falls by 0.29 percentage points for an immediate stimulus (from -0.14 to -0.43), and by 0.58 percentage points for a delayed stimulus (from -0.65 to -1.23). Apparently, larger capital losses on existing bonds from the government supplying more bonds decrease credit provision to the real economy disproportionally. As a consequence larger stimuli tend to become less effective.

We can draw two conclusions from Table 4. First, the conclusion from Table 3, that immediate stimuli are more effective than delayed stimuli, carries over to Table 4, and holds for both small and large fiscal stimuli. A second conclusion is that the size of the multiplier decreases by more for delayed stimuli than for immediate stimuli when increasing the size of the fiscal stimulus (0.58 percentage points vs 0.29 percentage points). Therefore, in an environment with undercapitalized financial intermediaries with risky long-term government debt on their balance sheets, the government should keep the size of the fiscal stimulus relatively small, and focus on fast implementation.

Cumulative multipliers for different size of the fiscal stimulus

Impact change in g_t	Immediate multiplier	Delayed multiplier
0.5% (base case)	-0.14	-0.65
1%	-0.17	-0.69
2%	-0.23	-0.81
3%	-0.31	-0.97
4%	-0.43	-1.23

Table 4: Discounted cumulative dynamic multipliers for different size of fiscal stimulus in response to a financial crisis that is initiated by a 5% increase in the diversion rate λ_t^k on impact. The reported multipliers correspond to the model version with long-term government debt and sovereign default risk. The delayed stimulus features an implementation lag of 4 quarters.

7 Discussion & robustness

In the absence of sovereign risk or long-term government debt, we find that our results are in line with the literature, as the cumulative (discounted) multiplier is positive, see Gornicka et al. (2020) among others. This changes, however, for the model version that features both long-term government bonds *and* sovereign risk, which decreases the multiplier by at least 0.60 percentage points. The difference with Gornicka et al. (2020) is most likely caused by the fact that the sample of Gornicka et al. (2020) includes all countries that are subject to an excessive deficit procedure of the European Commission, irrespective of whether countries were experiencing a sovereign debt crisis or not. Spain, for example, was in the middle of such a crisis in 2012, during which it almost lost access to bond markets when it tried to borrow billions of euros to recapitalize the Spanish banking system in May 2012. We predict that the same thing would have happened if Spain had tried to initiate a fiscal stimulus like the one studied in this paper, which would thus likely have been counterproductive, it could well have led to reduced rather than increased Spanish output. Therefore, we think our estimates of multipliers below zero are not unreasonable.

Finally, we perform robustness checks in Appendix E. We calculate the multiplier for several alternative parameter values such as the steady state diversion rate of corporate loans, the relative ratio of government bonds over corporate loans, the steady state leverage ratio, the coefficient related to households' quadratic adjustment costs from bond holdings, and the fraction of constrained households. We find that the multiplier changes very little in the absence of sovereign default risk, both for short-term debt and long-term debt, although the multiplier decreases when the fraction of constrained households is reduced. In addition, we also change several calibration targets for the function describing the probability of sovereign default (16). We find that the multiplier changes by more when government debt is long-term and subject to sovereign risk. However, the key conclusion of the paper, namely that the multiplier substantially decreases when sovereign default risk is introduced, continues to hold for all alternative parameter values.

8 Conclusion

In this paper we show that the effectiveness of fiscal stimuli is reduced when financial intermediaries are undercapitalized, and under specific circumstances to such an extent that the multiplier may actually turn negative. This happens when banks have large holdings of government bonds on their balance sheets, and the more so when these debt securities are subject to endogenous sovereign default risk. The mechanism we highlight is a new credit availability channel: deficit-financed stimuli lead to higher interest rates and possibly increasing sovereign risk. This in turn depresses bond prices and triggers capital losses on banks' existing holdings of sovereign debt; the resulting decline in capital ratios leads the banks to restrict loans to the corporate sector, with crowding out of investment as a result. On top of that, there is a feedback from banking troubles to government finances: tightening bank balance sheets do not only lead to more restrictive credit to the private sector but also depresses demand for government bonds, which in turn leads to a second round of higher rates and capital losses on banks' existing bond holdings, and a second round credit squeeze.

To make the general point we first construct a simple two period general equilibrium model with leverage constrained banks that hold long-term (risky) sovereign debt and extend loans to the real economy. We analytically show that in such circumstances credit provision to the real economy is crowded out by a debt-financed fiscal stimulus with crowding out of private investment as a consequence. We highlight that crowding out of private investment is *amplified* i) for longer maturity government debt, and ii) greater sensitivity of sovereign debt discounts to increasing levels of sovereign debt outstanding (endogenous sovereign default risk), as these two features lead to larger capital losses for banks holding government bonds.

To show the quantitative and empirical relevance of these claims we construct an infinite-horizon New Keynesian DSGE model of a small open economy member of a monetary union (Burriel et al., 2010). Unlike Burriel et al. (2010), there is an endogenous, partial response of interest rates since Spain is in fact not that small within the Eurozone, and may influence Eurozone aggregates sufficiently to trigger an ECB response. We extend this model by incorporating undercapitalized financial intermediaries with corporate loans and long-term government bonds subject to endogenous default risk on their balance sheets. We estimate critical parameters through Bayesian techniques using Spanish data while calibrating on first order moments, also to Spanish data.

We confirm in this empirical application that the effectiveness of fiscal stimuli is indeed reduced when the average duration of government debt is extended to a level that coincides with the maturity of outstanding Spanish sovereign debt, although the multiplier is still positive in the absence of sovereign risk, in line with Gornicka et al. (2020). The size of the multiplier is especially reduced when sovereign default risk is introduced in addition to the longer maturity sovereign debt. The combination of these two ingredients causes the multiplier to decrease by at least 0.60 percentage points with respect to the case where only one of these features is included.

A second set of results stresses the nonlinear nature of the model. Specifically, we find that the size of the financial crisis shock affects the effectiveness of fiscal stimuli in raising output: the larger the financial crisis, the smaller banks' net worth, and the smaller the cumulative multiplier. Quantitatively more important, and new in the literature, is that we find that the multiplier deteriorates with the size of the stimulus when sovereign debt is long-term and subject to default risk: the larger the stimulus, the larger the decrease in bond prices, and the larger the hit to banks' net worth. This second effect can cause the multiplier to fall by 0.58 percentage points with respect to smaller-sized stimuli.

A third set of results we wish to highlight is the impact of implementation delays on the size of the fiscal multiplier, an underresearched issue in the literature. We find that the size of the cumulative multiplier decreases with the lag between announcement and actual implementation of the stimulus by at least 0.30 percentage points for a stimulus with an implementation lag of four quarters (with respect to no lag). This is an important point as there typically is an implementation lag of at least several months in the real world, for example because of parliamentary approval procedures.

Our results are particularly relevant for Spain, where the banking system was severely undercapitalized after the bust of the housing boom of the 2000's, and Spanish sovereign debt holdings by banks were equivalent to 150% of Tier-1 capital; but this situation extended to Southern-Europe in general where banks had large holdings of domestic government debt and were severely undercapitalized after the Great Financial Crisis (GFC) (IMF, 2011; Hoshi and Kashyap, 2015).

To sum up, our results highlight the importance of i) implementing fiscal stimuli as soon as possible after they have been announced, as their effectiveness deteriorates with the time between announcement and implementation; ii) cleaning up commercial banks' balance sheets early on in an unfolding financial crisis, and before embarking upon fiscal stimuli, and (iii) keeping the size of fiscal stimuli small when commercial banks are undercapitalized, as the multiplier substantially decreases with size.

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Appendix “Financial Fragility and the Fiscal Multiplier”

A Data sources introduction

In this section we describe the data sources that we used in section 2. The data for Figure 1 were directly downloaded from the website of the World Bank, and did not need further processing.[25]

Figure 2 was computed from data from the European Banking Authority.[26] We added domestic government bonds of all maturities for all financial institutions that took part in the stress test, and divided by total capital for the financial institutions of this country that participated in the stress tests.

For Figure 3 we downloaded SNR CR Credit Default Swaps Premium Mid in Basis Points from Datastream, Thomson Reuters. We use the raw data, and did not perform any processing. Below we provide the codes for the respective countries:

- Republic of Italy Senior CR 5 Year E, Mnemonic ITG5EAC Code S183RD.
- Republic of Portugal Senior CR 5 Year E, Mnemonic PTG5EAC Code S18446.
- Kingdom of Spain Senior CR 5 Year E, Mnemonic ESG5EAC Code S164NN.

B Derivations Two period model

B.1 Details model setup

B.1.1 Households

Households care about consumption c in period $t = 0$ and $t = 1$ because consumption generates utility $u(c)$. The utility function $u(c)$ satisfies the regular conditions $u'(c) > 0$ and $u''(c) < 0$. Households discount the expected future cashflow in period $t = 1$ by the subjective discount factor β,

[25] The data can be found at http://data.worldbank.org/indicator/FB.AST.NPER.ZS.

[26] The data can be found at http://www.eba.europa.eu/risk-analysis-and-data/eu-wide-stress-testing/2011/results

and receive income in period $t = 0$ from an endowment $\mathcal{W}_0$, which they divide between consumption c_0 and savings through deposits d_0 at financial intermediaries and an internationally traded asset a_0. The interest rate on the internationally traded asset r_0^a is exogenous and equal to the world interest rate, while the interest rate on deposits r_0^d is determined in period $t = 0$. Principal and interst of oth assets are (re)paid in period $t = 1$. In addition to the repayment of deposits and the internationally traded assets, households also receive profits from financial intermediaries n_1 and profits from production firms $\Pi_1^f \equiv y_1 - \left(1 + r_0^k\right) k_0$. Period $t = 1$ income is used for consumption c_1 and to pay lump sum taxes τ_1 to the government. Although households own the production firms and financial intermediaries, they are not capable of influencing production or investment decisions and therefore take profits as given when choosing between consumption and savings in period $t = 0$. Households' optimization problem is given by:

$$
\begin{aligned}
\max_{\{c_0, c_1, d_0, a_0\}} \quad & u\left(c_0\right) - v\left(h_0\right) + E_0\left[\beta u\left(c_1\right)\right] \\
\text{s.t.} \quad & \\
c_0 + d_0 + a_0 \quad &= \quad \mathcal{W}_0, \\
c_1 + \tau_1 \quad &= \quad \left(1 + r_0^d\right) d_0 + \left(1 + r_0^a\right) a_0 + n_1 + \Pi_1^f,
\end{aligned}
$$

where E_0 denotes the expectations operator conditional on information in period $t = 0$. After setting up the accompanying Lagrangian, we obtain the following first order conditions:

$$
c_0 \quad : \quad u'\left(c_0\right) = \lambda_0, \tag{33}
$$

$$
c_1 \quad : \quad u'\left(c_1\right) = \lambda_1, \tag{34}
$$

$$
d_0 \quad : \quad E_0\left[\beta \Lambda_{0,1}\left(1 + r_0^d\right)\right] = 1, \tag{35}
$$

$$
a_0 \quad : \quad E_0\left[\beta \Lambda_{0,1}\left(1 + r_0^a\right)\right] = 1, \tag{36}
$$

where $\beta \Lambda_{0,1} = \beta \lambda_1 / \lambda_0$ is the households' stochastic discount factor, and λ_t the marginal utility from an additional unit of consumption in period t. By combining the first order condition for deposits (35) and the internationally traded asset (36), we immediately see that the interest rate on deposits r_0^d will in equilibrium be equal to the exogenous wordlr_0^a

B.1.2 Production sector

Production firms borrow in period $t = 0$ an amount s_0^k from financial intermediaries to acquire consumption goods from households, which they convert one-for-one into physical capital: $k_0 = s_0^k$.[27] Production firms employ the physical capital stock k_0 acquired in period $t = 0$ for production y_1 of consumption goods in period $t = 1$ using the following production technology:

$$y_1 = k_0^\alpha, \quad 0 < \alpha < 1, \tag{37}$$

The market for corporate loans is perfectly competitive, and the net interest rate r_0^k on loans is determined in period $t = 0$ and paid in period $t = 1$ to financial intermediaries, together with repayment of the principal. Because the market for corporate loans is perfectly competitive, both production firms and financial intermediaries take r_0^k as given when determining how much to borrow and lend, respectively. Period $t = 1$ profits are given by $\Pi_1^f \equiv y_1 - \left(1 + r_0^k\right) = k_0^\alpha - \left(1 + r_0^k\right)$. As there are no productivity shocks, production firms' period $t = 1$ profits Π_1^f are known at the end of period $t = 0$. Production firms will therefore choose to borrow an amount such that the marginal benefit from an additional unit of capital is equal to the marginal cost:

$$\alpha k_0^{\alpha-1} = 1 + r_0^k, \tag{38}$$

Therefore, the equilibrium interest rate will endogenously adjust to changes in lending by intermediaries until the market clears in period $t = 0$. Given production firms' first order condition for corporate loans, we can calculate their period $t = 1$ profits, which are equal to $\Pi_1^f = (1 - \alpha)\, k_0^\alpha$.

[27] In Gertler and Kiyotaki (2010); Gertler and Karadi (2011) as well as in our infinite-horizon model version, there are adjustment costs when converting final goods into physical capital, as a result of which the price of physical capital is typically different from 1. In the absence of such adjustment costs, the price of one unit of loans s_0^k in terms of consumption goods is therefore equal to 1.

B.1.3 Financial intermediaries

As mentioned in the main text, the objective of financial intermediaries is to maximize expected discounted net worth $E_0[\beta\Lambda_{0,1}n_1]$ in period $t = 1$, subject to intermediaries' balance sheet constraint (2) in period $t = 0$, intermediaries' law of motion for net worth (4) in period $t = 1$, and the incentive compatibility constraint (5). After substitution of intermediaries' net worth (4) in period $t = 1$ into $E_0[\beta\Lambda_{0,1}n_1]$:

$$E_0[\beta\Lambda_{0,1}n_1] = E_0\left\{\beta\Lambda_{0,1}\left[\left(1+r_0^k\right)k_0 + [1-p(b_0)]b_0 - \left(1+r_0^d\right)d_0\right]\right\}. \tag{39}$$

Next, we set up the accompanying Lagrangian of intermediaries' optimization problem:

$$\begin{aligned}\mathcal{L} &= (1+\mu_0)E_0\left\{\beta\Lambda_{0,1}\left[\left(1+r_0^k\right)k_0 + [1-p(b_0)]b_0 - \left(1+r_0^d\right)d_0\right]\right\}\\ &- \mu_0\left(\lambda_k k_0 + \lambda_b q_0^b b_0\right) + \chi_0\left(n_0 + d_0 - k_0 - q_0^b b_0\right),\end{aligned}$$

where μ_0 denotes the Lagrangian multiplier on intermediaries' incentive compatibility constraint (5), and χ_0 the Lagrangian multiplier on intermediaries' balance sheet constraint (2). Differentiation with respect to loans, bonds and deposits results in the following first order conditions:

$$\begin{aligned}k_0 &: (1+\mu_0)E_0\left[\beta\Lambda_{0,1}\left(1+r_0^k\right)\right] = \chi_0 + \lambda_k\mu_0,\\ b_0 &: (1+\mu_0)E_0\left\{\beta\Lambda_{0,1}[1-p(b_0)]\right\} = \chi_0 q_0^b + \lambda_b q_0^b\mu_0,\\ d_0 &: (1+\mu_0)E_0\left[\beta\Lambda_{0,1}\left(1+r_0^d\right)\right] = \chi_0.\end{aligned}$$

Substitution of the first order condition for deposits gives the following first order conditions for loans and bonds:

$$k_0 : E_0\left[\beta\Lambda_{0,1}\left(r_0^k - r_0^d\right)\right] = \frac{\lambda_k\mu_0}{1+\mu_0}, \tag{40}$$

$$b_0 : E_0\left\{\beta\Lambda_{0,1}\left[\frac{1-p(b_0)}{q_0^b} - 1 - r_0^d\right]\right\} = \frac{\lambda_b\mu_0}{1+\mu_0}, \tag{41}$$

Next, we substitute intermediaires' balance sheet constraint (2) to eliminate d_0 in intermediaries' expected net worth (39) in period $t = 1$:

$$\begin{aligned} E_0\left[\beta\Lambda_{0,1}n_1\right] &= E_0\left[\beta\Lambda_{0,1}\left(r_0^k - r_0^d\right)\right]k_0 + E_0\left\{\beta\Lambda_{0,1}\left[\frac{1-p\left(b_0\right)}{q_0^b} - 1 - r_0^d\right]\right\}q_0^b b_0 + E_0\left[\beta\Lambda_{0,1}\left(1+r_0^d\right)\right]n_0 \\ &= \frac{\mu_0}{1+\mu_0}\left(\lambda_k k_0 + \lambda_b q_0^b b_0\right) + n_0, \end{aligned} \tag{42}$$

where we substituted intermediaries' first order conditions (40) and (41) in the first two terms on the right hand side, respectively, and households' first order condition for deposits (35) in the third term. Substitution of expression (42) into intermediaries' incentive compatibility constraint (5) generates the desired expression for intermediaries' incentive compatibility constraint (6).

B.1.4 Aggregate resource constraints

The aggregate resource constraint in period $t = 0$ is given by:

$$c_0 + k_0 + g_0 + a_0 = n_0^{ex} + \mathcal{W}_0, \tag{43}$$

where n_0^{ex} denotes intermediaries net worth (7) in period $t = 0$, excluding the market value of their existing holdings of government bonds $q_0^b b_{-1}$, and $\mathcal{W}_0$ households' wealth in period $t = 0$.

The aggregate resource constraint in period $t = 1$ is given by:

$$c_1 = y_1 + \left(1 + r_0^a\right)a_0, \tag{44}$$

B.1.5 Equilibrium definition

The competitive equilibrium for the two-period economy is defined by the quantities $\{c_0, c_1, d_0, a_0, y_1, k_0, g_0, b_0, n_0\}$ and (shadow) prices $\{\lambda_0, \lambda_1, q_0^b, r_0^k, r_0^d, r_0^a, \mu_0\}$ such that:

1. Households optimize taking prices as given: (33) - (36), with $\beta\Lambda_{0,1} \equiv \beta\lambda_1/\lambda_0$ denoting households' stochastic discount factor.

2. Financial intermediaries optimize taking prices as given: (40) - (41) while taking into account

the balance sheet constraint (2), the incentive compatibility constraint (6), and intermediaries' net worth (7) in period $t = 0$.

3. Production firms optimize taking prices as given: (37) - (38).

4. The goods markets clear: (43) - (44).

5. The fiscal variables evolve according to: (1).

6. Exogenous variables are r_0^a, g_0.

B.2 Analysis of the equilibrium

We start by rewriting intermediaries' first order condition (40) in the following way:

$$\beta \Lambda_{0,1} \left(r_0^k - r_0^d \right) = \lambda_k \frac{\mu_0}{1+\mu_0},$$

where we have dropped the expectations operator. The reason why we can do so is that c_1 is effectively determined in period $t = 0$. We can see this by looking at the aggregate resource constraint (44), where we see that the right hand side is determined in period $t = 0$, since equation (37) shows that y_1 is effectively determined in period $t = 0$. Next, we use the above expression to solve for μ_0:

$$\mu_0 = \frac{\beta \Lambda_{0,1} \left(r_0^k - r_0^d \right)}{\lambda_k - \beta \Lambda_{0,1} \left(r_0^k - r_0^d \right)}.$$

Using households' first order condition for deposits (35), we can write the above expression as:

$$\mu_0 = \frac{r_0^k - r_0^d}{\lambda_k \left(1 + r_0^d\right) - \left(r_0^k - r_0^d \right)}.$$

Therefore, $1 + \mu_0$ is equal to:

$$1 + \mu_0 = \frac{\lambda_k \left(1 + r_0^d\right)}{\lambda_k \left(1 + r_0^d\right) - \left(r_0^k - r_0^d \right)}. \tag{45}$$

Now implicit differentiation of μ_0 with respect to g_0 gives the following expression:

$$\begin{aligned}
\frac{d\mu_0}{dg_0} &= \frac{\left(\frac{dr_0^k}{dg_0} - \frac{dr_0^d}{dg_0}\right)\left[\lambda_k\left(1+r_0^d\right) - \left(r_0^k - r_0^d\right)\right] - \left(r_0^k - r_0^d\right)\left[\lambda_k \cdot \frac{dr_0^d}{dg_0} - \left(\frac{dr_0^k}{dg_0} - \frac{dr_0^d}{dg_0}\right)\right]}{\left[\lambda_k\left(1+r_0^d\right) - \left(r_0^k - r_0^d\right)\right]^2}. \\
&= \frac{\lambda_k\left(1+r_0^d\right) \cdot \frac{dr_0^k}{dg_0} - \lambda_k\left(1+r_0^k\right) \cdot \frac{dr_0^d}{dg_0}}{\left[\lambda_k\left(1+r_0^d\right) - \left(r_0^k - r_0^d\right)\right]^2}. \\
&= \left(\frac{1+\mu_0}{\lambda_k\left(1+r_0^d\right)}\right)^2 \left[\lambda_k\left(1+r_0^d\right) \cdot \frac{dr_0^k}{dg_0} - \lambda_k\left(1+r_0^k\right) \cdot \frac{dr_0^d}{dg_0}\right]. \\
&= \frac{\left(1+\mu_0\right)^2}{\lambda_k}\left[\frac{1}{1+r_0^d} \cdot \frac{dr_0^k}{dg_0} - \frac{1+r_0^k}{\left(1+r_0^d\right)^2} \cdot \frac{dr_0^d}{dg_0}\right].
\end{aligned}$$

where we employed equation (45) when moving from the second to the third line. Finally, we implicitly differentiate equation (38) with respect to g_0 to obtain the following expression for $\frac{d\mu_0}{dg_0}$:

$$\frac{d\mu_0}{dg_0} = -C \cdot \frac{dk_0}{dg_0} - D \cdot \frac{dr_0^d}{dg_0}, \tag{46}$$

with C and D given by:

$$C = \frac{\left(1+\mu_0\right)^2}{\lambda_k} \cdot \frac{\alpha\left(1-\alpha\right)k_0^{\alpha-2}}{1+r_0^d} > 0, \tag{47}$$

$$D = \frac{\left(1+\mu_0\right)^2}{\lambda_k} \cdot \frac{1+r_0^k}{\left(1+r_0^d\right)^2} > 0. \tag{48}$$

Next, we derive two relations between the change in physical capital $\frac{dk_0}{dg_0}$ and the change in the bond price $\frac{dq_0^b}{dg_0}$. The first equation is obtained by substituting the government budget constraint (1) and the expression for intermediaries' net worth n_0 in period $t = 0$, equation (7), into intermediaries' binding incentive compatibility constraint (6):

$$\left(1+\mu_0\right)\left(n_0^{ex} + q_0^b b_{-1}\right) = \lambda_k k_0 + \lambda_b\left(q_0^b b_{-1} + g_0\right).$$

Implicit differentiation with respect to g_0, and employing expression (46) gives the following expression:

$$(\lambda_k + Cn_0) \cdot \frac{dk_0}{dg_0} = -\lambda_b - Dn_0 \cdot \frac{dr_0^d}{dg_0} + (1 + \mu_0 - \lambda_b)\, b_{-1} \cdot \frac{dq_0^b}{dg_0}, \tag{49}$$

which coincides with equation (9) in the main text.

A second equation that links the change in capital $\frac{dk_0}{dg_0}$ and the change in the bond price $\frac{dq_0^b}{dg_0}$ is obtained by combining intermediaries' first order conditions (40) - (41). Specifically, we solve for $\mu_0/(1+\mu_0)$ from intermediaries' first order condition for corporate loans (40), and substitute the resulting expression into intermediaries' first order condition for bonds (41), where we remember that we can drop the expectations operator as c_1 is determined in period $t = 0$. The result is a first order condition that relates the marginal cost from reducing corporate loans by one euro to the marginal benefit from increasing bonds by one euro:

$$\frac{1 - p\,(b_0)}{q_0^b} - \left(1 + r_0^d\right) = \frac{\lambda_b}{\lambda_k}\left(r_0^k - r_0^d\right). \tag{50}$$

Next, we eliminate r_0^k by substituting production firms' first order condition for capital (38), after which we implicitly differentiate the resulting expression with respect to g_0:

$$-\frac{p'\,(b_0)}{q_0^b} \cdot \frac{db_0}{dg_0} - \frac{1 - p\,(b_0)}{q_0^b} \cdot \frac{1}{q_0^b} \cdot \frac{dq_0^b}{dg_0} = \frac{\lambda_b}{\lambda_k}\alpha\,(\alpha - 1)\, k_0^{\alpha-2} \cdot \frac{dk_0}{dg_0} + \left(1 - \frac{\lambda_b}{\lambda_k}\right) \cdot \frac{dr_0^d}{dg_0}, \tag{51}$$

Now, we implicitly differentiate the government's budget constraint (1) with respect to g_0, and solve for $\frac{db_0}{dg_0}$:

$$\frac{db_0}{dg_0} = \frac{1}{q_0^b}\left[-\,(b_0 - b_{-1}) \cdot \frac{dq_0^b}{dg_0} + 1\right]. \tag{52}$$

Substitution of this expression allows us to rewrite equation (51) in the following way:

$$\frac{1 - p\,(b_0)}{q_0^b}\left[1 - (b_0 - b_{-1}) \cdot \frac{p'\,(b_0)}{1 - p\,(b_0)}\right] \cdot \frac{1}{q_0^b} \cdot \frac{dq_0^b}{dg_0} = -\frac{p'\,(b_0)}{\left(q_0^b\right)^2} - \left(1 - \frac{\lambda_b}{\lambda_k}\right) \cdot \frac{dr_0^d}{dg_0} + \frac{\lambda_b}{\lambda_k}\alpha\,(1 - \alpha)\, k_0^{\alpha-2} \cdot \frac{dk_0}{dg_0}. \tag{53}$$

Now that we have obtained two relations that link the change in capital $\frac{dk_0}{dg_0}$ and the change in the bond price $\frac{dq_0^b}{dg_0}$, we can combine the two to obtain a closed-form expression for the change in the bond price $\frac{dq_0^b}{dg_0}$. To do so, we substitute the expression for $\frac{dk_0}{dg_0}$ from equation (49) to get:

$$
\begin{aligned}
(F-G)\cdot\frac{1}{q_0^b}\cdot\frac{dq_0^b}{dg_0} &= -\frac{\lambda_b}{\lambda_k}\alpha\left(1-\alpha\right)k_0^{\alpha-2}\lambda_b-\frac{\lambda_b}{\lambda_k}\alpha\left(1-\alpha\right)k_0^{\alpha-2}Dn_0\cdot\frac{dr_0^d}{dg_0} \\
&- \left(\lambda_k+Cn_0\right)\left(1-\frac{\lambda_b}{\lambda_k}\right)\cdot\frac{dr_0^d}{dg_0}-\left(\lambda_k+Cn_0\right)\frac{p'\left(b_0\right)}{\left(q_0^b\right)^2}, \qquad (54)
\end{aligned}
$$

where F and G are given by:

$$
\begin{aligned}
F &= \left(\lambda_k+Cn_0\right)\frac{1-p\left(b_0\right)}{q_0^b}\left[1-\left(b_0-b_{-1}\right)\cdot\frac{p'\left(b_0\right)}{1-p\left(b_0\right)}\right]. \qquad (55) \\
G &= \frac{\lambda_b}{\lambda_k}\alpha\left(1-\alpha\right)k_0^{\alpha-2}\left(1+\mu_0-\lambda_b\right)q_0^b b_{-1}. \qquad (56)
\end{aligned}
$$

We see from equation (54) that $\frac{1}{q_0^b}\cdot\frac{dq_0^b}{dg_0}<0$ if $F>G$, since we assume that $0\leq\lambda_b\leq\lambda_k$. To show that $F>G$, we rewrite $F-G$ in the following way:

$$
F-G=F_1+F_2-G_1+G_2,
$$

where F_1, F_2, G_1, and G_2 are given by:

$$
\begin{aligned}
F_1 &= \lambda_k\frac{1-p\left(b_0\right)}{q_0^b}\left[1-\left(b_0-b_{-1}\right)\cdot\frac{p'\left(b_0\right)}{1-p\left(b_0\right)}\right]>0, \qquad (57) \\
F_2 &= Cn_0\frac{1-p\left(b_0\right)}{q_0^b}\left[1-\left(b_0-b_{-1}\right)\cdot\frac{p'\left(b_0\right)}{1-p\left(b_0\right)}\right]>0, \qquad (58) \\
G_1 &= \frac{\lambda_b}{\lambda_k}\alpha\left(1-\alpha\right)k_0^{\alpha-2}\left(1+\mu_0\right)q_0^b b_{-1}>0, \qquad (59) \\
G_2 &= \frac{\lambda_b}{\lambda_k}\alpha\left(1-\alpha\right)k_0^{\alpha-2}\lambda_b q_0^b b_{-1}>0 \qquad (60)
\end{aligned}
$$

It follows immediately that $F-G>0$ if we show that $F_2-G_1>0$. To show that $F_2-G_1>0$, we write out the expression below:

$$
\begin{aligned}
F_2 - G_1 &= Cn_0 \frac{1-p(b_0)}{q_0^b}\left[1-(b_0-b_{-1})\cdot\frac{p'(b_0)}{1-p(b_0)}\right] - \frac{\lambda_b}{\lambda_k}\alpha(1-\alpha)k_0^{\alpha-2}(1+\mu_0)q_0^b b_{-1} \\
&= \frac{(1+\mu_0)^2 n_0}{\lambda_k}\cdot\frac{\alpha(1-\alpha)k_0^{\alpha-2}}{1+r_0^d}\left(\frac{1-p(b_0)}{q_0^b}\right)\left[1-(b_0-b_{-1})\cdot\frac{p'(b_0)}{1-p(b_0)}\right] \\
&- \frac{\lambda_b}{\lambda_k}\alpha(1-\alpha)k_0^{\alpha-2}(1+\mu_0)q_0^b b_{-1} \\
&= \frac{(1+\mu_0)\alpha(1-\alpha)k_0^{\alpha-2}}{\lambda_k}\left\{(1+\mu_0)n_0\left(\frac{[1-p(b_0)]/q_0^b}{1+r_0^d}\right)\left[1-(b_0-b_{-1})\cdot\frac{p'(b_0)}{1-p(b_0)}\right] - \lambda_b q_0^b b_{-1}\right\}
\end{aligned}
$$

where we substituted the expression (47) for C when moving from the first to the second line. Now, we divide equation (50) by $1+r_0^d$ on the left and right hand side of the equation, and rearrange to obtain:

$$
\frac{[1-p(b_0)]/q_0^b}{1+r_0^d} = 1 + \frac{\lambda_b}{\lambda_k}\left(\frac{r_0^k - r_0^d}{1+r_0^d}\right) \geq 1.
$$

Using this (in)equality, we can write $F_2 - G_1$ as:

$$
\begin{aligned}
F_2 - G_1 &\geq \frac{(1+\mu_0)\alpha(1-\alpha)k_0^{\alpha-2}}{\lambda_k}\left\{(1+\mu_0)n_0\left[1-(b_0-b_{-1})\cdot\frac{p'(b_0)}{1-p(b_0)}\right] - \lambda_b q_0^b b_{-1}\right\} \\
&= \frac{(1+\mu_0)\alpha(1-\alpha)k_0^{\alpha-2}}{\lambda_k}\left\{\left(\lambda_k k_0 + \lambda_b q_0^b b_0\right)\left[1-(b_0-b_{-1})\cdot\frac{p'(b_0)}{1-p(b_0)}\right] - \lambda_b q_0^b b_{-1}\right\} \\
&= \frac{(1+\mu_0)\alpha(1-\alpha)k_0^{\alpha-2}}{\lambda_k}\lambda_k k_0\left[1-(b_0-b_{-1})\cdot\frac{p'(b_0)}{1-p(b_0)}\right] \\
&+ \frac{(1+\mu_0)\alpha(1-\alpha)k_0^{\alpha-2}}{\lambda_k}\left[\lambda_b q_0^b b_0 - \lambda_b q_0^b b_0(b_0-b_{-1})\cdot\frac{p'(b_0)}{1-p(b_0)} - \lambda_b q_0^b b_{-1}\right] \\
&= \frac{(1+\mu_0)\alpha(1-\alpha)k_0^{\alpha-2}}{\lambda_k}\lambda_k k_0\left[1-(b_0-b_{-1})\cdot\frac{p'(b_0)}{1-p(b_0)}\right] \\
&+ \frac{(1+\mu_0)\alpha(1-\alpha)k_0^{\alpha-2}}{\lambda_k}\left[\lambda_b q_0^b(b_0-b_{-1}) - \lambda_b q_0^b b_0(b_0-b_{-1})\cdot\frac{p'(b_0)}{1-p(b_0)}\right] \\
&= \frac{(1+\mu_0)\alpha(1-\alpha)k_0^{\alpha-2}}{\lambda_k}\lambda_k k_0\left[1-(b_0-b_{-1})\cdot\frac{p'(b_0)}{1-p(b_0)}\right] \\
&+ \frac{(1+\mu_0)\alpha(1-\alpha)k_0^{\alpha-2}}{\lambda_k}\lambda_b q_0^b(b_0-b_{-1})\left[1-b_0\cdot\frac{p'(b_0)}{1-p(b_0)}\right] > 0.
\end{aligned}
$$

where we substituted the binding incentive compatibility constraint (6) when moving from the first to the second line. Observe that this result is conditional on the default elasticity $b_0 \cdot \frac{p'(b_0)}{1-p(b_0)} < 1$, as this automatically implies that $(b_0 - b_{-1}) \cdot \frac{p'(b_0)}{1-p(b_0)} < 1$ since $(b_0 - b_{-1}) \cdot \frac{p'(b_0)}{1-p(b_0)} \leq b_0 \cdot \frac{p'(b_0)}{1-p(b_0)}$. The default elasticity being smaller than one is a reasonable assumption, as this implies that we are on the rising part of the debt-Laffer curve, which is an empirically plausible assumption for all but the most indebted countries (Claessens, 1990).

Now that we have proven that $F > G$, and therefore that the bond price always decreases in response to a government spending shock, we divide equation (54) by $F - G$ to obtain expression (8) from the main text, where A_1, A_2, and A_3 are given by:

$$A_1 = \frac{\frac{\lambda_b}{\lambda_k}\alpha\,(1-\alpha)\,k_0^{\alpha-2}}{F-G} > 0, \tag{61}$$

$$A_2 = \frac{\frac{\lambda_b}{\lambda_k}\alpha\,(1-\alpha)\,k_0^{\alpha-2} D n_0 + (\lambda_k + C n_0)\left(1 - \frac{\lambda_b}{\lambda_k}\right)}{F-G} > 0, \tag{62}$$

$$A_3 = \frac{\lambda_k + C n_0}{\left(q_0^b\right)^2 (F-G)} > 0. \tag{63}$$

C Derivations: infinite-horizon DSGE model

C.1 Households

We follow Gali et al. (2007), and assume that there are two types of households: constrained and unconstrained households. Each household type consists of a continuum of infinitely lived households with identical preferences, and (if applicable) identical asset endoments.

Both types of households derive utility from consumption and leisure, with habit formation in consumption, in order to capture realistic consumption dynamics (Christiano et al., 2005). Households optimize expected discounted utility:

$$E_t \left\{ \sum_{s=0}^{\infty} \beta^s \zeta_{t+s} \left[\log\left(c^q_{j,t+s} - \upsilon c^q_{j,t-1+s}\right) - \Psi \frac{\left(h^q_{j,t+s}\right)^{1+\varphi}}{1+\varphi} \right] \right\}, \quad \beta \in (0,1), \quad \upsilon \in [0,1), \quad \varphi \geq 0,$$

with $q \in \{r, u\}$, where r refers to constrained households, and u to unconstrained households. In addition, observe that $c^q_{j,t}$ denotes consumption per household of type q, $h^q_{j,t}$ are hours worked by the members of the household of type q that are workers, and ζ_t denotes a preference shock. We assume that the deep parameters β, υ, Ψ, and φ are the same for both types of households.

C.1.1 Constrained households

First, a fraction ν_r of households cannot save and consume their entire income, which consists of income from supplying labor $h^r_{j,t}$ at a wage rate w_t. This income is divided between consumption $c^r_{j,t}$ and lump sum taxes $\tau^r_{j,t}$. As a result, their budget constraint is given by:

$$c^r_{j,t} + \tau^r_{j,t} = w_t h^r_{j,t}, \tag{64}$$

where w_t denotes the real wage rate at which constrained households provide labor. Furthermore, lump sum taxes $\tau^r_{j,t}$ are allowed to be negative, in which case they constitute a transfer from the fiscal authority. A constrained household faces a perfectly competitive monopolistic labor market, in which it sets the nominal wage rate, and supplies as much labor as demanded by the labor agencies, which we will explain in section C.8

The constrained households set the nominal wage rate at which they are willing to supply labor, after which they provide any amount of labor demanded. Adjusting the nominal wage rate is subject to Calvo (1983) pricing frictions, as explained in Erceg et al. (2000). We will explain this in detail below. Apart from the first order condition for the nominal wage rate and the budget constraint (64), the first order condition for the constrained household's consumption level$c^r_{j,t}$ is given by:

$$\lambda^r_t = \zeta_t \left(c^r_{j,t} - \upsilon c^r_{j,t-1}\right)^{-1} - \upsilon\beta E_t \left[\zeta_{t+1}\left(c^r_{j,t+1} - \upsilon c^r_{j,t}\right)^{-1}\right], \tag{65}$$

where λ^r_t denotes the shadow value from an additional unit of consumption by the constrained household.

C.1.2 Unconstrained households

A fraction $1 - \nu_r$ of households is unconstrained, and can be subdivided between bankers and workers. Every period, a fraction f of the unconstrained household members is a banker running a financial intermediary. A fraction $1 - f$ of the unconstrained household members is a worker.

At the end of every period, all members of the unconstrained household pool their resources, and every member has the same consumption pattern. Hence there is perfect insurance within the unconstrained household, which facilitates aggregation.

Every period, the unconstrained household earns income from the labor of its working members and the profits of the firms that are owned by the household. In addition, unconstrained households keep short-term deposits in commercial banks, which are paid back with interest, as well as an internationally traded asset and long-term government bonds. Just as constrained households, the unconstrained household faces a perfectly competitive monopolistic labor market, in which it sets the nominal wage rate, and supplies as much labor as demanded by the labor agencies, see section C.8. The unconstrained household uses these incoming cash flows to buy consumption goods which are immediately consumed upon purchase, and make new deposits into financial intermediaries.[28] In addition, they acquire long-term domestic government bonds. We assume there is a risk-premium for the unconstrained household on Spanish deposits and the internationally traded assets. The risk-premium is decreasing in the net foreign assets that are held by unconstrained households.[29] In addition, the unconstrained household faces adjustment costs that are quadratic in their holdings of government bonds (Gertler and Karadi, 2013).

The budget constraint of unconstrained households is given by:

$$\begin{aligned} c_{j,t}^{u} + \tau_{j,t}^{u} + \frac{d_{j,t}}{\psi_t^{nfa}} + \frac{f_{j,t}}{\psi_t^{nfa}} + q_t^b s_{j,t}^{b,h} + \frac{1}{2}\kappa_b \left(s_{j,t}^{b,h} - \hat{s}^{b,h} \right)^2 &= w_t h_{j,t}^{u} + (1+r_t^d) d_{j,t-1} + (1+r_t^f) f_{j,t-1} \\ &+ (1+r_t^{b*}) q_t^b s_{j,t-1}^{b,h} + \Omega_t. \end{aligned} \tag{66}$$

Deposits $d_{j,t-1}$ are posted at financial intermediaries at the end of period $t-1$, and pay a net real return r_t^d and principal at time t. Similarly, net foreign assets $f_{j,t-1}$ are acquired at the end of period $t-1$, and pay a net real return r_t^f and principal at the beginning of period t. Government bonds $s_{j,t-1}^{b,h}$ are acquired at the end of period $t-1$ at a price q_{t-1}^b in the market for government debt, and pay a net return r_t^{b*}, which includes potential losses from a default by the domestic

[28]but not in the ones owned by the family, in order to prevent self-financing.

[29]We introduce this risk-premium to ensure the model is stationary in the sense that the net foreign asset position of the domestic economy eventually converges back to steady state (Schmitt-Grohe and Uribe, 2003).

government, see below. ψ_t^{nfa} denotes the risk-premium, which is determined at the moment the assets are acquired.[30] The risk-premium is given by the following functional form (Schmitt-Grohe and Uribe, 2003):

$$\psi_t^{nfa} = \exp\left[-\kappa_{nfa}\left(\frac{f_t - \bar{f}}{y_t^h}\right)\right], \tag{67}$$

where f_t denotes aggregate net foreign assets (in terms of the domestic consumer price index), $\bar{f}$ denotes aggregate net foreign assets in the steady state, and where y_t^h denotes domestic output.

Furthermore, w_t denotes the real wage rate which will be the same across constrained and unconstrained households, as we will see below. $\tau_{j,t}^u$ are lump sum taxes paid by the unconstrained household to the government, and Ω_t are the profits from the firms owned by the households. The profits of financial intermediaries are net of the startup capital for new bankers, as will be explained below.

The net real return on deposits and net foreign assets is given by:

$$1 + r_t^d = \frac{1 + r_{t-1}^n}{\pi_t}, \tag{68}$$

$$1 + r_t^f = \frac{1 + r_{t-1}^{nf}}{\pi_t}, \tag{69}$$

Now the first order conditions for the optimization problem of unconstrained households are then given by:

$$c_{j,t}^u \;:\; \lambda_t^u = \zeta_t \left(c_{j,t}^u - \upsilon c_{j,t-1}^u\right)^{-1} - \upsilon\beta E_t\left[\zeta_{t+1}\left(c_{j,t+1}^u - \upsilon c_{j,t}^u\right)^{-1}\right], \tag{70}$$

$$d_{j,t} \;:\; 1 = E_t\left[\beta\Lambda_{t,t+1}^u \psi_t^{nfa}\left(1 + r_{t+1}^d\right)\right], \tag{71}$$

$$f_{j,t} \;:\; 1 = E_t\left[\beta\Lambda_{t,t+1}^u \psi_t^{nfa}\left(1 + r_{t+1}^f\right)\right], \tag{72}$$

$$s_{j,t}^{b,h} \;:\; 1 = E_t\left[\beta\Lambda_{t,t+1}^u\left(\frac{\left(1 + r_{t+1}^{b*}\right) q_t^b}{q_t^b + \kappa_b\left(s_{j,t}^{b,h} - \hat{s}^{b,h}\right)}\right)\right], \tag{73}$$

where $r_{t+1}^{b*} \equiv \left(1 - p_{t+1}^{def}\vartheta_{def}\right)\left(1 + r_{t+1}^b\right) - 1$, and where λ_t^u denotes the shadow value from an

[30] We need to have that in addition to the internationally traded assets, deposits are also subject to the risk-premium, otherwise the Blanchard-Kahn conditions are not satisfied.

additional unit of consumption by the unconstrained household, and $\beta^i \Lambda^u_{t,t+i}$ the stochastic discount factor to convert cash flows from period $t+i$ in terms of utility in period t. Therefore, $\Lambda^u_{t,t+i} = \lambda^u_{t+i}/\lambda^u_t$ for $i \geq 0$.

C.1.3 Households' choice between domestic and foreign goods

Aggregate consumption c_t is given by the sum of aggregated consumption by constrained and unconstrained households, and is therefore given by:

$$c_t = \nu_r c^r_t + (1-\nu_r)\, c^u_t. \tag{74}$$

The consumption bundles c^r_t and c^u_t, however, are a composite of domestically and foreign produced goods. We assume that both household types have the same preferences for domestic goods $c^{q,h}_t$ and foreign goods $c^{q,f}_t$, where $q \in \{r, u\}$. We assume composite consumption c^q_t is given by a standard constant elasticity of substitution function:

$$c^q_t = \left[(1-\upsilon_c)^{\frac{1}{\eta_c}} \left(c^{q,h}_t \right)^{\frac{\eta_c - 1}{\eta_c}} + \upsilon_c^{\frac{1}{\eta_c}} \left(c^{q,f}_t \right)^{\frac{\eta_c - 1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c - 1}}, \tag{75}$$

where $1-\upsilon_c$ is the degree of steady state home bias, and η_c the elasticity of substitution between domestic and foreign goods. The problem of household type q is a static optimization problem in which the household of type q aims to minimize expenditures on domestic and foreign goods for a given level of composite consumption c^q_t:

$$\min_{\left\{c^h_t, c^f_t\right\}} P^h_t c^h_t + P^f_t c^f_t,$$

subject to equation (75), where P_t^h and P_t^f denote the nominal price of domestic and foreign goods, respectively. The resulting first order conditions are standard, and given by:

$$c_t^{q,h} \quad : \quad c_t^{q,h} = (1-\upsilon_c)\left(\frac{P_t^h}{P_t}\right)^{-\eta_c} c_t^q, \tag{76}$$

$$c_t^{q,f} \quad : \quad c_t^{q,f} = \upsilon_c\left(\frac{P_t^f}{P_t}\right)^{-\eta_c} c_t^q, \tag{77}$$

where P_t denotes the price level of the domestic consumption index (CPI). Since the composite consumption preferences (75) are the same for both types of households, we can aggregate straightforward to obtain that domestic aggregate consumption demand for domestic goods $c_t^h \equiv \nu_r c_t^{r,h} + (1-\nu_r)\, c_t^{u,h}$ and foreign goods $c_t^f \equiv \nu_r c_t^{r,f} + (1-\nu_r)\, c_t^{u,f}$ is given by:

$$c_t^h \quad : \quad c_t^h = (1-\upsilon_c)\left(p_t^h\right)^{-\eta_c} c_t, \tag{78}$$

$$c_t^f \quad : \quad c_t^f = \upsilon_c\left(p_t^f\right)^{-\eta_c} c_t, \tag{79}$$

where c_t is given by equation (74), and where $p_t^q \equiv P_t^q/P_t$ with $q \in \{h, f\}$.

Finally, substitution of the first order conditions (76) and (77) of households of type q into the consumption aggregation function (75) gives the following equation:

$$1 = (1-\upsilon_c)\left(p_t^h\right)^{1-\eta_c} + \upsilon_c\left(c_t^f\right)^{1-\eta_c}. \tag{80}$$

Solving for p_t^h from equation (78) and for p_t^f from equation (79), and substituting the resulting expressions into equation (80) gives the function that relates domestic and foreign goods into aggregate consumption c_t:

$$c_t = \left[(1-\upsilon_c)^{\frac{1}{\eta_c}}\left(c_t^h\right)^{\frac{\eta_c-1}{\eta_c}} + \upsilon_c^{\frac{1}{\eta_c}}\left(c_t^f\right)^{\frac{\eta_c-1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}}, \tag{81}$$

C.2 The Fiscal Authority

The fiscal authority (the government) levies lump sum taxes on households, issues bonds to finance its (exogeneous) expenditures g_t and services outstanding government liabilities. Government bonds have a parametrisable maturity structure as in Woodford (2001): nominal coupon payments on bonds issued in period $t-1$ equal $(1-\rho)^{j-1} x_c$ in period $t-1+j$, and therefore decay at a rate $1-\rho$ per period. Hence the price of a nominal bond B_{t-1} issued in period $t-1$ equals a fraction $1-\rho$ of the price q_t^b of a bond B_t issued in period t, where the price q_t^b is expressed in terms of the domestic consumer price index P_t. Outstanding nominal government liabilities at the beginning of period t are therefore equal to the coupon payments on outstanding bonds $x_c B_{t-1}$ and the market value of those outstanding bonds $(1-\rho) q_t^b B_{t-1}$. The expected duration is therefore equal to $1/[1-\beta(1-\rho)]$.[31] The nominal government budget constraint in the absence of sovereign default risk is therefore given by:

$$q_t^b B_t + P_t \tau_t = P_t^h g_t + x_c B_{t-1} + (1-\rho) q_t^b B_{t-1}.$$

Division by the domestic consumer price index P_t gives the budget constraint in real terms:

$$q_t^b b_t + \tau_t \quad = \quad p_t^h g_t + \left(1 + r_t^b\right) q_{t-1}^b b_{t-1},$$

where $b_t \equiv B_t/P_t$ is the real value of government bonds, and $1 + r_t^b$ is given by:

$$1 + r_t^b \quad = \quad \frac{x_c + (1-\rho) q_t^b}{\pi_t q_{t-1}^b}, \tag{82}$$

where $\pi_t = P_t/P_{t-1}$ is the gross inflation rate of the consumer price index.

[31] Duration is defined as: $\frac{\sum_{j=1}^{\infty} j \beta^{j-1} (1-\rho)^{j-1} x_c}{\sum_{j=1}^{\infty} \beta^{j-1} (1-\rho)^{j-1} x_c}$.

C.3 Financial intermediaries

Remember from Section 4.3 that intermediaries optimization problem is to maximize the continuation value (23), subject to the balance sheet constraint (21), the law of motion for net worth (22), and the incentive compatibility constraint (24). To find the solution, we follow Gertler and Kiyotaki (2010); Gertler and Karadi (2011), and assume that the solution for the value function has the following functional form, which we will check afterwards

$$V_{j,t} = \nu_t^k q_t^k s_{j,t}^k + \nu_t^b q_t^b s_{j,t}^b + \eta_t n_{j,t}. \tag{83}$$

Gertler and Kiyotaki (2010) show that the intermediary's optimization problem boils down to:

$$\max_{\{s_{j,t}^k, s_{j,t}^b\}} V_{j,t}, \quad \text{s.t.} \quad V_{j,t} \geq \lambda_t^k q_t^k s_{j,t}^k + \lambda_t^b q_t^b s_{j,t}^b$$

The Lagrangian for this problem is now given by:

$$\mathcal{L} = (1+\mu_t)\left(\nu_t^k q_t^k s_{j,t}^k + \nu_t^b q_t^b s_{j,t}^b + \eta_t n_{j,t}\right) - \mu_t\left(\lambda_t^k q_t^k s_{j,t}^k + \lambda_t^b q_t^b s_{j,t}^b\right),$$

where μ_t is the Lagrangian multiplier on the incentive compatibility constraint (24). Taking the derivative with resepct to corporate securities, government bonds, and the shadow value μ_t of the incentive compatibility constraint (24) gives the following first order conditions:

$$\begin{aligned}
s_{j,t}^k &: \quad (1+\mu_t)\nu_t^k - \lambda_t^k \mu_t = 0 \Longrightarrow \nu_t^k = \lambda_t^k \left(\frac{\mu_t}{1+\mu_t}\right) \\
s_{j,t}^b &: \quad (1+\mu_t)\nu_t^b - \lambda_t^b \mu_t = 0 \Longrightarrow \nu_t^b = \lambda_t^b \left(\frac{\mu_t}{1+\mu_t}\right) \\
\mu_t &: \quad \left(\nu_t^k q_t^k s_{j,t}^k + \nu_t^b q_t^b s_{j,t}^b + \eta_t n_{j,t} - \lambda_t^k q_t^k s_{j,t}^k - \lambda_t^b q_t^b s_{j,t}^b\right)\mu_t = 0
\end{aligned}$$

Taking the first order condition for corporate securities, we can solve for the shadow value μ_t of intermediaries' incentive compatibility constraint (24):

$$\mu_t = \frac{\nu_t^k}{\lambda_t^k - \nu_t^k}. \tag{84}$$

Combining the first order condition for corporate securities and government bonds gives the following equation:

$$\nu_t^b = \frac{\lambda_t^b}{\lambda_t^k}\nu_t^k, \tag{85}$$

Substitution of this expression into the guess for intermediaries' value function (83) gives:

$$V_{j,t} = \nu_t^k \left(q_t^k s_{j,t}^k + \frac{\lambda_t^b}{\lambda_t^k} q_t^b s_{j,t}^b \right) + \eta_t n_{j,t}. \tag{86}$$

Substitution of this expression into intermediaries' incentive compatibility constraint (24) then gives intermediaries' endogenous leverage constraint (25).

Next, we substitute the incentive compatibility constraint (25) into intermediaries' guess for the value function (86), which allows us to express the value function $V_{j,t}$ solely in terms of intermediaries' net worth $n_{j,t}$:

$$V_{j,t} = \left(\eta_t + \nu_t^k \phi_t\right) n_{j,t}. \tag{87}$$

Substitution of expression (87) into the right hand side of the Bellman equation (23) gives the following expression for the continuation value of the financial intermediary:

$$V_{j,t} = E_t \left\{ \beta \Lambda_{t,t+1}^u \left[1 - \theta + \theta \left(\eta_{t+1} + \nu_{t+1}^k \phi_{t+1} \right) \right] n_{j,t+1} \right\} = E_t \left[\Omega_{t+1} n_{j,t+1} \right],$$

where $\Omega_{t+1} = \beta \Lambda_{t,t+1}^u \left[1 - \theta + \theta \left(\eta_{t+1} + \nu_{t+1}^k \phi_{t+1} \right) \right]$ can be thought of as a stochastic discount factor that incorporates the financial friction (Gertler and Karadi, 2011). Now we substitute intermediaries' expression for next period's net worth (22) into the expression above:

$$
\begin{aligned}
V_{j,t} &= E_t\left[\Omega_{t+1} n_{j,t+1}\right] = E_t\left\{\Omega_{t+1}\left[\left(1+r_{t+1}^{k}\right) q_t^k s_{j,t}^k + \left(1+r_{t+1}^{b*}\right) q_t^b s_{j,t}^b - \left(1+r_{t+1}^{d}\right) d_{j,t}\right]\right\} \\
&= E_t\left\{\Omega_{t+1}\left[\left(r_{t+1}^{k}-r_{t+1}^{d}\right) q_t^k s_{j,t}^k + \left(r_{t+1}^{b*}-r_{t+1}^{d}\right) q_t^b s_{j,t}^b + \left(1+r_{t+1}^{d}\right) n_{j,t}\right]\right\}. \qquad (88)
\end{aligned}
$$

After comparing the conjectured solution (83) with expression (88), we find the following first order conditions for the shadow values η_t, ν_t^k and ν_t^b:

$$
\nu_t^k = E_t\left[\Omega_{t+1}\left(r_{t+1}^{k}-r_{t+1}^{d}\right)\right], \qquad (89)
$$

$$
\nu_t^b = E_t\left[\Omega_{t+1}\left(r_{t+1}^{b}-r_{t+1}^{d}-p_{t+1}^{def}\vartheta_{def}\left(1+r_{t+1}^{b}\right)\right)\right], \qquad (90)
$$

$$
\eta_t = E_t\left[\Omega_{t+1}\left(1+r_{t+1}^{d}\right)\right], \qquad (91)
$$

where we substituted $r_{t+1}^{b*} \equiv \left(1-p_{t+1}^{def}\vartheta_{def}\right)\left(1+r_{t+1}^{b}\right)-1$, which is the expected, default-inclusive return on government bonds.

C.4 Aggregation of financial variables

Aggregating the asset side of intermediaries' balance sheet (21) gives:

$$
p_t = q_t^k s_t^k + q_t^b s_t^b, \qquad (92)
$$

where p_t denotes the aggregate quantity of assets that are on the balance sheets of the financial intermediaries. Aggregating the liabilities side of intermediaries' balance sheet (21) gives:

$$
p_t = n_t + d_t, \qquad (93)
$$

Since ϕ_t does not depend on bank-specific variables, we can straightforward aggregate the leverage constraint (25) across financial intermediaries:

$$q_t^k s_t^k + \frac{\lambda_t^b}{\lambda_t^k} q_t^b s_t^b = \phi_t n_t, \tag{94}$$

where n_t denotes aggregate intermediary net worth.

The aggregate law of motion for net worth of existing financial intermediaries that are allowed to continue operating $n_{e,t}$ is given by:

$$\begin{aligned} n_{e,t} &= \theta\left[\left(1+r_t^k\right) q_{t-1}^k s_{t-1}^k + \left(1+r_t^{b*}\right) q_{t-1}^b s_{t-1}^b - \left(1+r_t^d\right) d_{t-1}\right] \\ &= \theta\left[\left(1+r_t^k\right) q_{t-1}^k s_{t-1}^k + (1-\vartheta_t)\left(\frac{x_c + (1-\rho)\, q_t^b}{\pi_t}\right) s_{t-1}^b - \left(1+r_t^d\right) d_{t-1}\right]. \end{aligned}$$

where θ is the exogenous probability that a financial intermediary is allowed to continue operating. A newly started financial intermediary j obtains an amount of new net worth which is equal to $\left(\frac{\chi}{1-\theta}\right) p_{j,t-1}$. In addition, we assume that each household uses its default proceeds to recapitalize their existing financial intermediaries. The households with an exiting banker do not use the proceeds to provide net worth to the newly starting banker, but only provide the amount $\left(\frac{\chi}{1-\theta}\right) p_{j,t-1}$. Even though the proceeds are randomly distributed among households, in the aggregate an amount of $\theta\left(\frac{\vartheta_t x_c + \vartheta_t(1-\rho) q_t^b}{\pi_t}\right) s_{t-1}^b$ will be added to aggregate net worth. Total new aggregate net worth therefore becomes:

$$\begin{aligned} n_t &= \theta\left[\left(1+r_t^k\right) q_{t-1}^k s_{t-1}^k + (1-\vartheta_t)\left(\frac{x_c + (1-\rho)\, q_t^b}{\pi_t}\right) s_{t-1}^b - \left(1+r_t^d\right) d_{t-1}\right] \\ &+ (1-\theta)\left(\frac{\chi}{1-\theta}\right) p_{t-1} + \theta\left(\frac{\vartheta_t x_c + \vartheta_t\,(1-\rho)\, q_t^b}{\pi_t}\right) s_{t-1}^b \\ &= \theta\left[\left(1+r_t^k\right) q_{t-1}^k s_{t-1}^k + \left(\frac{x_c + (1-\rho)\, q_t^b}{\pi_t}\right) s_{t-1}^b - \left(1+r_t^d\right) d_{t-1}\right] + \chi p_{t-1} \\ &= \theta\left[\left(1+r_t^k\right) q_{t-1}^k s_{t-1}^k + \left(1+r_t^b\right) q_{t-1}^b s_{t-1}^b - \left(1+r_t^d\right) d_{t-1}\right] + \chi p_{t-1} + n_t^g. \end{aligned}$$

As mentioned before, the proceeds from a sovereign default are randomly distributed to the households. Importantly, we assume that these proceeds are used by the household to recapitalize their respective financial intermediary.[32] Financial intermediaries, however, do not anticipate this recapitalization, as the households who perform the recap, receive a random payment. However, on an aggregate level, the financial intermediaries do not suffer ex post losses from the sovereign default because of the recap. Hence we can replace the default inclusive bond return r_t^{b*} by the default exclusive bond return r_t^b from equation (82). Note that this is only possible for the aggregate law of motion but not for the individual intermediaries' first order conditions!

C.5 Production side

C.5.1 Domestic Intermediate Goods Producers

There exists a continuum of domestic intermediate goods producers $i \in [0, 1]$. At the eend of period $t-1$, they issue securities $s_{i,t-1}^k$ at a price q_{t-1}^k to financial intermediaries. They use the proceeds to acquire $k_{i,t-1}$ units of physical capital at price q_{t-1}^k. Hence in equilibrium the number of securities $s_{i,t-1}^k$ will be equal to the number of units of physical capital $k_{i,t-1}$. Physical capital will be used for production in period t. In return for buying these securities, intermediate goods producers pledge future after-wage profits to the owners of these securities, in this case financial intermediaries. There are no financial frictions between intermediaries and intermediate goods producers, and hence intermediaries can costlessly monitor these loans. Therefore the promise by intermediate goods producers to pledge future after-wage profits is credible, and assures intermediaries of next period's after-wage profits made by intermediate goods producers. As these profits will vary with the business cycle, the claims of intermediaries on intermediate goods producers are therefore effectively state-contingent debt, see also Gertler and Kiyotaki (2010).

Intermediate goods producers employ the following production technology:

$$y_{i,t} \quad = \quad a_t \left(\xi_t k_{i,t-1}\right)^\alpha h_{i,t}^{1-\alpha},$$

where a_t denotes total factor productivity, and ξ_t capital quality (Gertler and Kiyotaki, 2010;

[32]We do so because otherwise a sovereign default would introduce a kink in intermediaries' net worth, which would force us to solve the model nonlinearily. Because we estimate the model with Bayesian techniques, we need to solve the model with first order perturbation approximation, which would not be possible in the presence of nonlinearities.

Gertler and Karadi, 2011). Both a_t and ξ_t follow a log-normal AR(1) process:

$$\log(x_t) \quad = \quad \rho_x \log(x_{t-1}) + \varepsilon_{x,t},$$

where $x \in \{a, \xi\}$. Innovations $\varepsilon_{x,t}$ follow a normal distribution: $\varepsilon_{x,t} \sim N(0, \sigma_x^2)$. After the shocks arrive at the beginning of period t, intermediate goods producers hire labor $h_{i,t}$ from labor agencies at a wage rate w_t, and starts producing using physical capital and labor. However, one unit of capital $k_{i,t-1}$ acquired in period $t-1$ is equivalent to $\xi_t k_{i,t-1}$ units of 'effective' capital in period t as a result of the capital quality shock (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011). After production in period t, output is sold at relative price m_t (which is expressed in terms of the domestic consumption price index) to retail firms, and sell the effective (depreciated) capital stock $(1-\delta)\xi_t k_{i,t-1}$ to capital producers at a price q_t^k. After wages have been paid, the remaining profits are paid out to financial intermediaries who make an effective return r_t^k on their securities holdings $q_{t-1}^k s_{i,t-1}^k$. After-wage profits are given by:

$$\Pi_{i,t} \quad = \quad m_t a_t \left(\xi_t k_{i,t-1}\right)^{\alpha} h_{i,t}^{1-\alpha} + q_t^k (1-\delta)\xi_t k_{i,t-1} - w_t h_{i,t}. \tag{95}$$

Intermediate goods producers maximize after-wage profits while taking prices m_t, w_t and q_t^k as given. Hiring in a perfectly competitive labor market implies that the wage rate will equal the marginal product of labor.

$$w_t = (1-\alpha) m_t y_{i,t} / h_{i,t}$$

Substitution of this first order condition in (95) results in the following after-wage profits:

$$\Pi_{i,t} \quad = \quad \alpha m_t a_t \left(\xi_t k_{i,t-1}\right)^{\alpha} h_{i,t}^{1-\alpha} + q_t^k (1-\delta)\xi_t k_{i,t-1}. \tag{96}$$

We can find the ex-post return on capital by setting (96) equal to $r_t^k q_{t-1}^k k_{i,t-1}$, as all after-wage profits are paid out to intermediate goods producers, and obtain the following expression for r_t^k:

$$1 + r_t^k = \frac{\alpha m_t a_t \xi_t^\alpha \left(k_{i,t-1}\right)^{\alpha-1} h_{i,t}^{1-\alpha} + q_t^k (1-\delta)\xi_t}{q_{t-1}^k} = \frac{\alpha m_t y_{i,t}/k_{i,t-1} + q_t^k (1-\delta)\xi_t}{q_{t-1}^k}$$

We obtain the factor demands by rearranging the first order condition for labor and the expression for the ex-post return on capital:

$$\begin{aligned} k_{i,t-1} &= \alpha m_t y_{i,t} / \left[q_{t-1}^k (1 + r_t^k) - q_t^k (1-\delta)\xi_t\right] \\ h_{i,t} &= (1-\alpha) m_t y_{i,t} / w_t \end{aligned}$$

Substitution of the factor demands into the production technology allows us to solve for the relative intermediate output price m_t:

$$m_t = \alpha^{-\alpha}(1-\alpha)^{\alpha-1} a_t^{-1} \left(w_t^{1-\alpha} \left[q_{t-1}^k (1 + r_t^k) - q_t^k (1-\delta)\xi_t \right]^\alpha \right). \tag{97}$$

C.5.2 Capital Producers

After intermediate goods producers have produced, they sell their remaining 'effective' capital stock $(1-\delta)\xi_t k_{i,t-1}$ to capital producers at a price q_t^k. In addition, capital producers purchase i_t goods at a price $p_t^i \equiv P_t^i / P_t$ for investment in new capital, which is a composite of domestic goods i_t^h and foreign goods i_t^f via a standard constant elasticity of substitution function:

$$i_t = \left[(1-\upsilon_i)^{\frac{1}{\eta_i}} \left(i_t^h\right)^{\frac{\eta_i - 1}{\eta_i}} + \upsilon_i^{\frac{1}{\eta_i}} \left(i_t^f\right)^{\frac{\eta_i - 1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i - 1}}, \tag{98}$$

where $1 - \upsilon_i$ is the degree of steady state home bias, and η_i the elasticity of substitution between domestic and foreign goods. The problem of capital producers is to choose between domestic and foreign goods with the aim of minimizing expenditures on domestic and foreign goods for a given level of composite investment i_t:

$$\min_{\{i_t^h, i_t^f\}} P_t^h i_t^h + P_t^f i_t^f,$$

subject to equation (98), where P_t^h and P_t^f denote the nominal price of domestic and foreign goods, respectively. The resulting first order conditions are standard, and given by:

$$i_t^h \quad : \quad i_t^h = (1-\upsilon_i)\left(\frac{P_t^h}{\mu_t^i}\right)^{-\eta_i} i_t, \tag{99}$$

$$i_t^f \quad : \quad i_t^f = \upsilon_i\left(\frac{P_t^f}{\mu_t^i}\right)^{-\eta_i} i_t, \tag{100}$$

where μ_t^i denotes the Lagrangian multiplier on the constraint (98). We can find an expression for μ_t^i by substituting the first order conditions (99) - (100) into the constraint (98), and solving for μ_t^i:

$$\mu_t^i = \left[(1-\upsilon_i)\left(P_t^h\right)^{1-\eta_i} + \upsilon_i\left(P_t^f\right)^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}}. \tag{101}$$

Hence we know from the New Keynesian literature that μ_t^i must be the price of the composite investment good i_t, which we will therefore call $P_t^i \equiv \mu_t^i$.

After having acquired domestic and foreign goods, old capital and newly acquired investment goods are then converted into new capital, and sold to intermediate goods producers at price q_t^k. However, capital producers face convex adjustment costs when converting investment goods whenever the growth rate i_t/i_{t-1} deviates from the long-run balanced growth path Λ_x. Hence one unit of investment goods translates into less than one unit of new capital unless $i_t = \Lambda_x i_{t-1}$. The new capital stock is therefore given by:

$$k_t \quad = \quad (1-\delta)\xi_t k_{t-1} + \zeta_t^i\left[1-\Psi(\iota_t)\right] i_t, \quad \Psi(\iota_t) = \frac{1}{2}\gamma_k\left(\iota_t - \Lambda_x\right)^2, \quad \iota_t = i_t/i_{t-1}, \tag{102}$$

where ζ_t^i denotes an investment-adjustment shock, and Λ_x is the long-run growth rate of output and investment along a balanced growth path.

Capital producers are profit-maximizers, and pay out their profits to households, who are the ultimate owners of capital producers. Profits Π_t^c in period t are given by:

$$\Pi_t^c = q_t^k k_t - q_t^k(1-\delta)\xi_t k_{t-1} - p_t^i i_t = q_t^k \zeta_t^i \left[1 - \Psi\left(\frac{i_t}{i_{t-1}}\right)\right] i_t - p_t^i i_t.$$

The capital producers' optimization problem is then given by:

$$\max_{\{i_{t+s}\}_{s=0}^{\infty}} E_t \left(\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s}^u \left\{ q_{t+s}^k \zeta_{t+s}^i \left[1 - \Psi\left(\frac{i_{t+s}}{i_{t-1+s}}\right)\right] i_{t+s} - p_{t+s}^i i_{t+s} \right\}\right)$$

The first order condition is found by differentiation with respect to investment:

$$p_t^i = q_t^k \zeta_t^i \left[1 - \Psi\left(\frac{i_t}{i_{t-1}}\right)\right] - q_t^k \zeta_t^i \frac{i_t}{i_{t-1}} \Psi'\left(\frac{i_t}{i_{t-1}}\right) + E_t\left[\beta \Lambda_{t,t+1}^u q_{t+1}^k \zeta_{t+1}^i \left(\frac{i_{t+1}}{i_t}\right)^2 \Psi'\left(\frac{i_{t+1}}{i_t}\right)\right],$$

which can be rewritten to find the price of capital:

$$\frac{p_t^i}{q_t^k} = \left[1 - \frac{1}{2}\gamma_k \left(\frac{i_t}{i_{t-1}} - \Lambda_x\right)^2\right] \zeta_t^i - \frac{\gamma_k i_t}{i_{t-1}} \left(\frac{i_t}{i_{t-1}} - \Lambda_x\right) \zeta_t^i + E_t\left[\beta \Lambda_{t,t+1}^u \frac{q_{t+1}^k}{q_t^k} \left(\frac{i_{t+1}}{i_t}\right)^2 \gamma_k \left(\frac{i_{t+1}}{i_t} - \Lambda_x\right) \zeta_{t+1}^i\right]. \tag{103}$$

C.5.3 Domestic Final Good Producers

Domestic final goods producers purchase from all domestic retail goods producers, of which there is a continuum of measure one, indexed by $j \in [0,1]$. Domestic final goods firms produce the domestic final good employing the following constant elasticity of substitution production technology:

$$y_t^h = \left[\int_0^1 \left(y_{j,t}^h\right)^{(\epsilon^p - 1)/\epsilon^p} dj\right]^{\epsilon^p/(\epsilon^p - 1)}, \tag{104}$$

where $y_{j,t}^h$ denotes the number of units purchased from domestic retail firm j, and where ϵ^p denotes the elasticity of substitution between the different domestic retail goods. Domestic final goods producers operate in an environment of perfect competition. Therefore, they take the price level of domestic final goods P_t^h and the price $P_{j,t}^h$ of domestic retail goods producer j as given, as well as the aggregate demand y_t^h for final goods. Domestic final goods producers choose the number of units $y_{k,t}^h$ acquired from domestic retail firms to maximize profits:

$$\max_{y_{j,t}^h} \; P_t^h y_t^h - \int_0^1 P_{j,t}^h y_{j,t}^h dj.$$

Differentiation with respect to $y_{j,t}$ results in the following demand function:

$$y_{j,t}^h = \left(\frac{P_{j,t}^h}{P_t^h}\right)^{-\epsilon^p} y_t^h. \tag{105}$$

The price level of the domestic final good can be found by substitution of (105) into the production technology of the domestic final goods producers (104):

$$\left(P_t^h\right)^{1-\epsilon^p} \quad = \quad \int_0^1 \left(P_{j,t}^h\right)^{1-\epsilon^p} dj. \tag{106}$$

C.5.4 Domestic Retail Firms

Domestic retail firm j acquires intermediate goods $y_{j,t}^i$ at a nominal price P_t^m, and converts these one-for-one into retail good j: $y_{j,t}^h = y_{j,t}^i$. It then sells these retail goods to domestic final goods producers at a nominal price $P_{j,t}^h$. Therefore, nominal profits of domestic retail firm j in period t are given by $\left(P_{j,t}^h - P_t^m\right) y_{j,t}^h$. Each domestic retail firm produces a unique retail good. Therefore, domestic retail firms operate in a market of perfect monopolistic competition, in which they set the price $P_{j,t}^h$ at which they sell domestic retail good j. As domestic retail firm j is a monopolist, it takes the demand for retail good j, equation (105), into account when setting the price. The goal of domestic retail firms is to maximize the expected sum of current and expected discounted future profits. However, domestic retail firms are subject to Calvo (1983) pricing frictions. As a result, each period only a fraction $1 - \psi_p$ of domestic retail firms is allowed to change the price at which they sell to domestic final goods producers, while a fraction ψ_p can only multiply its price from previous periods by a factor $\pi_t^{h,adj}$. This probability is i.i.d. in the cross-section and across time.

Therefore, the optimization problem of domestic retail firm j is described by:

$$\max_{P_{j,t}^h} E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s}^u \psi_p^s \left(\frac{P_{j,t}^h \prod_{k=1}^s \pi_{t+k}^{h,adj} - P_{t+s}^m}{P_{t+s}} \right) y_{j,t+s}^h \right],$$

where $y_{j,t}^h$ is subject to the demand function for retail good j (105), P_t the general domestic consumer price index, and $\beta^s \Lambda_{t,t+s}^u$ the stochastic discount factor of unconstrained households, who are the ultimate owners of domestic retail firms. Substitution of the demand function (105) gives the following maximization problem:

$$\max_{P_{j,t}^h} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s}^u \psi_p^s \left[p_{t+s}^h \left(\frac{P_{j,t}^h \prod_{k=1}^s \pi_{t+k}^{h,adj}}{P_{t+s}^h} \right)^{1-\epsilon^p} y_{t+s}^h - m_{t+s} \left(\frac{P_{j,t}^h \prod_{k=1}^s \pi_{t+k}^{h,adj}}{P_{t+s}^h} \right)^{-\epsilon^p} y_{t+s}^h \right] \right\},$$

where $p_t^h \equiv P_t^h / P_t$ and $m_t \equiv P_t^m / P_t$. Sine the only firm-specific variable is the price $P_{j,t}^h$, all domestic retail firms that are able to change prices, will choose the same price in equilibrium, which we denote by $P_t^{h,new}$. Now we take the derivative with respect to $P_{j,t}^h$ to find the following first order condition:

$$\begin{aligned} & (\epsilon^p - 1) E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s}^u \psi_p^s p_{t+s}^h \left(\frac{P_t^{h,new} \prod_{k=1}^s \pi_{t+k}^{h,adj}}{P_{t+s}^h} \right)^{1-\epsilon^p} \frac{1}{P_t^{h,new}} y_{t+s}^h \right] \\ = \; & \epsilon^p E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s}^u \psi_p^s m_{t+s} \left(\frac{P_t^{h,new} \prod_{k=1}^s \pi_{t+k}^{h,adj}}{P_{t+s}^h} \right)^{-\epsilon^p} \frac{1}{P_t^{h,new}} y_{t+s}^h \right], \end{aligned}$$

which we can rewrite as:

$$\begin{aligned} & (\epsilon^p - 1) \left(\frac{P_t^{h,new}}{P_t^h} \right)^{1-\epsilon^p} E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s}^u \psi_p^s p_{t+s}^h \left(\frac{P_t^h}{P_{t+s}^h} \prod_{k=1}^s \pi_{t+k}^{h,adj} \right)^{1-\epsilon^p} y_{t+s}^h \right] \\ = \; & \epsilon^p \left(\frac{P_t^{h,new}}{P_t^h} \right)^{-\epsilon^p} E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s}^u \psi_p^s m_{t+s} \left(\frac{P_t^h}{P_{t+s}^h} \prod_{k=1}^s \pi_{t+k}^{h,adj} \right)^{-\epsilon^p} y_{t+s}^h \right]. \end{aligned}$$

Defining the relative price of retail firms that are allowed to reset prices as $\pi_t^{h,new} \equiv P_t^{h,new} / P_t^h$, and gross inflation of domestic final goods as $\pi_t^h \equiv P_t^h / P_{t-1}^h$, we can rewrite the above first order condition:

$$\pi_t^{h,new} \equiv \frac{P_t^{h,new}}{P_t^h} = \left(\frac{\epsilon^p}{\epsilon^p - 1}\right) \frac{E_t\left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s}^u \psi_p^s m_{t+s} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}^h}{\pi_{t+k}^{h,adj}}\right)^{\epsilon^p} y_{t+s}^h\right]}{E_t\left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s}^u \psi_p^s p_{t+s}^h \prod_{k=1}^{s} \left(\frac{\pi_{t+k}^h}{\pi_{t+k}^{h,adj}}\right)^{\epsilon^p - 1} y_{t+s}^h\right]},$$

which we can rewrite in the following way:

$$\pi_t^{h,new} = \left(\frac{\epsilon^p}{\epsilon^p - 1}\right) \frac{\Xi_{1,t}}{\Xi_{2,t}}, \tag{107}$$

$$\Xi_{1,t} = \lambda_t^u m_t y_t^h + E_t\left[\beta\psi_p \left(\frac{\pi_{t+1}^h}{\pi_{t+1}^{h,adj}}\right)^{\epsilon^p} \Xi_{1,t+1}\right], \tag{108}$$

$$\Xi_{2,t} = \lambda_t^u p_t^h y_t^h + E_t\left[\beta\psi_p \left(\frac{\pi_{t+1}^h}{\pi_{t+1}^{h,adj}}\right)^{\epsilon^p - 1} \Xi_{2,t+1}\right]. \tag{109}$$

Next, we write out the expression for the price level of domestic final goods (106):

$$\left(P_t^h\right)^{1-\epsilon^p} = (1-\psi_p)\left(P_t^{h,new}\right)^{1-\epsilon^p} + \psi_p(1-\psi_p)\left(P_{t-1}^{h,new}\pi_t^{h,adj}\right)^{1-\epsilon^p} + \psi_p^2(1-\psi_p)\left(P_{t-2}^{h,new}\pi_{t-1}^{h,adj}\pi_t^{h,adj}\right)^{1-\epsilon^p} + ... \tag{110}$$

Iterating thsi expression one period backward, and multiplying by $\psi_p\left(\pi_t^{h,adj}\right)^{1-\epsilon^p}$ gives the following expression:

$$\psi_p\left(\pi_t^{h,adj}\right)^{1-\epsilon^p}\left(P_{t-1}^h\right)^{1-\epsilon^p} = \psi_p(1-\psi_p)\left(P_{t-1}^{h,new}\pi_t^{h,adj}\right)^{1-\epsilon^p} + \psi_p^2(1-\psi_p)\left(P_{t-2}^{h,new}\pi_{t-1}^{h,adj}\pi_t^{h,adj}\right)^{1-\epsilon^p} + ...$$

We se that the above expression coincides with the right hand side of equation of (110) starting from the second term. Therefore, we can rewrite equation (110) in the following way:

$$\left(P_t^h\right)^{1-\epsilon^p} = (1-\psi_p)\left(P_t^{h,new}\right)^{1-\epsilon^p} + \psi_p\left(\pi_t^{h,adj}\right)^{1-\epsilon^p}\left(P_{t-1}^h\right)^{1-\epsilon^p}. \tag{111}$$

Division of the left and right hand side by $\left(P_t^h\right)^{1-\epsilon^p}$ gives the following final expression:

$$1 = (1-\psi_p)\left(\pi_t^{h,new}\right)^{1-\epsilon^p} + \psi_p\left(\frac{\pi_t^h}{\pi_t^{h,adj}}\right)^{\epsilon^p-1}. \tag{112}$$

Next, we calculate the dispersion $\mathcal{D}_t^h \equiv \int_0^1 \left(\frac{P_{j,t}^h}{P_t^h}\right)^{-\epsilon^p} dj$:

$$\mathcal{D}_t^h = (1-\psi_p)\left(\frac{P_t^{h,new}}{P_t^h}\right)^{-\epsilon^p} + \psi_p(1-\psi_p)\left(\frac{P_{t-1}^{h,new}\pi_t^{h,adj}}{P_t^h}\right)^{-\epsilon^p} + \psi_p^2(1-\psi_p)\left(\frac{P_{t-2}^{h,new}\pi_{t-1}^{h,adj}\pi_t^{h,adj}}{P_t^h}\right)^{-\epsilon^p} \ldots.. \tag{113}$$

Iterating this expression one period back, multiplying by $\psi_p\left(\pi_t^{h,adj}\right)^{-\epsilon^p}\left(\frac{P_{t-1}^h}{P_t^h}\right)^{-\epsilon^p}$ gives the following expression:

$$\psi_p\left(\pi_t^{h,adj}\right)^{-\epsilon^p}\left(\frac{P_{t-1}^h}{P_t^h}\right)^{-\epsilon^p}\mathcal{D}_{t-1}^h = \psi_p(1-\psi_p)\left(\frac{P_{t-1}^{h,new}\pi_t^{h,adj}}{P_t^h}\right)^{-\epsilon^p} + \psi_p^2(1-\psi_p)\left(\frac{P_{t-2}^{h,new}\pi_{t-1}^{h,adj}\pi_t^{h,adj}}{P_t^h}\right)^{-\epsilon^p} \ldots..$$

We see that the right hand side of the expression of the above equation coincides with the right hand side of equation (113) starting from the second term. Therefore, we can write equation (113) in the following way:

$$\mathcal{D}_t^h = (1-\psi_p)\left(\frac{P_t^{h,new}}{P_t^h}\right)^{-\epsilon^p} + \psi_p\left(\pi_t^{h,adj}\right)^{-\epsilon^p}\left(\frac{P_{t-1}^h}{P_t^h}\right)^{-\epsilon^p}\mathcal{D}_{t-1}^h,$$

which we can rewrite as:

$$\mathcal{D}_t^h = (1-\psi_p)\left(\pi_t^{h,new}\right)^{-\epsilon^p} + \psi_p\left(\frac{\pi_t^h}{\pi_t^{h,adj}}\right)^{\epsilon^p}\mathcal{D}_{t-1}^h, \tag{114}$$

Finally, we assume that the price adjustment factor $\pi_t^{h,adj}$ depends on the previous period's gross inflation rate of final domestic goods π_{t-1}^h in the following way:

$$\pi_t^{h,adj} = \left(\pi_{t-1}^h\right)^{\gamma_p}. \tag{115}$$

C.6 Import Sector

We model the import sector in similar fashion as Burriel et al. (2010). Specifically, the import sector features the same staggered price-setting structure as for domestic production. That is, retail import firms acquire foreign goods from abroad , and convert the foreign goods one-for-one into retail import goods. Retail import firms produce a unique import retail good. Final import firms purchase retail import goods from all different retail import firms, and compute the final import good using a constant elasticity of substitution production function. As a result, retail import firms operate in a monopolistically competitive market, and are able to set the price at which they sell to final import firms. Final import firms take prices and demand for the final import good as given, and choose how many retail import goods to acquire from each reatil import firm.

C.6.1 Final import goods

Final import goods producers produce final import goods y_t^f. To do so, they acquire retail import goods $y_{m,t}^f$ from retail import firms at price $P_{m,t}^f$, of which there is a continuum $m \in [0,1]$ of measure one. Final import firms convert these retail import goods into final import goods using the following production technology:

$$y_t^f = \left[\int_0^1 \left(y_{m,t}^f\right)^{\left(\epsilon^f-1\right)/\epsilon^f} dm\right]^{\epsilon^f/\left(\epsilon^f-1\right)}. \tag{116}$$

Final import firms operate in a perfectly competitive market. Therefore, they take prices and demand for final import goods y_t^f as given when choosing how many retail import goods $y_{m,t}^f$ to buy from retail import good firm m. After converting retail import goods into final import goods, final import firms sell the final import goods y_t^f at a price P_t^f to domestic consumers and domestic capital goods producers. Therefore, final import firms' optimization problem is given by:

$$\max_{y_{m,t}^f} \; P_t^f y_t^f - \int_0^1 P_{m,t}^f y_{m,t}^f dm, \tag{117}$$

subject to final import firms' production technology (116). The resulting first order condition for the volume of retail import goods $y^f_{m,t}$ is subsequently given by:

$$y^f_{m,t} = \left(\frac{P^f_{m,t}}{P^f_t}\right)^{-\epsilon^f} y^f_t \,, \tag{118}$$

Substitution of the demand function (118) into final import firms' production technology (116) shows that the price level of final import goods P^f_t is given by:

$$P^f_t = \left[\int_0^1 \left(P^f_{m,t}\right)^{1-\epsilon^f} dm\right]^{1/\left(1-\epsilon^f\right)} . \tag{119}$$

C.6.2 Retail import goods

Retail import firms acquire foreign goods $y^{f*}_{m,t}$ from foreign production firms at price P^{f*}_t, and convert these one-for-one into retail import goods that are sold to final import producers, i.e. $y^f_{m,t} = y^{f*}_{m,t}$ at price $P^f_{m,t}$. Therefore, the profits of retail import producer m in period t is equal to $\left(P^f_{m,t} - P^{f*}_t\right) y^f_{m,t}$. As retail import firms produce a unique retail import good, they are effectively operating in an environment of monopolistic competition. Therefore, each retail import producer is capable of setting the price $P^f_{m,t}$ for its retail import good, thereby taking into account the demand function (118) of final import producers. The goal of retail import producers is to maximize the sum of current and expected future discounted profits. Retail import producers, however, are subject to Calvo (1983) pricing frictions. As a result, each retail import producer faces a probability ψ_f that it will not be allowed to freely choose a new price $P^f_{m,t}$ at which it is selling its retail import goods. In that case it can index the old price at a rate $\pi^{f,adj}_t$. Therefore, retail import producer m's maximization problem is given by the following expression:

$$\max_{P^f_{m,t}} \; E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda^u_{t,t+s} \psi^s_f \left(\frac{P^f_{m,t} \prod_{j=1}^{s} \pi^{f,adj}_{t+j} - P^{f*}_{t+s}}{P_{t+s}}\right) y^f_{m,t+s}\right],$$

subject to final import producers' demand function (118), and where $\beta^s \Lambda^u_{t,t+s}$ denotes the stochastic discount factor of unconstrained households. After substitution of the demand function (118), we obtain the following maximization objective:

$$\max_{P^f_{m,t}} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \Lambda^u_{t,t+s} \psi^s_f \left[p^f_{t+s} \left(\frac{P^f_{m,t} \prod_{j=1}^s \pi^{f,adj}_{t+j}}{P^f_{t+s}} \right)^{1-\epsilon^f} y^f_{t+s} - p^{f*}_{t+s} \left(\frac{P^f_{m,t} \prod_{j=1}^s \pi^{f,adj}_{t+j}}{P^f_{t+s}} \right)^{-\epsilon^f} y^f_{t+s} \right] \right\},$$

where $p^f_t \equiv P^f_t / P_t$ and $p^{f*}_t \equiv P^{f*}_t / P_t$. Taking the first order condition with respect to $P^f_{m,t}$ gives the following equation:

$$\begin{aligned} & \left(\epsilon^f - 1\right) E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda^u_{t,t+s} \psi^s_f p^f_{t+s} \left(\frac{P^{f,new}_{m,t} \prod_{j=1}^s \pi^{f,adj}_{t+j}}{P^f_{t+s}} \right)^{1-\epsilon^f} \frac{1}{P^{f,new}_{m,t}} y^f_{t+s} \right] \\ = \quad & \epsilon^f E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda^u_{t,t+s} \psi^s_f p^{f*}_{t+s} \left(\frac{P^{f,new}_{m,t} \prod_{j=1}^s \pi^{f,adj}_{t+j}}{P^f_{t+s}} \right)^{-\epsilon^f} \frac{1}{P^{f,new}_{m,t}} y^f_{t+s} \right], \end{aligned}$$

where $P^{f,new}_{m,t}$ denotes the optimal price chosen by retail import producer m. We can rewrite the above expression further as:

$$\begin{aligned} & \left(\epsilon^f - 1\right) \left(\frac{P^{f,new}_{m,t}}{P^f_t} \right)^{1-\epsilon^f} E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda^u_{t,t+s} \psi^s_f p^f_{t+s} \left(\frac{P^f_t}{P^f_{t+s}} \prod_{j=1}^{s} \pi^{f,adj}_{t+j} \right)^{1-\epsilon^f} y^f_{t+s} \right] \\ = \quad & \epsilon^f \left(\frac{P^{f,new}_{m,t}}{P^f_t} \right)^{-\epsilon^f} E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda^u_{t,t+s} \psi^s_f p^{f*}_{t+s} \left(\frac{P^f_t}{P^f_{t+s}} \prod_{j=1}^{s} \pi^{f,adj}_{t+j} \right)^{-\epsilon^f} y^f_{t+s} \right], \end{aligned}$$

which can be rewritten as:

$$\pi^{f,new}_t \equiv \frac{P^{f,new}_{m,t}}{P^f_t} = \left(\frac{\epsilon^f}{\epsilon^f - 1} \right) \frac{E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda^u_{t,t+s} \psi^s_f p^{f*}_{t+s} \prod_{j=1}^s \left(\frac{\pi^f_{t+j}}{\pi^{f,adj}_{t+j}} \right)^{\epsilon^f} y^f_{t+s} \right]}{E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda^u_{t,t+s} \psi^s_f p^f_{t+s} \prod_{j=1}^s \left(\frac{\pi^f_{t+j}}{\pi^{f,adj}_{t+j}} \right)^{\epsilon^f - 1} y^f_{t+s} \right]},$$

where $\pi_t^f \equiv P_t^f / P_{t-1}^f$ denotes the gross inflation rate of the price of final import producers. Finally, we can rewrite the above expression in the following way:

$$\pi_t^{f,new} = \left(\frac{\epsilon^f}{\epsilon^f - 1}\right) \frac{\Xi_{1,t}^f}{\Xi_{2,t}^f}, \tag{120}$$

$$\Xi_{1,t}^f = \lambda_t^u p_t^{f*} y_t^f + E_t \left[\beta \psi_f \left(\frac{\pi_{t+1}^f}{\pi_{t+1}^{f,adj}}\right)^{\epsilon^f} \Xi_{1,t+1}^f\right], \tag{121}$$

$$\Xi_{2,t}^f = \lambda_t^u p_t^f y_t^f + E_t \left[\beta \psi_f \left(\frac{\pi_{t+1}^f}{\pi_{t+1}^{f,adj}}\right)^{\epsilon^f - 1} \Xi_{2,t+1}^f\right], \tag{122}$$

Next, we are able to rewrite the expression for the price level of final import goods (119):

$$\left(P_t^f\right)^{1-\epsilon^f} = (1-\psi_f)\left(P_{m,t}^{f,new}\right)^{1-\epsilon^f} + \psi_f (1-\psi_f)\left(P_{m,t-1}^{f,new} \pi_t^{f,adj}\right)^{1-\epsilon^f} + \psi_f^2 (1-\psi_f)\left(P_{m,t-2}^{f,new} \pi_{t-1}^{f,adj} \pi_t^{f,adj}\right)^{1-\epsilon^f} .... \tag{123}$$

Shifting this equation one period back, and multiplying the left and right hand side by $\psi_f \left(\pi_t^{f,adj}\right)^{1-\epsilon^f}$ gives the following expression:

$$\psi_f \left(\pi_t^{f,adj}\right)^{1-\epsilon^f} \left(P_{t-1}^f\right)^{1-\epsilon^f} = \psi_f (1-\psi_f)\left(P_{m,t-1}^{f,new} \pi_t^{f,adj}\right)^{1-\epsilon^f} + \psi_f^2 (1-\psi_f)\left(P_{m,t-2}^{f,new} \pi_{t-1}^{f,adj} \pi_t^{f,adj}\right)^{1-\epsilon^f} + ...$$

We immediately see that the above expression coincides with the remaining terms on the right hand side of equation (123) after the first term. Therefore, we can write equation (123) in the following way:

$$\left(P_t^f\right)^{1-\epsilon^f} = (1-\psi_f)\left(P_{m,t}^{f,new}\right)^{1-\epsilon^f} + \psi_f \left(\pi_t^{f,adj}\right)^{1-\epsilon^f} \left(P_{t-1}^f\right)^{1-\epsilon^f}.$$

Division on the left and right hand side by $\left(P_t^f\right)^{1-\epsilon^f}$ gives the following equation:

$$1 = (1-\psi_f)\left(\pi_t^{f,new}\right)^{1-\epsilon^f} + \psi_f \left(\frac{\pi_t^f}{\pi_t^{f,adj}}\right)^{\epsilon^f - 1}. \tag{124}$$

Next, we calculate the dispersion $\mathcal{D}_t^f \equiv \int_0^1 \left(\frac{P_{m,t}^f}{P_t^f}\right)^{-\epsilon^f} dm$:

$$\mathcal{D}_t^f = (1-\psi_f)\left(\frac{P_{m,t}^{f,new}}{P_t^f}\right)^{-\epsilon^f} + \psi_f(1-\psi_f)\left(\frac{P_{m,t-1}^{f,new}\pi_t^{f,adj}}{P_t^f}\right)^{-\epsilon^f} + \psi_f^2(1-\psi_f)\left(\frac{P_{m,t-2}^{f,new}\pi_{t-1}^{f,adj}\pi_t^{f,adj}}{P_t^f}\right)^{-\epsilon^f} \ldots.. \tag{125}$$

Iterating this expression one period back, multiplying by $\psi_f\left(\pi_t^{f,adj}\right)^{-\epsilon^f}\left(\frac{P_{t-1}^f}{P_t^f}\right)^{-\epsilon^f}$ gives the following expression:

$$\psi_f\left(\pi_t^{f,adj}\right)^{-\epsilon^f}\left(\frac{P_{t-1}^f}{P_t^f}\right)^{-\epsilon^f}\mathcal{D}_{t-1}^f = \psi_f(1-\psi_f)\left(\frac{P_{m,t-1}^{f,new}\pi_t^{f,adj}}{P_t^f}\right)^{-\epsilon^f} + \psi_f^2(1-\psi_f)\left(\frac{P_{m,t-2}^{f,new}\pi_{t-1}^{f,adj}\pi_t^{f,adj}}{P_t^f}\right)^{-\epsilon^f} \ldots..$$

We see that the right hand side of the expression of the above equation coincides with the right hand side of equation (125) starting from the second term. Therefore, we can write equation (125) in the following way:

$$\mathcal{D}_t^f = (1-\psi_f)\left(\frac{P_{m,t}^{f,new}}{P_t^f}\right)^{-\epsilon^f} + \psi_f\left(\pi_t^{f,adj}\right)^{-\epsilon^f}\left(\frac{P_{t-1}^f}{P_t^f}\right)^{-\epsilon^f}\mathcal{D}_{t-1}^f,$$

which we can rewrite as:

$$\mathcal{D}_t^f = (1-\psi_f)\left(\pi_t^{f,new}\right)^{-\epsilon^f} + \psi_f\left(\frac{\pi_t^f}{\pi_t^{f,adj}}\right)^{\epsilon^f}\mathcal{D}_{t-1}^f, \tag{126}$$

Finally, we assume indexation $\pi_t^{f,adj}$ depends on previous period inflation of final import goods π_{t-1}^f in the following way:

$$\pi_t^{f,adj} = \left(\pi_{t-1}^f\right)^{\gamma_f}. \tag{127}$$

C.7 Export Sector

We model the export sector in similar fashion as Burriel et al. (2010). Specifically, the export sector features the same staggered price-setting structure as for domestic production. That is, retail export firms acquire domestically produced goods, and convert these one-for-one into retail export goods.

Retail export firms produce a unique export retail good. Final export firms purchase retail export goods from all retail export firms, and produce the final export good using a constant elasticity of substitution production function. As a result, retail export firms operate in a monopolistically competitive market. Therefore, they are able to set the price at which they sell their respective retail export good, while taking the demand for their good into account while setting the price. Final export firms operate in a perfectly competitive market, and therefore take all prices, as well as the aggregate demand for final export goods as given when determining how much to buy from each retail export firm.

C.7.1 Final export goods

Final export goods producers producer final export goods y_t^x. To do so, they acquire retail export goods from $y_{q,t}^x$ from retail export firms at price $P_{q,t}^x$, of which there exists a continuum $q \in [0,1]$ of measure one. Retail export goods are converted into final export goods using the following production technology:

$$y_t^x = \left[\int_0^1 \left(y_{q,t}^x\right)^{(\epsilon^x-1)/\epsilon^x} dq\right]^{\epsilon^x/(\epsilon^x-1)} . \tag{128}$$

Final export firms operate in a perfectly competitive market. Therefore, they take prices and demand for final export goods y_t^x as given when choosing how many retail export goods $y_{q,t}^x$ to buy from retail import good firm q. After converting retail export goods into final export goods, final export firms sell the final export goods y_t^x at a price P_t^x to foreign agents. Therefore, final export firms' optimization problem is given by:

$$\max_{y_{q,t}^x} P_t^x y_t^x - \int_0^1 P_{q,t}^x y_{q,t}^x dq, \tag{129}$$

subject to final export firms' production technology (128). The resulting first order condition for the volume of retail import goods $y_{q,t}^x$ is subsequently given by:

$$y_{q,t}^x = \left(\frac{P_{q,t}^x}{P_t^x}\right)^{-\epsilon^x} y_t^x, \tag{130}$$

Substitution of the demand function (130) into final export firms' production technology (128) shows that the price level of final export goods P_t^x is given by:

$$P_t^x = \left[\int_0^1 \left(P_{q,t}^x\right)^{1-\epsilon^x} dq\right]^{1/(1-\epsilon^x)}. \tag{131}$$

C.7.2 Retail export goods

Retail export firms acquire final domestic goods $x_{q,t}$ from domestic production firms at price P_t^h, and convert these one-for-one into retail export goods that are sold to final export producers, i.e. $y_{q,t}^x = x_{q,t}$ at price $P_{q,t}^x$. Therefore, the profits of retail export producer q in period t is equal to $\left(P_{q,t}^x - P_t^h\right) y_{q,t}^x$. As retail export firms produce a unique retail export good, they are effectively operating in an environment of monopolistic competition. Therefore, each retail export producer is capable of setting the price $P_{q,t}^x$ for its retail export good, thereby taking into account the demand function (130) of final export producers. The goal of retail export producers is to maximize the sum of current and expected future discounted profits. Retail export producers, however, are subject to Calvo (1983) pricing frictions. As a result, each retail export producer faces a probability ψ_x that it will not be allowed to freely choose a new price $P_{q,t}^x$ at which it is selling its retail export goods. In that case it can index the old price at a rate $\pi_t^{x,adj}$. Therefore, retail export producer q's maximization problem is given by the following expression:

$$\max_{P_{q,t}^x} E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s}^u \psi_x^s \left(\frac{P_{q,t}^x \prod_{j=1}^s \pi_{t+j}^{x,adj} - P_{t+s}^h}{P_{t+s}}\right) y_{q,t+s}^x\right],$$

subject to final export producers' demand function (130), and where $\beta^s \Lambda_{t,t+s}^u$ denotes the stochastic discount factor of unconstrained households. After substitution of the demand function (130), we obtain the following maximization objective:

$$\max_{P_{q,t}^x} E_t \left\{\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s}^u \psi_x^s \left[p_{t+s}^x \left(\frac{P_{q,t}^x \prod_{j=1}^s \pi_{t+j}^{x,adj}}{P_{t+s}^x}\right)^{1-\epsilon^x} y_{t+s}^x - p_{t+s}^h \left(\frac{P_{q,t}^x \prod_{j=1}^s \pi_{t+j}^{x,adj}}{P_{t+s}^x}\right)^{-\epsilon^x} y_{t+s}^x\right]\right\},$$

where $p_t^x \equiv P_t^x/P_t$ and $p_t^h \equiv P_t^h/P_t$. Taking the first order condition with respect to $P_{q,t}^x$ gives the following equation:

$$
\begin{aligned}
& (\epsilon^x - 1)\, E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s}^u \psi_x^s p_{t+s}^x \left(\frac{P_{q,t}^{x,new} \prod_{j=1}^{s} \pi_{t+j}^{x,adj}}{P_{t+s}^x} \right)^{1-\epsilon^x} \frac{1}{P_{q,t}^{x,new}} y_{t+s}^x \right] \\
= \; & \epsilon^x E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s}^u \psi_x^s p_{t+s}^h \left(\frac{P_{q,t}^{x,new} \prod_{j=1}^{s} \pi_{t+j}^{x,adj}}{P_{t+s}^x} \right)^{-\epsilon^x} \frac{1}{P_{q,t}^{x,new}} y_{t+s}^x \right],
\end{aligned}
$$

where $P_{q,t}^{x,new}$ denotes the optimal price chosen by retail import producer m. We can rewrite the above expression further as:

$$
\begin{aligned}
& (\epsilon^x - 1) \left(\frac{P_{q,t}^{x,new}}{P_t^x} \right)^{1-\epsilon^x} E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s}^u \psi_x^s p_{t+s}^x \left(\frac{P_t^x}{P_{t+s}^x} \prod_{j=1}^{s} \pi_{t+j}^{x,adj} \right)^{1-\epsilon^x} y_{t+s}^x \right] \\
= \; & \epsilon^x \left(\frac{P_{q,t}^{x,new}}{P_t^x} \right)^{-\epsilon^x} E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s}^u \psi_x^s p_{t+s}^h \left(\frac{P_t^x}{P_{t+s}^x} \prod_{j=1}^{s} \pi_{t+j}^{x,adj} \right)^{-\epsilon^x} y_{t+s}^x \right],
\end{aligned}
$$

which can be rewritten as:

$$
\pi_t^{x,new} \equiv \frac{P_{q,t}^{x,new}}{P_t^x} = \left(\frac{\epsilon^x}{\epsilon^x - 1} \right) \frac{E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s}^u \psi_x^s p_{t+s}^h \prod_{j=1}^{s} \left(\frac{\pi_{t+j}^x}{\pi_{t+j}^{x,adj}} \right)^{\epsilon^x} y_{t+s}^x \right]}{E_t \left[\sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s}^u \psi_x^s p_{t+s}^f \prod_{j=1}^{s} \left(\frac{\pi_{t+j}^x}{\pi_{t+j}^{x,adj}} \right)^{\epsilon^x - 1} y_{t+s}^x \right]},
$$

where $\pi_t^x \equiv P_t^x/P_{t-1}^x$ denotes the gross inflation rate of the price of final export producers. Finally, we can rewrite the above expression in the following way:

$$
\pi_t^{x,new} = \left(\frac{\epsilon^x}{\epsilon^x - 1} \right) \frac{\Xi_{1,t}^x}{\Xi_{2,t}^x}, \tag{132}
$$

$$
\Xi_{1,t}^x = \lambda_t^u p_t^h y_t^x + E_t \left[\beta \psi_x \left(\frac{\pi_{t+1}^x}{\pi_{t+1}^{x,adj}} \right)^{\epsilon^x} \Xi_{1,t+1}^x \right], \tag{133}
$$

$$
\Xi_{2,t}^x = \lambda_t^u p_t^x y_t^x + E_t \left[\beta \psi_x \left(\frac{\pi_{t+1}^x}{\pi_{t+1}^{x,adj}} \right)^{\epsilon^x - 1} \Xi_{2,t+1}^x \right], \tag{134}
$$

Next, we are able to rewrite the expression for the price level of final export goods (131):

$$\left(P_t^x\right)^{1-\epsilon^x} = \left(1-\psi_x\right)\left(P_{q,t}^{x,new}\right)^{1-\epsilon^x} + \psi_x\left(1-\psi_x\right)\left(P_{q,t-1}^{x,new}\pi_t^{x,adj}\right)^{1-\epsilon^x} + \psi_x^2\left(1-\psi_x\right)\left(P_{q,t-2}^{x,new}\pi_{t-1}^{x,adj}\pi_t^{x,adj}\right)^{1-\epsilon^x} \dots \quad (135)$$

Shifting this equation one period back, and multiplying the left and right hand side by $\psi_x\left(\pi_t^{x,adj}\right)^{1-\epsilon^x}$ gives the following expression:

$$\psi_x\left(\pi_t^{x,adj}\right)^{1-\epsilon^x}\left(P_{t-1}^x\right)^{1-\epsilon^x} = \psi_x\left(1-\psi_x\right)\left(P_{q,t-1}^{x,new}\pi_t^{x,adj}\right)^{1-\epsilon^x} + \psi_x^2\left(1-\psi_x\right)\left(P_{q,t-2}^{x,new}\pi_{t-1}^{x,adj}\pi_t^{x,adj}\right)^{1-\epsilon^x} + \dots$$

We immediately see that the above expression coincides with the remaining terms on the right hand side of equation (135) after the first term. Therefore, we can write equation (135) in the following way:

$$\left(P_t^x\right)^{1-\epsilon^x} = \left(1-\psi_x\right)\left(P_{q,t}^{x,new}\right)^{1-\epsilon^x} + \psi_x\left(\pi_t^{x,adj}\right)^{1-\epsilon^x}\left(P_{t-1}^x\right)^{1-\epsilon^x}.$$

Division on the left and right hand side by $\left(P_t^x\right)^{1-\epsilon^x}$ gives the following equation:

$$1 = \left(1-\psi_x\right)\left(\pi_t^{x,new}\right)^{1-\epsilon^x} + \psi_x\left(\frac{\pi_t^x}{\pi_t^{x,adj}}\right)^{\epsilon^x-1}. \quad (136)$$

Finally, we calculate the dispersion $\mathcal{D}_t^x \equiv \int_0^1\left(\frac{P_{q,t}^x}{P_t^x}\right)^{-\epsilon^x} dq$:

$$\mathcal{D}_t^x = \left(1-\psi_x\right)\left(\frac{P_{q,t}^{x,new}}{P_t^x}\right)^{-\epsilon^x} + \psi_x\left(1-\psi_x\right)\left(\frac{P_{q,t-1}^{x,new}\pi_t^{x,adj}}{P_t^x}\right)^{-\epsilon^x} + \psi_x^2\left(1-\psi_x\right)\left(\frac{P_{q,t-2}^{x,new}\pi_{t-1}^{x,adj}\pi_t^{x,adj}}{P_t^x}\right)^{-\epsilon^x} \dots\dots \quad (137)$$

Iterating this expression one period back, multiplying by $\psi_x\left(\pi_t^{x,adj}\right)^{-\epsilon^x}\left(\frac{P_{t-1}^x}{P_t^x}\right)^{-\epsilon^x}$ gives the following expression:

$$\psi_x\left(\pi_t^{x,adj}\right)^{-\epsilon^x}\left(\frac{P_{t-1}^x}{P_t^x}\right)^{-\epsilon^x}\mathcal{D}_{t-1}^x = \psi_x\left(1-\psi_x\right)\left(\frac{P_{q,t-1}^{x,new}\pi_t^{x,adj}}{P_t^x}\right)^{-\epsilon^x} + \psi_x^2\left(1-\psi_x\right)\left(\frac{P_{q,t-2}^{x,new}\pi_{t-1}^{x,adj}\pi_t^{x,adj}}{P_t^x}\right)^{-\epsilon^x} \dots\dots$$

We see that the right hand side of the expression of the above equation coincides with the right hand side of equation (137) starting from the second term. Therefore, we can write equation (137) in the following way:

$$\mathcal{D}_t^x = (1-\psi_x)\left(\frac{P_{q,t}^{x,new}}{P_t^x}\right)^{-\epsilon^x} + \psi_x \left(\pi_t^{x,adj}\right)^{-\epsilon^x}\left(\frac{P_{t-1}^x}{P_t^x}\right)^{-\epsilon^x}\mathcal{D}_{t-1}^x,$$

which we can rewrite as:

$$\mathcal{D}_t^x = (1-\psi_x)\left(\pi_t^{x,new}\right)^{-\epsilon^x} + \psi_x \left(\frac{\pi_t^x}{\pi_t^{x,adj}}\right)^{\epsilon^x}\mathcal{D}_{t-1}^x, \tag{138}$$

Finally, we assume indexation $\pi_t^{x,adj}$ depends on previous period inflation of final export goods π_{t-1}^x in the following way:

$$\pi_t^{x,adj} = \left(\pi_{t-1}^x\right)^{\gamma_x}. \tag{139}$$

C.8 Labor Market

C.8.1 Labor Agencies

Following Erceg et al. (2000), labor agencies operate in a perfectly competitive market in which they acquire different labor types $h_{i,t}$ from a continuum of labor unions $i \in [0,1]$ to produce final homogenous labor h_t. Final labor is a constant elasticity of substitution function of labor offered by labor unions:

$$h_t = \left[\int_0^1 h_{i,t}^{(\epsilon^w-1)/\epsilon^w} di\right]^{\epsilon^w/(\epsilon^w-1)} \tag{140}$$

Labor agencies sell final labor h_t to intermediate goods producers at an aggregate nominal wage rate W_t. Because labor agencies operates in an envrionment of perfect competition, they take the nominal aggregate wage W_t, and the nominal wage rate $W_{i,t}$ of labor union i, as well as the aggregate labor demand h_t as given. They maximize profits by adjusting their demand for labor $h_{i,t}$ of type i, subject to the labor technology (140):

$$\max_{\{h_{i,t}\}} W_t h_t - \int_0^1 W_{i,t} h_{i,t} di. \tag{141}$$

The maximization problem of labor agencies results in the following first order condition for the demand for labor of type i:

$$h_{i,t} = \left(\frac{W_{i,t}}{W_t} \right)^{-\epsilon^w} h_t. \tag{142}$$

Substitution of the labor demand curve (142) into the labor technology (140) provides us with the aggregate wage-index W_t:

$$W_t^{1-\epsilon^w} = \int_0^1 W_{i,t}^{1-\epsilon^w} di. \tag{143}$$

C.8.2 Labor Unions

Each labor union i represents a unique, differentiated labor of type $h_{i,t}$. Therefore, labor union i is a monopolist when it comes to offering labor of type i to labor agencies.

The labor union that represents labor of type i is the sole union for that type of labor, as a result of which it has the power to set the nominal wage rate $W_{i,t}$ for labor of type i. However, labor unions operate in a competitive monopolistic market, as the representative labor agency has the possibility to shift (part of) its labor demand to other labor unions, see equation (140). Therefore, labor union i takes the demand schedule (142) into account when setting the wage rate $W_{i,t}$, after which households of type i provide as much labor as demanded by the labor agency.

When setting the nominal wage rate $W_{i,t}$, labor union i takes into account that there is an exogenous probability of ψ_w that it is not allowed to choose the nominal wage rate next period, while it is allowed to change with probability $1 - \psi_w$ (Calvo, 1983; Erceg et al., 2000). In case labor union i cannot choose its desired wage rate, it is allowed to partially index the wage rate by multiplying it with the wage adjustment ω_t^{adj}. Constrained and unconstrained households are uniformly distributed across worker types. Labor unions allocate labor demand uniformly between members from constrained and unconstrained households. Therefore, workers from constrained and workers from unconstrained households that belong to the same union will provide the same amount of labor in equilibrium (Gali et al., 2007), i.e. $h_{i,t}^r = h_{i,t}^u = h_{i,t}$. In setting the nominal wage rate $\tilde{W}_{i,t}$ (when labor union i is allowed to choose the wage rate), labor union i weighs off the effects on total wage income, and the anticipated effect on the disutility from providing labor. This results in the following optimization problem for labor union i in setting the optimal nominal wage rate $\tilde{W}_{i,t}$:

$$\max_{\{\tilde{W}_{i,t}\}} E_t \left\{ \sum_{s=0}^{\infty} (\beta\psi_w)^s \left[\lambda_{t+s} \frac{\tilde{W}_{i,t}\left(\Pi_{j=1}^{j=s}\omega_{t+j}^{adj}\right)}{P_{t+s}} h_{i,t+s} - \chi \frac{h_{i,t+s}^{1+\varphi}}{1+\varphi} \right] \right\},$$

where λ_{t+s} is given by (Gali et al., 2007):

$$\lambda_{t+s} = \nu_r \lambda_t^r + (1-\nu_r)\lambda_t^u, \tag{144}$$

where λ_t^r refers to the shadow value of constrained households' budget constraint (65), and λ_t^u to the shadow value of unconstrained households' budget constraint (70). This weighted average shadow value is used by labor union i to discount the future real wage income $\frac{\tilde{W}_{i,t}\left(\Pi_{j=1}^{j=s}\omega_{t+j}^{adj}\right)}{P_{t+s}} h_{i,t+s}$. As labor union i is the monopolistic representative of households of labor type i, it takes the labor demand schedule (142) into account when setting the new nominal wage rate $\tilde{W}_{i,t}$. Therefore, we substitute equation (142) into the labor union's optimization problem, which results in the following reformulation of the labor union's optimization problem:

$$\max_{\{\tilde{W}_{i,t}\}} E_t \left\{ \sum_{s=0}^{\infty} (\beta\psi_w)^s \left[\lambda_{t+s} w_{t+s} \left(\frac{\tilde{W}_{i,t}}{W_{t+s}} \Pi_{j=1}^{j=s}\omega_{t+j}^{adj} \right)^{1-\epsilon^w} h_{t+s} - \chi \left(\frac{\tilde{W}_{i,t}}{W_{t+s}} \Pi_{j=1}^{j=s}\omega_{t+j}^{adj} \right)^{-\epsilon^w(1+\varphi)} \frac{h_{t+s}^{1+\varphi}}{1+\varphi} \right] \right\},$$

where $w_t \equiv W_t/P_t$ denotes the real wage rate. Differentiation with respect to $\tilde{W}_{i,t}$ gives the following first order condition:

$$\begin{aligned} & (\epsilon^w - 1) E_t \left[\sum_{s=0}^{\infty} (\beta\psi_w)^s \lambda_{t+s} w_{t+s} \left(\frac{\tilde{W}_{i,t}}{W_{t+s}} \Pi_{j=1}^{j=s}\omega_{t+j}^{adj} \right)^{1-\epsilon^w} \frac{1}{\tilde{W}_{i,t}} h_{t+s} \right] \\ = \quad & \epsilon^w (1+\varphi) E_t \left[\sum_{s=0}^{\infty} (\beta\psi_w)^s \chi \left(\frac{\tilde{W}_{i,t}}{W_{t+s}} \Pi_{j=1}^{j=s}\omega_{t+j}^{adj} \right)^{-\epsilon^w(1+\varphi)} \frac{1}{\tilde{W}_{i,t}} \frac{h_{t+s}^{1+\varphi}}{1+\varphi} \right]. \end{aligned}$$

This can be rewritten in the following way:

$$
\begin{aligned}
& \left(\epsilon^w-1\right)\left(\frac{\tilde{W}_{i,t}}{W_t}\right)^{1-\epsilon^w} E_t\left[\sum_{s=0}^{\infty}\left(\beta \psi_w\right)^s \lambda_{t+s} w_{t+s}\left(\frac{W_t}{W_{t+s}} \Pi_{j=1}^{j=s} \omega_{t+j}^{adj}\right)^{1-\epsilon^w} h_{t+s}\right] \\
= \quad & \epsilon^w(1+\varphi)\left(\frac{\tilde{W}_{i,t}}{W_t}\right)^{-\epsilon^w(1+\varphi)} E_t\left[\sum_{s=0}^{\infty}\left(\beta \psi_w\right)^s \chi\left(\frac{W_t}{W_{t+s}} \Pi_{j=1}^{j=s} \omega_{t+j}^{adj}\right)^{-\epsilon^w(1+\varphi)} \frac{h_{t+s}^{1+\varphi}}{1+\varphi}\right].
\end{aligned}
$$

Which we can rewrite in the following way:

$$
\left(\frac{\tilde{W}_{i,t}}{W_t}\right)^{1+\epsilon^w \varphi}=\chi\left(\frac{\epsilon^w}{\epsilon^w-1}\right) \frac{E_t\left[\sum_{s=0}^{\infty}\left(\beta \psi_w\right)^s \Pi_{j=1}^{j=s}\left(\frac{\omega_{t+j}}{\omega_{t+j}^{adj}}\right)^{\epsilon^w(1+\varphi)} h_{t+s}^{1+\varphi}\right]}{E_t\left[\sum_{s=0}^{\infty}\left(\beta \psi_w\right)^s \lambda_{t+s} w_{t+s} \Pi_{j=1}^{j=s}\left(\frac{\omega_{t+j}}{\omega_{t+j}^{adj}}\right)^{\epsilon^w-1} h_{t+s}\right]},
$$

where $\omega_{t+j}=W_{t+j}/W_{t-1+j}$ is the rate of gross nominal wage inflation from period $t-1+j$ to period $t+j$, which is related to the real wage rate $w_t \equiv W_t/P_t$ in the following way:

$$
\omega_t \equiv \frac{W_t}{W_{t-1}}=\left(\frac{w_t}{w_{t-1}}\right) \pi_t. \tag{145}
$$

Observe that except $\tilde{W}_{i,t}$ there is no labor union specific variable in the first order condition for labor union i. Therefore, all labor unions will choose the same wage nominal rate in equilibrium. As a result, we can drop the subscript i, and refer to the newly chosen wage rate as $\tilde{W}_t$. Now, we define $\omega_t^{new} \equiv \tilde{W}_t/W_t$, and rewrite the above equation in the following way:

$$
\omega_t^{new} = \chi\left(\frac{\epsilon^w}{\epsilon^w-1}\right) \frac{\Xi_{1,t}^w}{\Xi_{2,t}^w}, \tag{146}
$$

$$
\Xi_{1,t}^w = h_t^{1+\varphi}+E_t\left[\beta \psi_w\left(\frac{\omega_{t+1}}{\omega_{t+1}^{adj}}\right)^{\epsilon^w(1+\varphi)} \Xi_{1,t+1}^w\right], \tag{147}
$$

$$
\Xi_{2,t}^w = \lambda_t w_t h_t+E_t\left[\beta \psi_w\left(\frac{\omega_{t+1}}{\omega_{t+1}^{adj}}\right)^{\epsilon^w-1} \Xi_{2,t+1}^w\right], \tag{148}
$$

Now we look at the aggregate wage rate (143) in the economy:

$$W_t^{1-\epsilon^w} = (1-\psi_w)\left(\tilde{W}_t\right)^{1-\epsilon^w} + \psi_w(1-\psi_w)\left(\tilde{W}_{t-1}\omega_t^{adj}\right)^{1-\epsilon^w} + \psi_w^2(1-\psi_w)\left(\tilde{W}_{t-2}\omega_{t-1}^{adj}\omega_t^{adj}\right)^{1-\epsilon^w} \ldots.. \tag{149}$$

Iterating one period back, and multiplying the left and right hand side by $\psi_w\left(\omega_t^{adj}\right)^{1-\epsilon^w}$ gives the following expression:

$$\psi_w\left(\omega_t^{adj}\right)^{1-\epsilon^w} W_{t-1}^{1-\epsilon^w} = \psi_w(1-\psi_w)\left(\tilde{W}_{t-1}\omega_t^{adj}\right)^{1-\epsilon^w} + \psi_w^2(1-\psi_w)\left(\tilde{W}_{t-2}\omega_{t-1}^{adj}\omega_t^{adj}\right)^{1-\epsilon^w} + \ldots..$$

We see that the right hand side of the above expression is equal to the right hand side of equation (149) starting from the second term. Therefore, we can rewrite equation (149) in the following way:

$$W_t^{1-\epsilon^w} = (1-\psi_w)\left(\tilde{W}_t\right)^{1-\epsilon^w} + \psi_w\left(\omega_t^{adj}\right)^{1-\epsilon^w} W_{t-1}^{1-\epsilon^w}. \tag{150}$$

Division of the left and right hand side by $W_t^{1-\epsilon^w}$ gives our final expression for the law of moton for the aggregate wage rate:

$$1 = (1-\psi_w)\left(\omega_t^{new}\right)^{1-\epsilon^w} + \psi_w\left(\frac{\omega_t}{\omega_t^{adj}}\right)^{\epsilon^w-1}. \tag{151}$$

Next, we introduce the wage-dispersion parameter $\mathcal{D}_t^w \equiv \int_0^1\left(\frac{W_{i,t}}{W_t}\right)^{-\epsilon^w} di$:

$$\mathcal{D}_t^w = (1-\psi_w)\left(\frac{\tilde{W}_t}{W_t}\right)^{-\epsilon^w} + \psi_w(1-\psi_w)\left(\frac{\tilde{W}_{t-1}\omega_t^{adj}}{W_t}\right)^{-\epsilon^w} + \psi_w^2(1-\psi_w)\left(\frac{\tilde{W}_{t-2}\omega_{t-1}^{adj}\omega_t^{adj}}{W_t}\right)^{-\epsilon^w}. \tag{152}$$

Iterating one period back, and multiplying the left and right hand side by $\psi_w\left(\frac{W_{t-1}}{W_t}\omega_t^{adj}\right)^{-\epsilon^w}$ gives the following expression:

$$\psi_w\left(\frac{W_{t-1}}{W_t}\omega_t^{adj}\right)^{-\epsilon^w}\mathcal{D}_{t-1}^w = \psi_w(1-\psi_w)\left(\frac{\tilde{W}_{t-1}\omega_t^{adj}}{W_t}\right)^{-\epsilon^w} + \psi_w^2(1-\psi_w)\left(\frac{\tilde{W}_{t-2}\omega_{t-1}^{adj}\omega_t^{adj}}{W_t}\right)^{-\epsilon^w} + \ldots..$$

Hence we see that the right hand side of the above expression is equal to the right hand side of equation (152) starting from the second term. Therefore, we can rewrite equation (152) in the following way:

$$\mathcal{D}_t^w = (1-\psi_w)\left(\frac{\tilde{W}_t}{W_t}\right)^{-\epsilon^w} + \psi_w\left(\frac{W_{t-1}}{W_t}\omega_t^{adj}\right)^{-\epsilon^w}\mathcal{D}_{t-1}^w, \tag{153}$$

which we can rewrite as:

$$\mathcal{D}_t^w = (1-\psi_w)\left(\omega_t^{new}\right)^{-\epsilon^w} + \psi_w\left(\frac{\omega_t}{\omega_t^{adj}}\right)^{\epsilon^w}\mathcal{D}_{t-1}^w, \tag{154}$$

Finally, the wage indexation ω_t^{adj} depends on previous period nominal wage inflation ω_{t-1} in the following way:

$$\omega_t^{adj} = \omega_{t-1}^{\gamma_w}. \tag{155}$$

C.8.3 Aggregation

We start by remembering that $y_{j,t}^i = y_{j,t}^h = y_t^h\left(P_{j,t}^h/P_t^h\right)^{-\epsilon^p}$, for all j. Therefore, factor demands by intermediate firm i can be rewritten as:

$$h_{i,t} = (1-\alpha)m_t y_{j,t}^h/w_t, \quad k_{i,t-1} = \alpha m_t y_{j,t}^h/\left[q_{t-1}^k(1+r_t^k) - q_t^k(1-\delta)\xi_t\right]$$

Aggregate labor and capital are found by aggregating over all intermediate goods producing firms i:

$$h_t = (1-\alpha)m_t y_t^h \mathcal{D}_t^h/w_t, \quad k_{t-1} = \alpha m_t y_t^h \mathcal{D}_t^h/\left[q_{t-1}^k(1+r_t^k) - q_t^k(1-\delta)\xi_t\right]$$

where $\mathcal{D}_t^h = \int_0^1\left(P_{j,t}^h/P_t^h\right)^{-\epsilon^p}dj$ denotes price dispersion (114). Computation of the aggregate capital-labor ratio reveals that it equals the individual capital-labor ratio:

$$k_{t-1}/h_t \quad = \quad \alpha(1-\alpha)^{-1}w_t/\left[q_{t-1}^k(1+r_t^k) - q_t^k(1-\delta)\xi_t\right] = k_{i,t-1}/h_{i,t}. \tag{156}$$

Therefore, we see that the aggregate capital-labor ratio k_{t-1}/h_t is exactly equal to intermediate firm i's capital labor ratio $k_{i,t-1}/h_{i,t}$. Next, we remember from Appendix C.5.1 that the return on corporate loans and the wage rate are given by:

$$
\begin{aligned}
1+r_t^k &= \frac{\alpha m_t a_t \xi_t^\alpha \left(k_{i,t-1}\right)^{\alpha-1} h_{i,t}^{1-\alpha} + q_t^k (1-\delta)\xi_t}{q_{t-1}^k}, \\
w_t &= (1-\alpha) m_t a_t \left(\xi_t k_{i,t-1}\right)^\alpha h_{i,t}^{-\alpha}.
\end{aligned}
$$

Given that intermediate gods producers choose the same capital-labor ratio, we can immediately write:

$$
1+r_t^k = \frac{\alpha m_t a_t \xi_t^\alpha \left(k_{t-1}\right)^{\alpha-1} h_t^{1-\alpha} + q_t^k (1-\delta)\xi_t}{q_{t-1}^k}, \tag{157}
$$

$$
w_t = (1-\alpha) m_t a_t \left(\xi_t k_{t-1}\right)^\alpha h_t^{-\alpha}. \tag{158}
$$

Next, aggregation of $y_{i,t} = a_t \left(\xi_t k_{i,t-1}\right)^\alpha h_{i,t}^{1-\alpha}$ over all firms i delivers aggregate supply:

$$
\int_0^1 a_t \left(\xi_t k_{i,t-1}\right)^\alpha h_{i,t}^{1-\alpha} di = a_t \left(\frac{\xi_t k_{t-1}}{h_t}\right)^\alpha \int_0^1 h_{i,t} di = a_t \left(\xi_t k_{t-1}\right)^\alpha {h_t}^{1-\alpha}
$$

while aggregation over $y_{j,t}^h$ gives:

$$
\int_0^1 y_{j,t}^h dj = y_t^h \int_0^1 \left(P_{j,t}^h / P_t^h\right)^{-\epsilon^p} dj = y_t^h \mathcal{D}_t^h,
$$

which results in the following relation for aggregate supply y_t^h:

$$
y_t^h \mathcal{D}_t^h = a_t \left(\xi_t k_{t-1}\right)^\alpha {h_t}^{1-\alpha}. \tag{159}
$$

Next, we aggregate the relation for the production technology of retail import producers, i.e.

$y^{f*}_{m,t} = y^{f}_{m,t}$:

$$y^{f*}_t \equiv \int_0^1 y^{f*}_{m,t} dm = \int_0^1 y^{f}_{m,t} dm$$
$$= \int_0^1 \left(\frac{P^f_{m,t}}{P^f_t} \right)^{-\epsilon^m} y^f_t \, dm = \mathcal{D}^f_t y^f_t, \tag{160}$$

where we used the demand function (118) to replace $y^f_{m,t}$. Total imports im_t (in terms of the consumer price index) are given by:

$$im_t \equiv \frac{\int_0^1 P^{f*}_t y^{f*}_{m,t} dm}{P_t} = p^{f*}_t \int_0^1 y^{f*}_{m,t} dm = p^{f*}_t y^{f*}_t. \tag{161}$$

Now we aggregate the relation for the production technology of retail export producers, i.e. $x_{q,t} = y^x_{q,t}$:

$$x_t \equiv \int_0^1 x_{q,t} dq = \int_0^1 y^x_{q,t} dq$$
$$= \int_0^1 \left(\frac{P^x_{q,t}}{P^x_t} \right)^{-\epsilon^x} y^x_t \, dq = \mathcal{D}^x_t y^x_t, \tag{162}$$

where we used the demand function (130) to replace $y^x_{q,t}$. Total exports ex_t (in terms of the consumer price index) are given by:

$$ex_t \equiv \frac{P^x_t y^x_t}{P_t} = p^x_t y^x_t. \tag{163}$$

Finally, we look at the aggregate demand for final exports from foreign countries y^x_t, which we asssume takes the following functional form (Burriel et al., 2010):

$$y^x_t = \upsilon_x S^{\gamma*}_t y^*_t, \tag{164}$$

where S_t denotes the terms of trade, see below, and y^*_t denotes output from the rest of the monetary union.

C.9 Domestic output, imports & net international asset position

Domestic output y_t^h is absorbed by domestic households for consumption c_t^h, by capital producers for investment i_t^h, by the government g_t, and by retail export firms x_t. Therefore, the aggregate resource constraint of the domestic economy is given by:

$$y_t^h = c_t^h + i_t^h + g_t + x_t. \tag{165}$$

Final imported goods y_t^f must be equal in equilibrium to the demand for final imported goods by domestic consumers and domestic capital producers:

$$y_t^f = c_t^f + i_t^f. \tag{166}$$

The trade balance τ_t^b in terms of the domestic consumer price index P_t is given by the difference between exports (163) and imports (161):

$$\tau_t^b \equiv ex_t - im_t = p_t^x y_t^x - p_t^{f*} y_t^{f*}. \tag{167}$$

The law of motion for the internationally traded assets held by domestic households (divided by the risk-premium ψ_t^{nfa} and expressed in terms of the domestic consumer price index P_t) is given by sum of the trade balance τ_t^b and gross real interest payments on previous period holdings of the internationally traded asset $\left(1 + r_t^f\right) f_{t-1}$ the following equation

$$\frac{f_t}{\psi_t^{nfa}} = \tau_t^b + \left(1 + r_t^f\right) f_{t-1}. \tag{168}$$

C.10 Relations between prices and inflation rates

The terms of trade S_t is defined as the nominal price of imported foreign goods P_t^{f*} over the nominal price of domestic final export goods P_t^x:

$$S_t \equiv \frac{P_t^{f*}}{P_t^x}, \tag{169}$$

The real exchange rate $\mathcal{Q}_t$ is defined as the aggregate nominal foreign price level P_t^* over the aggregate domestic consumer price index P_t:

$$\mathcal{Q}_t \equiv \frac{P_t^*}{P_t}, \tag{170}$$

Now we define the relative price p_t^{f*} of the imported foreign good in terms of the domestic consumer price index:

$$p_t^{f*} \equiv \frac{P_t^{f*}}{P_t}.$$

Because of our assumption that the domestic economy is a small country within the monetary union, we can assume that the price of the imported foreign good is equal to the aggregate foreign price level: $P_t^{f*} = P_t^*$. This allows us to write p_t^{f*} in the following way:

$$p_t^{f*} = \frac{P_t^*}{P_t} = \mathcal{Q}_t. \tag{171}$$

Now we define the relative price p_t^x of the domestic export good in terms of the domestic consumer price index:

$$p_t^x \equiv \frac{P_t^x}{P_t} = \frac{P_t^x}{P_t^{f*}}\frac{P_t^{f*}}{P_t} = \frac{\mathcal{Q}_t}{S_t}. \tag{172}$$

Next, we derive the following relation for the change in the real exchange rate:

$$\frac{\mathcal{Q}_t}{\mathcal{Q}_{t-1}} = \frac{P_t^*/P_t}{P_{t-1}^*/P_{t-1}} = \frac{\pi_t^*}{\pi_t}. \tag{173}$$

Next, we express the inflation rates π_t^h, π_t^f, π_t^x, and π_t^i and in the following way:

$$\pi_t^h = \frac{P_t^h}{P_{t-1}^h} = \frac{P_t^h}{P_t} \cdot \frac{P_t}{P_{t-1}} \cdot \frac{P_{t-1}}{P_{t-1}^h} = \left(\frac{p_t^h}{p_{t-1}^h}\right) \pi_t. \tag{174}$$

$$\pi_t^f = \frac{P_t^f}{P_{t-1}^f} = \frac{P_t^f}{P_t} \cdot \frac{P_t}{P_{t-1}} \cdot \frac{P_{t-1}}{P_{t-1}^f} = \left(\frac{p_t^f}{p_{t-1}^f}\right) \pi_t. \tag{175}$$

$$\pi_t^x = \frac{P_t^x}{P_{t-1}^x} = \frac{P_t^x}{P_t} \cdot \frac{P_t}{P_{t-1}} \cdot \frac{P_{t-1}}{P_{t-1}^x} = \left(\frac{p_t^x}{p_{t-1}^x}\right) \pi_t. \tag{176}$$

$$\pi_t^i = \frac{P_t^i}{P_{t-1}^i} = \frac{P_t^i}{P_t} \cdot \frac{P_t}{P_{t-1}} \cdot \frac{P_{t-1}}{P_{t-1}^i} = \left(\frac{p_t^i}{p_{t-1}^i}\right) \pi_t. \tag{177}$$

$$\pi_t^{f*} = \frac{P_t^{f*}}{P_{t-1}^{f*}} = \frac{P_t^{f*}}{P_t} \cdot \frac{P_t}{P_{t-1}} \cdot \frac{P_{t-1}}{P_{t-1}^{f*}} = \left(\frac{p_t^{f*}}{p_{t-1}^{f*}}\right) \pi_t. \tag{178}$$

C.11 Exogenous processes

Below, we describe the exogenous processes:

$$\log(a_t) = \rho_a \log(a_{t-1}) + \varepsilon_{a,t}, \tag{179}$$

$$\log(\xi_t) = \rho_\xi \log(\xi_{t-1}) + \varepsilon_{\xi,t}, \tag{180}$$

$$\log(\tilde{g}_t/\bar{g}) = \rho_g \log(\tilde{g}_{t-1}/\bar{g}) + \varepsilon_{g,t}, \tag{181}$$

$$\log\left(\lambda_t^k/\bar{\lambda}_k\right) = \rho_{\lambda_k} \log\left(\lambda_{t-1}^k/\bar{\lambda}_k\right), \tag{182}$$

$$\lambda_t^b = \left(\bar{\lambda}_b/\bar{\lambda}_k\right) \lambda_t^b, \tag{183}$$

$$\log(y_t^*/\bar{y}^*) = \rho_{y*} \log\left(y_{t-1}^*/\bar{y}^*\right) + \varepsilon_{y*,t}, \tag{184}$$

$$\log(\pi_t^*/\bar{\pi}^*) = \rho_{\pi*} \log\left(\pi_{t-1}^*/\bar{\pi}^*\right) + \varepsilon_{\pi*,t}, \tag{185}$$

$$\log(\epsilon_t^c) = \rho_c \log\left(\epsilon_{t-1}^c\right) + \varepsilon_{c,t}, \tag{186}$$

$$\log\left(\epsilon_t^i\right) = \rho_i \log\left(\epsilon_{t-1}^i\right) + \varepsilon_{i,t}, \tag{187}$$

$$\log(\epsilon_t^p/\bar{\epsilon}_p) = \rho_{\epsilon_p} \log\left(\epsilon_{t-1}^p/\bar{\epsilon}_p\right) + \varepsilon_{\epsilon_p,t}, \tag{188}$$

$$\log(\epsilon_t^w/\bar{\epsilon}_w) = \rho_{\epsilon_w} \log\left(\epsilon_{t-1}^w/\bar{\epsilon}_w\right) + \varepsilon_{\epsilon_w,t}. \tag{189}$$

While most of theabove processes are standard, observe that the elasticity of substitution between different domestic retail goods (188) and the elasticity of substitution between labor from different labor unions (189) are time-varying.

C.12 Equilibrium Conditions

Let $\{c^r_{t-1}, c^u_{t-1}, s^{b,h}_{t-1}, f_{t-1}, d_{t-1}, s^{k,p}_{t-1}, s^{b,p}_{t-1}, n_{t-1}, k_{t-1}, i_{t-1}, b_{t-1}, y^{MU}_{t-1}, r^n_{t-1}, r^{n,f}_{t-1}, \pi_{t-1}, \mathcal{D}^p_{t-1}, \omega_{t-1}, \mathcal{Q}_{t-1}$ $, p^h_{t-1}, p^f_{t-1}, p^x_{t-1}, p^i_{t-1}, p^{f*}_{t-1}, \pi^h_{t-1}, \pi^f_{t-1}, \pi^x_{t-1}\}$ be the endogenous state-variables, while $\{z_t, \xi_t, \tilde{g}_t, y^*_t, \pi^*_t, \epsilon^c_t, \epsilon^i_t, \epsilon^p_t, \epsilon^w_t\}$ be the exogenous state-variables. A recursive competitive equilibrium is a sequence of quantities and prices $\{c_t, c^h_t, c^f_t, c^r_t, c^u_t, \lambda_t, \lambda^r_t, \lambda^u_t, h_t, s^{b,h}_t, f_t, \nu^k_t, \nu^b_t, \eta_t, \phi_t, \mu_t, n_t, s^k_t, s^b_t,$ $p_t, d_t, q^k_t, q^b_t, r^k_t, r^b_t, r^d_t, w_t, m_t, \mathcal{Q}_t, S_t, p^h_t, p^f_t, p^x_t, p^i_t, p^{f*}_t, \pi_t, \pi^h_t, \pi^{h,new}_t, \pi^{h,adj}_t, \Xi_{1,t}, \Xi_{2,t}, \mathcal{D}^h_t, \omega_t, \omega^{new}_t, \omega^{adj}_t, \Xi^w_{1,t},$ $\Xi^w_{2,t}, \mathcal{D}^w_t, \pi^f_t, \pi^{f*}_t, \pi^{f,new}_t, \pi^{f,adj}_t, \Xi^f_{1,t}, \Xi^f_{2,t}, \mathcal{D}^f_t, \pi^x_t, \pi^{x,new}_t, \pi^{x,adj}_t, \Xi^x_{1,t}, \Xi^x_{2,t}, \mathcal{D}^x_t, \pi^i_t, i_t, i^h_t, i^f_t, k_t, y^h_t, b_t, g_t, \tau_t, \tau^r_t, \tau^u_t,$ $r^n_t, r^{nf}_t, r^f_t, p^{def}_t, y^f_t, y^{f*}_t, y^x_t, x_t, ex_t, im_t, \tau^b_t, \psi^{nfa}_t, \pi^{MU}_t, y^{MU}_t\}$, and exogenous shocks $\{a_t, \xi_t, \tilde{g}_t, \lambda^k_t, \lambda^b_t, y^*_t, \pi^*_t, \epsilon^c_t, \epsilon^i_t, \epsilon^p_t, \epsilon^w_t\}$ such that:

1. Households optimize taking prices as given: (64) - (65), (70) - (73), (74), (78) - (79), and (81).

2. Financial intermediaries optimize taking prices as given: (84), (85), (89) - (91), (92) - (94), the endogenous leverage ratio (25), and the aggregate law of motion for net worth (26).

3. Capital producers optimize taking prices as given: (98) - (100) and (102) - (103).

4. Intermediate goods producers optimize taking prices as given, from which we can find the ex post return on corporate loans (157), the wage rate (158), and the aggregate supply relation (159).

5. Domestic retail goods producers that are allowed to choose prices optimize taking the input price m_t and the price of domestic final goods p^h_t as given: (107) - (109), (227), (114), and (115).

6. Retail import goods producers that are allowed to choose prices optimize taking the input price p^{f*}_t and the price of final import goods p^f_t as given: (120) - (122), (124), (126), and (127)

7. Retail export goods producers that are allowed to choose prices optimize taking the input price p_t^h and the price of the final export good p_t^x as given: (132) - (134), (136), (138), and (139)

8. Labor unions optimize taking aggregate labor demand and the aggregate nominal wage rate as given: (144), (145), (146) - (148), and (151), (154), (155).

9. Asset markets clear: (28) - (29).

10. The market for domestic goods clears: (31).

11. The market for final import goods clears: (166).

12. The relation between the demand and supply for final import goods: (160).

13. The relation between the demand and supply for final export goods: (162).

14. The relation between imports, exports, the foreign demand for final export goods, the trade balance, the law of motion for the internationally traded asset, and the risk-premium on the internationally traded asset hold: (161), (163), (164), (167), (268), and (67).

15. The fiscal variables evolve according to: (13) - (14), the level of lump sum taxes (15) on households of type $i \in \{r, u\}$, the default-exclusive return on government bonds (82), the probability of default (16), as well as the process for governemnt spending (19).

16. The monetary variables evolve according to: the Taylor rule on the nominal interest rate on deposits (10) and equation (11) with $x \in \{\pi, y\}$.

17. The relation between the ex post real interest rate and the nominal interest rate on deposits and foreign assets hold (68) - (69).

18. The relation between the relative prices, the real exchange rate, and the terms of trade: (171) - (173).

19. The relation between the relative prices and gross inflation rates: (174) - (178).

20. Exogenous processes evolve according to (179) - (189).

C.13 Introducing trend growth into the model

We introduce a trend X_t into the model that grows over time by a factor $\mu_t^x \equiv X_t/X_{t-1}$, similar to Burriel et al. (2010) and many other papers in the literature. Specifically, we transform all quantities q_t by writing them as $q_t \equiv X_t\tilde{q}_t$. We change the process for productivity a_t by assuming it can be written in the following way:

$$a_t \equiv \tilde{a}_t X_t^{1-\alpha},$$

where $\tilde{a}_t$ follows a regular AR(1) process. Therefore, we effectively have labor-augmenting technology growth (Pfeifer, 2018). We will see below that substitution of the above transformations results in first order conditions that feature the trend growth μ_t^x and quantities $\tilde{q}_t$. In addition, we find that the wage rate and the shadow value of households' budget constraint are also transformed, and can be written as $w_t = X_t\tilde{w}_t$, $\lambda_t = \tilde{\lambda}_t/X_t$, $\lambda_t^r = \tilde{\lambda}_t^r/X_t$, and $\lambda_t^u = \tilde{\lambda}_t^u/X_t$. Next, we define the process for μ_t^x (Burriel et al., 2010; Pfeifer, 2018):

$$\log \mu_t^x = \Lambda_x + \varepsilon_{x,t}, \tag{190}$$

where $\varepsilon_{x,t}$ is normally distributed with mean zero and standard deviation σ_x, and where Λ_x denotes trend growth. Now, we give the resulting, adjusted equilibrium definition, as well as the adjusted first order conditions.

A recursive competitive equilibrium is a sequence of quantities and prices $\{\tilde{c}_t, \tilde{c}_t^h, \tilde{c}_t^f, \tilde{c}_t^r, \tilde{c}_t^u, \tilde{\lambda}_t, \tilde{\lambda}_t^r, \tilde{\lambda}_t^u,$ $h_t, \tilde{s}_t^{b,h}, \tilde{f}_t, \nu_t^k, \nu_t^b, \eta_t, \phi_t, \mu_t, \tilde{n}_t, \tilde{s}_t^k, \tilde{s}_t^b, \tilde{p}_t, \tilde{d}_t, q_t^k, q_t^b, r_t^k, r_t^b, r_t^d, \tilde{w}_t, m_t, \mathcal{Q}_t, S_t, p_t^h, p_t^f, p_t^x, p_t^i, p_t^{f*}, \pi_t, \pi_t^h, \pi_t^{h,new}, \pi_t^{h,adj},$ $\Xi_{1,t}, \Xi_{2,t}, \mathcal{D}_t^h, \omega_t, \omega_t^{new}, \omega_t^{adj}, \Xi_{1,t}^w, \Xi_{2,t}^w, \mathcal{D}_t^w, \pi_t^f, \pi_t^{f*}, \pi_t^{f,new}, \pi_t^{f,adj}, \Xi_{1,t}^f, \Xi_{2,t}^f, \mathcal{D}_t^f, \pi_t^x, \pi_t^{x,new}, \pi_t^{x,adj}, \Xi_{1,t}^x, \Xi_{2,t}^x, \mathcal{D}_t^x, \pi_t^i,$ $\tilde{i}_t, \tilde{i}_t^h, \tilde{i}_t^f, \tilde{k}_t, \tilde{y}_t^h, \tilde{b}_t, \tilde{g}_t, \tilde{\tau}_t, \tilde{\tau}_t^r, \tilde{\tau}_t^u, r_t^n, r_t^{nf}, r_t^f, p_t^{def}, \tilde{y}_t^f, \tilde{y}_t^{f*}, \tilde{y}_t^x, \tilde{x}_t, \tilde{ex}_t, \tilde{im}_t, \tilde{\tau}_t^b, \psi_t^{nfa}, \pi_t^{MU}, \tilde{y}_t^{MU}\}$, exogenous shocks $\{\tilde{a}_t, \xi_t, \tilde{\tilde{g}}_t, \lambda_t^k, \lambda_t^b, \tilde{y}_t^*, \pi_t^*, \epsilon_t^c, \epsilon_t^i, \epsilon_t^p, \epsilon_t^w, \mu_t^x\}$, and observation variables $\{y_t^{obs}, c_t^{obs}, g_t^{obs}, ex_t^{obs}, im_t^{obs},$ $w_t^{obs}, h_t^{obs}, \pi_t^{obs}, R_t^{n,obs}, R_t^{k,obs}, R_t^n, R_t^k\}$ such that the following first order conditions hold.

C.13.1 Domestic households (constrained)

$$\tilde{c}_t^r + \tilde{\tau}_t^r = \tilde{w}_t h_t, \tag{191}$$

$$\tilde{\lambda}_t^r = \zeta_t \left(\tilde{c}_t^r - \upsilon \frac{\tilde{c}_{t-1}^r}{\mu_t^x} \right)^{-1} - \upsilon \beta E_t \left[\zeta_{t+1} \left(\mu_{t+1}^x \tilde{c}_{j,t+1}^r - \upsilon \tilde{c}_{j,t}^r \right)^{-1} \right], \tag{192}$$

C.13.2 Domestic households (unconstrained)

$$\tilde{\lambda}_t^u = \zeta_t \left(\tilde{c}_{j,t}^u - \upsilon \frac{\tilde{c}_{j,t-1}^u}{\mu_t^x} \right)^{-1} - \upsilon \beta E_t \left[\zeta_{t+1} \left(\mu_{t+1}^x \tilde{c}_{j,t+1}^u - \upsilon \tilde{c}_{j,t}^u \right)^{-1} \right], \tag{193}$$

$$1 = E_t \left[\beta \tilde{\Lambda}_{t,t+1}^u \psi_t^{nfa} \left(1 + r_{t+1}^d \right) \right], \tag{194}$$

$$1 = E_t \left[\beta \tilde{\Lambda}_{t,t+1}^u \psi_t^{nfa} \left(1 + r_{t+1}^f \right) \right], \tag{195}$$

$$1 = E_t \left[\beta \tilde{\Lambda}_{t,t+1}^u \left(\frac{\left(1 - p_{t+1}^{def} \vartheta_{def} \right) \left(1 + r_{t+1}^b \right) q_t^b}{q_t^b + \kappa_b \left(\tilde{s}_t^{b,h} - \hat{s}^{b,h} \right)} \right) \right], \tag{196}$$

$$\psi_t^{nfa} = \exp \left[-\kappa_{nfa} \left(\frac{\tilde{f}_t - \bar{\tilde{f}}}{\tilde{y}_t^h} \right) \right], \tag{197}$$

where $\tilde{\Lambda}_{t,t+1}^u = \frac{\tilde{\lambda}_{t+i}^u}{\mu_{t+1}^x \tilde{\lambda}_t^u}$.

C.13.3 Domestic households (aggregate)

$$\tilde{c}_t = \nu_r \tilde{c}_t^r + (1 - \nu_r) \tilde{c}_t^u. \tag{198}$$

$$\tilde{c}_t^h = (1 - \upsilon_c) \left(p_t^h \right)^{-\eta_c} \tilde{c}_t, \tag{199}$$

$$\tilde{c}_t^f = \upsilon_c \left(p_t^f \right)^{-\eta_c} \tilde{c}_t, \tag{200}$$

$$\tilde{c}_t = \left[(1 - \upsilon_c)^{\frac{1}{\eta_c}} \left(\tilde{c}_t^h \right)^{\frac{\eta_c - 1}{\eta_c}} + \upsilon_c^{\frac{1}{\eta_c}} \left(\tilde{c}_t^f \right)^{\frac{\eta_c - 1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c - 1}}, \tag{201}$$

C.13.4 Central Bank

$$r_t^n = (1-\rho_r)\left[\kappa_\pi\left(\pi_t^{MU}-\bar{\pi}^{MU}\right)+\kappa_y \log\left(\frac{\tilde{y}_t^{MU}}{\tilde{y}_{t-1}^{MU}}\right)\right]+\rho_r r_{t-1}^n+\varepsilon_{r,t}, \tag{202}$$

$$\pi_t^{MU} = \left(\pi_t^*\right)^{1-n}\left(\pi_t\right)^n, \tag{203}$$

$$\tilde{y}_t^{MU} = \left(\tilde{y}_t^*\right)^{1-n}\left(\tilde{y}_t^h\right)^n, \tag{204}$$

C.13.5 Fiscal authority

$$q_t^b\tilde{b}_t+\tilde{\tau}_t = p_t^h\tilde{g}_t+\left(1+r_t^b\right)q_{t-1}^b\frac{\tilde{b}_{t-1}}{\mu_t^x}, \tag{205}$$

$$1+r_t^b = \frac{x_c+(1-\rho)\,q_t^b}{\pi_t q_{t-1}^b}, \tag{206}$$

$$\tilde{\tau}_t = \nu_r\tilde{\tau}_t^r+(1-\nu_r)\,\tilde{\tau}_t^u. \tag{207}$$

$$\tilde{\tau}_t^r = \bar{\tilde{\tau}}^r+\zeta_b\left(\tilde{b}_{t-1}-\bar{\tilde{b}}\right)/\mu_t^x. \tag{208}$$

$$\tilde{\tau}_t^u = \bar{\tilde{\tau}}^u+\zeta_b\left(\tilde{b}_{t-1}-\bar{\tilde{b}}\right)/\mu_t^x. \tag{209}$$

$$p_t^{def} = F_\beta\left(\frac{\tilde{b}_t}{4\bar{\tilde{y}}}\frac{1}{\bar{\tilde{b}}_{max}};\alpha_b,\beta_b\right). \tag{210}$$

$$\tilde{g}_t = \bar{\tilde{g}}_t+\varsigma(\lambda_{t-l}^k-\bar{\lambda}^k), \tag{211}$$

C.13.6 Financial intermediaries

$$\nu_t^k = E_t\left[\tilde{\Omega}_{t+1}\left(r_{t+1}^k - r_{t+1}^d\right)\right], \tag{212}$$

$$\nu_t^b = E_t\left[\tilde{\Omega}_{t+1}\left(r_{t+1}^b - r_{t+1}^d - p_{t+1}^{def}\vartheta_{def}\left(1 + r_{t+1}^b\right)\right)\right], \tag{213}$$

$$\eta_t = E_t\left[\tilde{\Omega}_{t+1}\left(1 + r_{t+1}^d\right)\right], \tag{214}$$

$$\nu_t^b = \left(\frac{\lambda_t^b}{\lambda_t^k}\right)\nu_t^k, \tag{215}$$

$$\phi_t = \frac{\eta_t}{\lambda_t^k - \nu_t^k} \tag{216}$$

$$\tilde{n}_t = \theta\left[(r_t^k - r_t^d)q_{t-1}^k\tilde{s}_{t-1}^k + (r_t^b - r_t^d)q_{t-1}^b\tilde{s}_{t-1}^b + (1 + r_t^d)\tilde{n}_{t-1}\right]/\mu_t^x + \frac{\chi\tilde{p}_{t-1}}{\mu_t^x} \tag{217}$$

$$\tilde{p}_t = q_t^k\tilde{s}_t^k + q_t^b\tilde{s}_t^b, \tag{218}$$

$$\tilde{p}_t = \tilde{n}_t + \tilde{d}_t, \tag{219}$$

$$q_t^k\tilde{s}_t^k + \frac{\lambda_t^b}{\lambda_t^k}q_t^b\tilde{s}_t^b = \phi_t\tilde{n}_t, \tag{220}$$

$$\mu_t = \frac{\nu_t^k}{\lambda_t^k - \nu_t^k}. \tag{221}$$

$$1 + r_t^d = \frac{1 + r_{t-1}^n}{\pi_t}, \tag{222}$$

$$1 + r_t^f = \frac{1 + r_{t-1}^{nf}}{\pi_t}, \tag{223}$$

where $\tilde{\Omega}_{t,t+1} = \beta\tilde{\Lambda}_{t,t+1}^u\left(1 - \theta + \theta\left(\eta_{t+1} + \nu_{t+1}^k\phi_{t+1}\right)\right)$

C.13.7 Domestic retail firms

$$\pi_t^{h,new} = \left(\frac{\epsilon^p}{\epsilon^p - 1}\right)\frac{\Xi_{1,t}}{\Xi_{2,t}}, \tag{224}$$

$$\Xi_{1,t} = \tilde{\lambda}_t^u m_t \tilde{y}_t^h + E_t\left[\beta\psi_p\left(\frac{\pi_{t+1}^h}{\pi_{t+1}^{h,adj}}\right)^{\epsilon^p}\Xi_{1,t+1}\right], \tag{225}$$

$$\Xi_{2,t} = \tilde{\lambda}_t^u p_t^h \tilde{y}_t^h + E_t\left[\beta\psi_p\left(\frac{\pi_{t+1}^h}{\pi_{t+1}^{h,adj}}\right)^{\epsilon^p-1}\Xi_{2,t+1}\right]. \tag{226}$$

$$1 = (1-\psi_p)\left(\pi_t^{h*}\right)^{1-\epsilon^p} + \psi_p\left(\frac{\pi_t^h}{\pi_t^{h,adj}}\right)^{\epsilon^p-1}. \tag{227}$$

$$\mathcal{D}_t^h = (1-\psi_p)\left(\pi_t^{h*}\right)^{-\epsilon^p} + \psi_p\left(\frac{\pi_t^h}{\pi_t^{h,adj}}\right)^{\epsilon^p}\mathcal{D}_{t-1}^h, \tag{228}$$

$$\pi_t^{h,adj} = \left(\pi_{t-1}^h\right)^{\gamma_p}. \tag{229}$$

C.13.8 Export sector

$$\pi_t^{x,new} = \left(\frac{\epsilon^x}{\epsilon^x - 1}\right)\frac{\Xi_{1,t}^x}{\Xi_{2,t}^x}, \tag{230}$$

$$\Xi_{1,t}^x = \tilde{\lambda}_t^u p_t^h \tilde{y}_t^x + E_t\left[\beta\psi_x\left(\frac{\pi_{t+1}^x}{\pi_{t+1}^{x,adj}}\right)^{\epsilon^x}\Xi_{1,t+1}^x\right], \tag{231}$$

$$\Xi_{2,t}^x = \tilde{\lambda}_t^u p_t^x \tilde{y}_t^x + E_t\left[\beta\psi_x\left(\frac{\pi_{t+1}^x}{\pi_{t+1}^{x,adj}}\right)^{\epsilon^x-1}\Xi_{2,t+1}^x\right], \tag{232}$$

$$1 = (1-\psi_x)\left(\pi_t^{x*}\right)^{1-\epsilon^x} + \psi_x\left(\frac{\pi_t^x}{\pi_t^{x,adj}}\right)^{\epsilon^x-1}. \tag{233}$$

$$\mathcal{D}_t^x = (1-\psi_x)\left(\pi_t^{x*}\right)^{-\epsilon^x} + \psi_x\left(\frac{\pi_t^x}{\pi_t^{x,adj}}\right)^{\epsilon^x}\mathcal{D}_{t-1}^x, \tag{234}$$

$$\pi_t^{x,adj} = \left(\pi_{t-1}^x\right)^{\gamma_x}. \tag{235}$$

C.13.9 Import sector

$$\pi_t^{f,new} = \left(\frac{\epsilon^f}{\epsilon^f - 1}\right)\frac{\Xi_{1,t}^f}{\Xi_{2,t}^f}, \tag{236}$$

$$\Xi_{1,t}^f = \tilde{\lambda}_t^u p_t^{f*} \tilde{y}_t^f + E_t\left[\beta\psi_f\left(\frac{\pi_{t+1}^f}{\pi_{t+1}^{f,adj}}\right)^{\epsilon^f}\Xi_{1,t+1}^f\right], \tag{237}$$

$$\Xi_{2,t}^f = \tilde{\lambda}_t^u p_t^f \tilde{y}_t^f + E_t\left[\beta\psi_f\left(\frac{\pi_{t+1}^f}{\pi_{t+1}^{f,adj}}\right)^{\epsilon^f-1}\Xi_{2,t+1}^f\right], \tag{238}$$

$$1 = (1-\psi_f)\left(\pi_t^{f,new}\right)^{1-\epsilon^f} + \psi_f\left(\frac{\pi_t^f}{\pi_t^{f,adj}}\right)^{\epsilon^f-1}. \tag{239}$$

$$\mathcal{D}_t^f = (1-\psi_f)\left(\pi_t^{f,new}\right)^{-\epsilon^f} + \psi_f\left(\frac{\pi_t^f}{\pi_t^{f,adj}}\right)^{\epsilon^f}\mathcal{D}_{t-1}^f, \tag{240}$$

$$\pi_t^{f,adj} = \left(\pi_{t-1}^f\right)^{\gamma_f}. \tag{241}$$

C.13.10 Wage setting

$$\tilde{\lambda}_{t+s} = \nu_r\tilde{\lambda}_t^r + (1-\nu_r)\tilde{\lambda}_t^u, \tag{242}$$

$$\omega_t = \mu_t^x\left(\frac{\tilde{w}_t}{\tilde{w}_{t-1}}\right)\pi_t. \tag{243}$$

$$\omega_t^{new} = \chi\left(\frac{\epsilon^w}{\epsilon^w - 1}\right)\frac{\Xi_{1,t}^w}{\Xi_{2,t}^w}, \tag{244}$$

$$\Xi_{1,t}^w = h_t^{1+\varphi} + E_t\left[\beta\psi_w\left(\frac{\omega_{t+1}}{\omega_{t+1}^{adj}}\right)^{\epsilon^w(1+\varphi)}\Xi_{1,t+1}^w\right], \tag{245}$$

$$\Xi_{2,t}^w = \tilde{\lambda}_t\tilde{w}_t h_t + E_t\left[\beta\psi_w\left(\frac{\omega_{t+1}}{\omega_{t+1}^{adj}}\right)^{\epsilon^w-1}\Xi_{2,t+1}^w\right], \tag{246}$$

$$1 = (1-\psi_w)\left(\omega_t^{new}\right)^{1-\epsilon^w} + \psi_w\left(\frac{\omega_t}{\omega_t^{adj}}\right)^{\epsilon^w-1}. \tag{247}$$

$$\mathcal{D}_t^w = (1-\psi_w)\left(\omega_t^{new}\right)^{-\epsilon^w} + \psi_w\left(\frac{\omega_t}{\omega_t^{adj}}\right)^{\epsilon^w}\mathcal{D}_{t-1}^w, \tag{248}$$

$$\omega_t^{adj} = \omega_{t-1}^{\gamma_w}. \tag{249}$$

C.13.11 Capital producers

$$\tilde{i}_t = \left[(1-\upsilon_i)^{\frac{1}{\eta_i}} \left(\tilde{i}_t^h\right)^{\frac{\eta_i-1}{\eta_i}} + \upsilon_i^{\frac{1}{\eta_i}} \left(\tilde{i}_t^f\right)^{\frac{\eta_i-1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i-1}}, \tag{250}$$

$$\tilde{i}_t^h = (1-\upsilon_i) \left(\frac{p_t^h}{p_t^i}\right)^{-\eta_i} \tilde{i}_t, \tag{251}$$

$$\tilde{i}_t^f = \upsilon_i \left(\frac{p_t^f}{p_t^i}\right)^{-\eta_i} \tilde{i}_t, \tag{252}$$

$$\tilde{k}_t = (1-\delta)\xi_t \frac{\tilde{k}_{t-1}}{\mu_t^x} + \zeta_t^i \left[1 - \frac{1}{2}\gamma_k \left(\frac{\mu_t^x \tilde{i}_t}{\tilde{i}_{t-1}} - \Lambda_x \right)^2 \right] \tilde{i}_t, \tag{253}$$

$$\begin{aligned} \frac{p_t^i}{q_t^k} &= \left[1 - \frac{1}{2}\gamma_k \left(\frac{\mu_t^x \tilde{i}_t}{\tilde{i}_{t-1}} - \Lambda_x \right)^2 \right] \zeta_t^i - \frac{\gamma_k \mu_t^x \tilde{i}_t}{\tilde{i}_{t-1}} \left(\frac{\mu_t^x \tilde{i}_t}{\tilde{i}_{t-1}} - \Lambda_x \right) \zeta_t^i \\ &+ E_t \left[\beta \tilde{\Lambda}_{t,t+1}^u \frac{q_{t+1}^k}{q_t^k} \left(\frac{\mu_{t+1}^x \tilde{i}_{t+1}}{\tilde{i}_t} \right)^2 \gamma_k \left(\frac{\mu_{t+1}^x \tilde{i}_{t+1}}{\tilde{i}_t} - \Lambda_x \right) \zeta_{t+1}^i \right]. \end{aligned} \tag{254}$$

C.13.12 Intermediate goods producers

$$1 + r_t^k = \frac{\alpha m_t \tilde{a}_t \left(\mu_t^x\right)^{1-\alpha} \xi_t^\alpha \left(\tilde{k}_{t-1}\right)^{\alpha-1} h_t^{1-\alpha} + q_t^k (1-\delta)\xi_t}{q_{t-1}^k}, \tag{255}$$

$$\tilde{w}_t = (1-\alpha)\, m_t \tilde{a}_t \left(\mu_t^x\right)^{-\alpha} \left(\xi_t \tilde{k}_{t-1}\right)^\alpha h_t^{-\alpha}, \tag{256}$$

$$\tilde{y}_t^h \mathcal{D}_t^h = \tilde{a}_t \left(\mu_t^x\right)^{-\alpha} \left(\xi_t \tilde{k}_{t-1}\right)^\alpha {h_t}^{1-\alpha}. \tag{257}$$

C.13.13 Domestic market clearing

$$\tilde{k}_t = \tilde{s}_t^k, \tag{258}$$

$$\tilde{b}_t = \tilde{s}_t^b + \tilde{s}_t^{b,h}, \tag{259}$$

$$\tilde{y}_t^h = \tilde{c}_t^h + \tilde{i}_t^h + \tilde{g}_t + \tilde{x}_t, \tag{260}$$

$$\tilde{y}_t^f = \tilde{c}_t^f + \tilde{i}_t^f. \tag{261}$$

C.13.14 International trade

$$\tilde{y}_t^{f*} = \mathcal{D}_t^f \tilde{y}_t^f, \tag{262}$$

$$\tilde{im}_t = p_t^{f*} \tilde{y}_t^{f*}. \tag{263}$$

$$\tilde{x}_t = \mathcal{D}_t^x \tilde{y}_t^x, \tag{264}$$

$$\tilde{ex}_t = p_t^x \tilde{y}_t^x. \tag{265}$$

$$\tilde{y}_t^x = \upsilon_x S_t^{\gamma*} \tilde{y}_t^*, \tag{266}$$

$$\tilde{\tau}_t^b = p_t^x \tilde{y}_t^x - p_t^{f*} \tilde{y}_t^{f*}, \tag{267}$$

$$\frac{\tilde{f}_t}{\psi_t^{nfa}} = \tilde{\tau}_t^b + \left(1 + r_t^f\right) \frac{\tilde{f}_{t-1}}{\mu_t^x}. \tag{268}$$

C.13.15 Remaining prices

$$p_t^{f*} = \mathcal{Q}_t, \tag{269}$$

$$p_t^x = \frac{\mathcal{Q}_t}{S_t}, \tag{270}$$

$$\frac{\mathcal{Q}_t}{\mathcal{Q}_{t-1}} = \frac{\pi_t^*}{\pi_t}, \tag{271}$$

$$\pi_t^h = \left(\frac{p_t^h}{p_{t-1}^h}\right) \pi_t. \tag{272}$$

$$\pi_t^f = \left(\frac{p_t^f}{p_{t-1}^f}\right) \pi_t. \tag{273}$$

$$\pi_t^x = \left(\frac{p_t^x}{p_{t-1}^x}\right) \pi_t. \tag{274}$$

$$\pi_t^i = \left(\frac{p_t^i}{p_{t-1}^i}\right) \pi_t. \tag{275}$$

$$\pi_t^{f*} = \left(\frac{p_t^{f*}}{p_{t-1}^{f*}}\right) \pi_t. \tag{276}$$

C.14 Exogenous processes

$$\log(\tilde{a}_t) = \rho_a \log(\tilde{a}_{t-1}) + \varepsilon_{a,t}, \quad (277)$$

$$\log(\xi_t) = \rho_\xi \log(\xi_{t-1}) + \varepsilon_{\xi,t}, \quad (278)$$

$$\log\left(\tilde{g}_t/\bar{\tilde{g}}\right) = \rho_g \log\left(\tilde{g}_{t-1}/\bar{\tilde{g}}\right) + \varepsilon_{g,t}, \quad (279)$$

$$\log\left(\lambda_t^k/\bar{\lambda}_k\right) = \rho_{\lambda_k} \log\left(\lambda_{t-1}^k/\bar{\lambda}_k\right), \quad (280)$$

$$\lambda_t^b = \left(\bar{\lambda}_b/\bar{\lambda}_k\right)\lambda_t^b, \quad (281)$$

$$\log(\tilde{y}_t^*/\bar{\tilde{y}}^*) = \rho_{y*} \log\left(\tilde{y}_{t-1}^*/\bar{\tilde{y}}^*\right) + \varepsilon_{y*,t}, \quad (282)$$

$$\log(\pi_t^*/\bar{\pi}^*) = \rho_{\pi*} \log\left(\pi_{t-1}^*/\bar{\pi}^*\right) + \varepsilon_{\pi*,t}, \quad (283)$$

$$\log(\epsilon_t^c) = \rho_c \log\left(\epsilon_{t-1}^c\right) + \varepsilon_{c,t}, \quad (284)$$

$$\log\left(\epsilon_t^i\right) = \rho_i \log\left(\epsilon_{t-1}^i\right) + \varepsilon_{i,t}, \quad (285)$$

$$\log(\epsilon_t^p/\bar{\epsilon}_p) = \rho_{\epsilon_p} \log\left(\epsilon_{t-1}^p/\bar{\epsilon}_p\right) + \varepsilon_{\epsilon_p,t}, \quad (286)$$

$$\log(\epsilon_t^w/\bar{\epsilon}_w) = \rho_{\epsilon_w} \log\left(\epsilon_{t-1}^w/\bar{\epsilon}_w\right) + \varepsilon_{\epsilon_w,t}. \quad (287)$$

$$\mu_t^x = \Lambda_x + \varepsilon_{x,t}. \quad (288)$$

C.14.1 Observation equations

$$
\begin{aligned}
y_t^{obs} &= \log \tilde{y}_t - \log \tilde{y}_{t-1} + \log \mu_t^x, && (289)\\
c_t^{obs} &= \log \tilde{c}_t - \log \tilde{c}_{t-1} + \log \mu_t^x, && (290)\\
g_t^{obs} &= \log \tilde{g}_t - \log \tilde{g}_{t-1} + \log \mu_t^x, && (291)\\
ex_t^{obs} &= \log \tilde{ex}_t - \log \tilde{ex}_{t-1} + \log \mu_t^x, && (292)\\
im_t^{obs} &= \log \tilde{im}_t - \log \tilde{im}_{t-1} + \log \mu_t^x, && (293)\\
w_t^{obs} &= \log \tilde{w}_t - \log \tilde{w}_{t-1} + \log \mu_t^x, && (294)\\
h_t^{obs} &= \log h_t - \log h_{t-1}, && (295)\\
\pi_t^{obs} &= \log \pi_t - \log \bar{\pi}, && (296)\\
R_t^{n,obs} &= R_t^n - \bar{R}^n, && (297)\\
R_t^{k,obs} &= R_t^k - \bar{R}^k, && (298)\\
R_t^n &= 1 + r_t^n, && (299)\\
R_t^k &= \frac{1 + r_t^k}{\pi_t}, && (300)
\end{aligned}
$$

D Calibration & estimation

We start this section by discussing the data, after which we discuss the calibrated parameters and the priors of the variables we are estimating. We conclude by reporting the posterior distributions.

D.1 Data

The frequency of our model is quarterly. We therefore construct quarterly time series from Spanish data which proxy ten variables in our model: real output per capita, real consumption per capita, real government spending per capita, real exports per capita, real imports per capita, real wages, hours worked per capita, gross inflation of the consumer price index, the 3-month Spanish interest rate, and the real return on loans to non-financial corporations.

We obtain the national accounts data for output, consumption, government spending, exports, and imports from Eurostat. Subsequently, we calculate the gross inflation rate of the consumer price index by dividing nominal consumption by real consumption.

We use the publicly available database used by Burriel et al. (2010) to obtain time series for the size of the population above 16 years old, total nominal wage payments, total hours worked, and a time series for the Spanish Non transferable three-month deposit rate.[33] We use the time series for the population above 16 years to convert real output, consumption, government spending, exports, and imports into their per capita equivalent. We divide total nominal wage payments by total hours worked to obtain a time series for the nominal wage rate per hour worked, which we divide by the consumer price index to obtain the real wage rate per hour worked. Next, we divide total hours worked by the population size to obtain a time series for hours worked per capita.

Next, we obtain a time series for the nominal interest rate on loans to Spanish non-financial corporations from the website of the Bank of Spain. This is a monthly time series, of which we take the end of quarter value as representing the interest rate for that particular quarter.[34] Next, we take the time series for the nominal interest rate on loans to non-financial corporations, and divide by 400 to convert from annual percentages into quarterly decimals, after which we divide 1 plus the resulting time series by the gross inflation rate of the consumer price index to obtain the gross real interest rate on loans to non-financial corporations.

We take the log of real output per capita, real consumption per capita, real government spending per capita, real exports per capita, real imports per capita, the real wage rate per hour worked, and hours worked per capita. Afterwards, we take the difference between the resulting time series and their lags to obtain the unfiltered growth rate of these variables. As there is no trend growth for hours worked per capita in our model, we demean this time series before feeding it into the

[33] The database of Burriel et al. (2010) was downloaded in May 2018, and can be obtained through the following link: http://www.sepg.pap.hacienda.gob.es/sitios/sepg/en-GB/Presupuestos/Documentacion/Paginas/BasedatosmodeloREMS.aspx. An explanation of the computation of the time series can be found in Bosc et al. (2007).

[34] The time series was obtained through the following link: https://www.bde.es/webbde/en/estadis/infoest/temas/sb_tiif.html. On this webpage, go to the header "Other supplementary information", and download data from "Main economic indicators of Spain. Interest rates (table 2.10 of the SB)", where "SB" is an abbreviation for "Statistical Bulletin". Within this table, we download the time series "Credit institutions. New Business (CBE 4/2002), Loans, Synthetic rate Non-financial corporations" as a time series for the nominal interest rate on loans to non-financial corporations.

Bayesian estimation.

Finally, we compute a time series for the leverage ratio by downloading time series for total assets and capital & reserves for "Other monetary financial institutions" (OMFIs).[35] Again, these are monthly data, of which we use the end of quarter value as representing the value for that particular quarter.[36] We get a time series for the leverage ratio by dividing total assets over capital & reserves.

D.2 Calibration

For the calibration, we either take parameter values from the literature, or target key first order moments, which can be found in Table 5. These include steady state labor supply, terms of trade, inflation, the investment-output ratio, the government spending-output ratio, the government debt over output ratio, the fraction of bonds held by financial intermediaries, the coupon payment on bonds, the average duration of outstanding Spanish government debt, the probability of default, the "maximum" level of government debt as a percentage of output, the relative diversion rate for government bonds over corporate loans, and the leverage ratio. We take the average over 2003Q1-2010Q4 wherever possible.

In line with much of the macroeconomic literature, we set the subjective discount factor β equal to 0.99. We set steady state consumption of constrained households equal to that of unconstrained households by adjusting steady state lump sum taxes $\bar{\tau}^r$ and $\bar{\tau}^u$ (Gali et al., 2007). We set steady state labor supply equal to 1/3, which coincides with the fraction per day spent on work (8/24). To hit this last target, we manually adjust the coefficient Ψ in front of the term with disutility from labor supply. We set the coefficient in front of unconstrained households' adjustment costs for government bonds (κ_b) equal to 0.015, which is close to the posterior mean of 0.009 found in Kühl (2018). Observe, however, that Kühl (2018) estimate a closed economy model with the help

[35] OMFIs refer to credit institutions and specialised lending institutions with access to the balance sheet of the ECB.

[36] The data were downloaded from the webpage https://www.bde.es/webbde/en/estadis/infoest/temas/sb_ifiescbs.html#ec, where we went to the header "A2) Other monetary financial institutions, Balance sheets according to euro area returns", and then under "Breakdown of liabilities" we downloaded "Summary (table 8.3 of the SB)", where "SB" refers to "Statistical Bulletin". We used the time series for "Capital and reserves" and "Total liabilities", the last of which coincides with total assets.

Target	Definition	Value	Data
$\bar{h}$	Labor supply	1/3	8 hours of work per day
$\bar{S}$	Terms of trade	1	Literature
$\bar{\pi}$	Domestic inflation	1.005	2% annual net inflation
$\bar{\pi}^*$	Foreign inflation	1.005	2% annual net inflation
$\bar{i}/\bar{y}$	Investment ratio	0.226	2003Q1-2010Q4 average
$\bar{g}/\bar{y}$	Government spending ratio	0.178	2003Q1-2010Q4 average
$\bar{q}_b\bar{b}/\bar{y}$	Government debt ratio	3.2	Maastricht criteria
$\bar{s}^b/\bar{b}$	Fraction of bonds held by int.	0.25	2003Q1-2010Q4
x_c	Coupon payment bonds	4.1%	2003Q1-2010Q4 average 10y-yield
$1/\left[1-\beta\left(1-\rho\right)\right]$	Duration bonds	20	Bank of Spain
$\bar{p}_{def}$	Probability of default	0.0050	Schabert and van Wijnbergen (2014)
$\bar{q}_b b_{max}/\bar{y}$	"Maximum" gov't debt ratio	2.4	Maastricht criteria
$\bar{\lambda}_b/\bar{\lambda}_k$	Relative diversion rate	0.5	Gertler and Karadi (2013)
$\bar{\phi}$	Leverage ratio	6.4848	2003Q1-2010Q4 average

Table 5: List of steady state calibration targets and source of calibration for the model version without financial frictions and sovereign default risk.

of aggregate Eurozone data, whereas we estimate a small open economy model with the help of Spanish data. To check the extent to which this choice affects our results, we perform a robustness check with alternative values for κ_b. As we target the steady state fraction of bonds held by financial intermediaries, see below, and manually set κ_b, we adjust $\hat{s}^{b,h}$ to ensure that the first two targets are hit. The parameter κ_{nfa} and the net international investment position $\bar{f}$ in the expression for the international risk premium (67) are determined in the following way. First, we set $\kappa_{nfa} = 0.01$, which is small enough to ensure that the international risk premium has no discernible effects on the transition dynamics (Eggertsson et al., 2014). Second, we set the steady state net international investment position equal to -34% of quarterly domestic output, or -8.6% of annual domestic output. This is in line with the fact that Spain was a net debtor to the rest of the world during our estimation period.

We take the calibrated value for α from Burriel et al. (2010), as well as the posterior means for the steady state elasticity of substitution between domestic retail goods $\bar{\epsilon}^p$, the steady state elasticity of substitution between different labor unions $\bar{\epsilon}^w$, the consumption elasticity of substitution between domestic and foreign final goods η_c, and the investment elasticity of substitution between domestic and foreign final goods η_i. We set the steady state elasticity of substitution between retail import goods $\bar{\epsilon}^f$ and the steady state elasticity of substitution between retail export goods $\bar{\epsilon}^x$ equal to 8. As Spain is part of the European Union's single market, we set the steady state share of foreign goods in the consumption and investment bundles equal to 0.5. We follow the literature by setting the steady state terms of trade $\bar{S} = 1$, and follow many in the literature by setting γ^* in equation (164) equal to 1.

The steady state gross inflation rate of the domestic and foreign consumer price index, respectively, are set such that annual net inflation is equal to 2%, which is equal to the inflation target of the European Central Bank. Therefore, we can immediately infer that steady state inflation in the monetary union $\bar{\pi}^{MU}$ coincides with a 2% annual net inflation rate. Furthermore, we set the Taylor rule parameters κ_π and κ_y to values that are standard in the literature. We set the weight n of Spanish macro developments in the Taylor rule (10) equal to 0.10, as Spanish GDP comprises approximately 10% of Eurozone GDP. We set the interest rate smoothing parameter ρ_r equal to zero, as there will already be limited feedback from Spanish macrodevelopments on the nominal interest rate set by the central bank. Finally, we manually set the standard deviation of the monetary policy shock σ_r equal to 25 basis points, which is the standard change when the ECB adjusts its policy rate.

Steady state investment over GDP is computed as the average ratio of private investment over GDP in the sample period (2003Q1-2010Q4), which is equal to 22.6%. Similarly, we find the average ratio of government consumption over GDP over the same period to be 17.8%. We use the sovereign debt database of Merler and Pisani-Ferry (2012) to calculate the average fraction of Spanish government bonds held by Spanish OMFIs, which is equal to approximately 25% over the period 2003Q1-2010Q4.[37] While consolidated gross debt of the central government was on average 45.5% of Spanish GDP during our estimation period, we set the steady state debt-GDP ratio $\bar{q}^b\bar{b}$ equal to 80% of annual GDP, which implies banks' steady state sovereign debt holdings equal 135% of their net worth. This is substantially below the sovereign debt exposure of 150% of Tier-1 capital in Figure 2, and would have been even lower had we used the 45.5% number. Setting steady state debt equal to 80% of annual GDP therefore keeps the middle between the 45.5% average debt-GDP

[37] "OMFIs" is an abbreviation for "Other Monetary Financial Institutions", which are credit institutions and specialised lending institutions with access to the ECB balance sheet.

ratio and a much higher debt-GDP ratio necessary to match the 150% of Tier-1 capital. In addition, the higher debt ratio is also more in line with the stock of outstanding Spanish sovereign debt during the European sovereign debt crisis of 2011-2013, during which the average level of public debt was equal to 86.8% of Spanish GDP.

The fixed cash flow payment x_c is set to 4.1%, which is close to the average interest rate on Spanish government bonds according to the database of the Spanish Treasury over the period 2003-2010.[38] Our modeling of long-term bonds allows us to calculate the average duration of the bonds (Woodford, 1998, 2001).[39] We set the average duration equal to 20 quarters (5 years), which implies that $\rho = 0.04$. Unfortunately, we cannot directly observe the average duration from the data. However, it is possible to find the average maturity of Spanish government debt over the period 2003-2010, which is approximately 6 years.[40] Therefore, an average duration of 5 years seems reasonable. We set the feedback from the level of government debt to the level of lump sum taxes raised on households ζ_b in equation (15) equal to 0.05. Although the presence of balance-sheet-constrained financial intermediaries and households subject to quadratic adjustment costs breaks Ricardian equivalence, this does not affect our results qualitatively as taxes are not distortionary, see also Bocola (2016).

We set the haircut parameter ϑ equal to 0.5 following Corsetti et al. (2013). Regarding the calibration of the default probability function (16), we apply the following targets: we set $\bar{q}^b\bar{b}_{max}$ equal to 60% of annual output, in line with the Maastricht criterium. As mentioned in the main text, $\bar{b}_{max}$ is not an actual maximum level of debt, since steady state government debt $\bar{q}^b\bar{b}$ is equal to 80% of GDP in our calibration. Instead of thinking of $\bar{q}^b\bar{b}_{max}$ as an upper limit for the amount of government debt, one can think of it as the maximum level of debt prescribed by the Maastricht Treaty, which says that government debt as a percentage of GDP should not be above 60%. However, there is no economic reason why government debt cannot be above 60% of GDP,

[38]The average interest rate on government debt can be found via https://www.tesoro.es/en/deuda-publica/estadisticas-mensuales under the header "Euro - denominated Government Debt: Average interest rate outstanding debt".

[39]Average duration is calculated as $\frac{\sum_{j=1}^{\infty}\beta^j j(1-\rho)^j}{\sum_{j=1}^{\infty}\beta^j(1-\rho)^j} = \frac{1}{1-\beta(1-\rho)}$

[40]The average maturity on government debt can be found via https://www.tesoro.es/en/deuda-publica/estadisticas-mensuales under the header "Government Debt: Average maturity".

which is confirmed by the fact that many Eurozone countries have debt levels above 60% of GDP.

To find the parameters α_b and β_b, we use the following targets: we set the steady state default probability equal to $\bar{p}_{def} = 0.0050$, which implies a 2% annual default probability (Schabert and van Wijnbergen, 2014). This value is also in line with 5-year CDS spreads on Spanish government bonds at the end of 2010. We also target the first derivative of the default probability function (16) with respect to government bonds b_t, and set it equal to 0.2 in the steady state. This results in a spread of 100 basis points (annually) between the steady state return on government bonds for the model version which includes long-term bonds and sovereign default risk on the one hand, and the steady state return on bonds for the model version without sovereign risk on the other hand. This difference is rather conservative when compared with a spread of 200 basis points between 10 year Spanish government bonds and 10 year German Bunds at the end of 2010. By setting these two targets, we find $\alpha_b = 33.4251$ and $\beta_b = 15.9481$. In addition, we will see in our simulations that the probability of default will increase by 5 basis points in response to a fiscal stimulus of 0.5% of quarterly output on impact. Therefore, the default elasticity, as defined by $\frac{b_t}{1-p_t^{def}} \cdot \frac{\Delta p_t^{def}}{\Delta b_t} = 0.003.$[41] This is relatively small, as Schabert and van Wijnbergen (2014), for example, work with a default elasticity of 0.01.

We set the ratio of the diversion rate for government bonds over the diversion rate of corporate loans λ_t^b/λ_t^k equal to 0.5 in all periods, as in Gertler and Karadi (2013).[42] We calculate the average of the time series for the leverage ratio that was described in the previous section over the period 2003Q1-2010Q4, and divide the resulting value by 2 to obtain $\bar{\phi} = 6.4848$ in equation (25). In doing so, we follow Gertler and Karadi (2013), who explain that the loans to the private sector are state-contingent in our setup, and thus more equity-like. Therefore, net worth of financial intermediaries

[41]The underlying calculation is $\frac{b_t}{1-p_t^{def}} \cdot \frac{\Delta p_t^{def}}{\Delta b_t} = \frac{3.2}{1-0.005} \cdot \frac{0.0005}{0.5} = 0.003.$

[42]Note that λ_t^k and λ_t^b are not legal capital requirements, in which case λ_t^b should be equal to zero according to Basel III regulations, but are rather constraints imposed by depositors on financial intermediaries within a market transaction. In fact, literally following the original interpretation of Gertler and Karadi (2011) could lead one to argue that λ_t^b should be larger than λ_t^k, as government bonds are typically more liquid than corporate securities, and can therefore more easily be diverted by bank managers. We, however, think of the incentive compatibility constraint as capturing in reduced form financial frictions that give rise to a return difference between assets and deposit funding. As this spread is typically larger for corporate securities than for government bonds, we set $\lambda_t^k > \lambda_t^b$, in line with Gertler and Karadi (2013) and many others in this literature. As there was a substantial spread between the yield on Spanish government debt and the deposit rate during the European sovereign debt crisis, which is the relevant period to look at for our research question, we set $\lambda_t^b > 0$

will be more volatile, everything else equal, than when intermediaries provide corporate loans with a fixed principal, which is the dominant form of private sector credit in reality. Therefore, a lower steady state leverage ratio compensates for the higher volatility induced by our equity-like corporate loans. Our leverage ratio results in a sovereign debt exposure of 135% of net worth, a number which is conservative relative to that in Figure 2.[43] We set θ, the probability that financial intermediaries are allowed to continue operating, equal to 0.92, which corresponds to an average lifetime of bankers equal to 12.5 quarters. This is below typical values found in the literature (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011, 2013). We do so, however, as otherwise the parameter χ, which is the aggregate fraction of previous period assets that is provided to new bankers, would be negative.

Parameters that are manually adjusted to match the calibration targets include the duration parameter of bonds ρ, the depreciation parameter δ, the disutility weight of labor Ψ, the diversion rate $\bar{\lambda}_b$ of bankers, the fraction χ of previous period aggregate assets that goes to new bankers, the steady state level of lump sum taxes $\bar{\tau}^r$ and $\bar{\tau}^u$, and the steady state level of the net foreign asset position $\bar{f}$. We are capable of targeting first order moments, as well as estimating parameters that affect some of those same first order moments by writing a separate steady state file which dynare reads into during the estimation, see Pfeifer (2018).

Finally, we set the persistence parameter ρ_{λ_k}, which determines how fast the diversion rate of corporate loans λ_t^k returns to its steady state after a one-off MIT shock, equal to 0.7. This implies that output is back at the pre-crisis level of output after 20 quarters, or 5 years. Table 6 contains all the parameter values that are not estimated.

[43]We calculate steady state net worth from equation (25). From this equation, we immediately see that the leverage ratio directly affects the level of steady state net worth, and thus the steady state ratio of intermediaries' bond holdings over net worth $\bar{q}^b\bar{s}^b/\bar{n}$.

Parameter	Definition	Value
Domestic households		
β	Subjective discount factor	0.990
Ψ	Disutility weight of labour	23.7411
κ_b	Gov't bond adjustment costs	0.015
$\hat{s}^{b,h}$	Gov't bond reference level	2.4552
υ_c	Import share dom. consumption bundle	0.5
η_c	Consumption elast. of subst. dom. & for. goods	7.512
κ_{nfa}	International risk premium	0.01
Foreign households		
γ^*	Elasticity demand for dom. export goods	1
Financial Intermediaries		
θ	Survival rate of bankers	0.92
λ_b	Steady state diversion rate domestic bonds	0.3210
χ	Transfer share to new bankers	0.0010
Production Sector		
α	Capital share	0.3621
δ	Depreciation	0.0612
υ_i	Import share dom. investment bundle	0.5
η_i	Investment elast. of subst. dom. & for. goods	7.851
Elasticity of substitution		
$\bar{\epsilon}^p$	Elasticity of substit. (goods)	8.577
$\bar{\epsilon}^f$	Elasticity of substit. (imports)	8
$\bar{\epsilon}^x$	Elasticity of substit. (exports)	8
$\bar{\epsilon}^w$	Elasticity of substit. (labor)	7.758
Government debt		
ρ	Government debt maturity parameter	0.04
x_c	Nominal coupon payment to bondholder	0.041
Sovereign default		
ϑ^{cf}_{def}	Haircut (cash flow)	0.5
ϑ^{p}_{def}	Haircut (principal)	0.5
α_b	First parameter beta-cdf	33.4251
β_b	Second parameter beta-cdf	15.9481
Policy Parameters		
ζ_b	Gov. debt feedback on taxes	0.05
$\bar{\pi}^{MU}$	Inflation rate target monetary union	1.005
κ_π	Inflation feedback parameter	1.500
κ_y	Output feedback parameter	0.125
n	Weight Spain Taylor rule	0.1
ρ_r	Interst rate smoothing par.	0
σ_r	Std. dev. interest rate shock	0.0025
Shock process		
ρ_{λ_k}	Diversion rate shock	0.7

Table 6: List of calibrated parameter values and source of calibration.

D.3 Bayesian estimation of remaining parameters

Tables 7 and 8 show the priors and posteriors for the parameters that are estimated using Bayesian techniques. Specifically, Table 7 contains the deep parameters that affect the steady state of the model, while Table 8 contains the parameters that determine the persistence and the standard deviation of the exogenous processes.

We follow Darracq-Paris and Kühl (2017) for the priors of habit formation, the Frisch-elasticity, and the probability of not being able to choose new prices and wages ("Calvo"). An exception is the probability ψ_p of not being able to choose a new price for domestic retail firms, for which we set the prior mean at 0.8. We do so, as setting the prior mean at 0.5 would result in a posterior mean of ψ_p that is unrealistically low with respect to the literature. We choose a relatively large standard deviation for the prior distribution of the remaining parameters, as we have no additional information that warrants a tighter prior.

	Parameter	Distrib.	Mean	Std. dev.	Mean	5%	Mode	95%
υ	Habit formation	Normal	0.7	0.1	0.6511	0.5391	0.6981	0.7640
φ	Frisch-elasticity	Gamma	2	0.75	2.0665	1.0198	4.7999	3.0630
ψ_p	Calvo (domestic)	Beta	0.8	0.1	0.4868	0.3532	0.4397	0.6178
ψ_w	Calvo (wages)	Beta	0.5	0.1	0.3507	0.2161	0.3668	0.4913
ψ_f	Calvo (imports)	Beta	0.5	0.1	0.0835	0.0471	0.0795	0.1144
ψ_x	Calvo (exports)	Beta	0.5	0.1	0.5557	0.3939	0.5772	0.7386
γ_p	Indexation (dom.)	Beta	0.5	0.2	0.2603	0.0305	0.1243	0.4777
γ_w	Indexation (wages)	Beta	0.5	0.2	0.1025	0.0073	0.0334	0.1886
γ_f	Indexation (imports)	Beta	0.5	0.2	0.1578	0.0118	0.1063	0.2931
γ_x	Indexation (exports)	Beta	0.5	0.2	0.5234	0.1996	0.5187	0.8516
υ_x	Exports demand	Beta	0.5	0.2	0.4910	0.4250	0.4909	0.5585
Λ_x	Trend growth	Invg.	0.01	0.05	0.0048	0.0027	0.0073	0.0066
$\bar{\lambda}_k$	Div. rate corp. loans	Beta	0.5	0.1	0.6420	0.5023	0.6507	0.7679
ν_r	Fraction constr. HH	Beta	0.5	0.2	0.2003	0.0734	0.2522	0.3148
γ	Invest. adj. cost	Gamma	2.5	1	0.8189	0.4002	0.6749	1.2434

Table 7: Priors (columns 3-5) and posteriors (columns 6-9) of the parameters that are estimated with Bayesian techniques. The results are based on 2 chains, each with 500,000 draws based on the Metropolis-Hastings algorithm. "Invg." is an abbreviation for the inverse Gamma distribution. "Calvo" refers to the probability of not being able to change nominal prices or wages. "Domestic" refers to domestic retail goods.

Tables 7 and 8 also report the summary statistics of the posterior distribution of the parameters.

	Parameter	Distrib.	Mean	Std. dev.	Mean	5%	Mode	95%
AR coef.								
ρ_z	Productivity	Beta	0.8	0.1	0.9426	0.8938	0.9728	0.9887
ρ_ξ	Capital quality	Beta	0.8	0.1	0.8580	0.7996	0.8551	0.9273
ρ_g	Gov't spending	Beta	0.8	0.1	0.9501	0.9160	0.9572	0.9883
ρ_c	Preference	Beta	0.8	0.1	0.7566	0.6251	0.7531	0.8927
ρ_i	Invest. adj.	Beta	0.8	0.1	0.7246	0.5706	0.7646	0.8894
ρ_p	Price-elasticity	Beta	0.8	0.1	0.8018	0.7110	0.8784	0.9011
ρ_w	Wage-elasticity	Beta	0.8	0.1	0.7192	0.5417	0.9090	0.9070
ρ_{y^*}	For. output	Beta	0.8	0.1	0.8246	0.7135	0.8524	0.9534
ρ_{π^*}	For. inflation	Beta	0.8	0.1	0.6242	0.5245	0.6164	0.7235
Std. dev.								
σ_z	Productivity	Invg.	0.01	0.05	0.0074	0.0049	0.0068	0.0097
σ_ξ	Capital quality	Invg.	0.01	0.05	0.0033	0.0022	0.0028	0.0043
σ_g	Gov't spending	Invg.	0.01	0.05	0.0152	0.0088	0.0133	0.0216
σ_c	Preference	Invg.	0.01	0.05	0.0252	0.0162	0.0284	0.0339
σ_i	Invest. adj.	Invg.	0.01	0.05	0.0170	0.0097	0.0135	0.0244
σ_p	Price-elasticity	Invg.	0.01	0.05	0.1157	0.0858	0.0996	0.1467
σ_w	Wage-elasticity	Invg.	0.01	0.05	1.0418	0.4674	0.9671	1.6188
σ_x	Trend	Invg.	0.01	0.05	0.0117	0.0059	0.0109	0.0177
σ_{y^*}	For. output	Invg.	0.01	0.05	0.0267	0.0208	0.0259	0.0327
σ_{π^*}	For. inflation	Invg.	0.01	0.05	0.0060	0.0047	0.0057	0.0072

Table 8: Priors (columns 3-5) and posteriors (columns 6-9) of the parameters that are estimated with Bayesian techniques. The results are based on 2 chains, each with 500,000 draws based on the Metropolis-Hastings algorithm. "Invg." is an abbreviation for the inverse Gamma distribution.

After having estimated the model, we applied the convergence statistics proposed by Brooks and Gelman (1998) to check that convergence was reached.

We see from Table 7 that the estimation is informative for a key parameter determining the strength of financial frictions, namely the diversion rate for corporate loans $\bar{\lambda}_k$. We find that the posterior mean is equal to 0.6420, which is above the prior mean of 0.5. It is also substantially higher than values used in the literature, where the diversion rate for corporate loans is typically below 0.4 (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011). By setting $\bar{\lambda}_k = 0.6420$ in our simulations, we find that $\bar{\lambda}_b = 0.3210$, which is substantially above the value employed in papers where financial intermediaries have both corporate loans and government bonds on their balance sheet. Gertler and Karadi (2013) and Karadi and Nakov (2021), for example, employ a value for $\bar{\lambda}_b$ that is less than 0.2. Therefore, our estimation suggests that financial frictions are relatively important for the Spanish economy.

We also find that the posterior mean of the fraction of constrained households is equal to 0.2003, which is in line with the literature. In addition, the estimation is informative on the Calvo parameters determining the probability of not being able to choose a new price, and the indexation parameters. An exception is in the export sector, where the estimation is uninformative for ψ_x and γ_x. The estimation is also informative on the underying trend growth Λ_x of productivity, as well as on the parameter γ determining the degree to which capital goods producers face investment adjustment costs.

Finally, observe that the estimation is informative on almost all parameters relating to the exogenous processes, except for the persistence ρ_p of a shock to the elasticity of substitution among domestic retail goods, and the persistence ρ_{y^*} of foreign output.

E Robustness checks

In this section we perform a robustness check by reporting discounted cumulative multipliers for alternative parameter values. Table 9 displays the robustness checks for the case where government debt is short-term (1 quarter maturity) and sovereign risk absent, while Table 10 reports the case where government debt is long-term and sovereign risk absent. Finally, Table 11 shows the results for the base case from the main text, where government debt is long-term and subject to sovereign default risk. We report immediate stimuli, as well as delayed stimuli, which have an implementation lag of four quarters (like in the main text). Our key focus is to vary parameters or calibration targets relating to financial frictions and the default probability function. The goal of this section is to show that the results we obtained in the main text carry over qualitatively for these alternative parameter values.

Let us first look in Table 9 at the case where government debt is short-term and not subject to default risk. Specifically, we look at the impact of changes in the diversion rate of corporate loans $\bar{\lambda}_k$, changes in the relative diversion rate of bonds over corporate loans $\bar{\lambda}_b/\bar{\lambda}_k$, changes in the steady state leverage ratio $\bar{\phi}$, changes in the coefficient in front of households' quadratic adjustment costs κ_b in equation (73), and changes in the fraction of constrained households ν_r. We see in Table 9 that changes in these parameter values hardly affect the size of the multiplier, both for immediate stimuli, as well as for delayed stimuli. Only the multiplier from an immediate stimulus for $\nu_r = 0.05$ is 0.05 percentage points below the base case. This result is unsurprising, as fewer hand-to-mouth consumers implies that the indirect effect of the stimulus that leads to more consumption by constrained households is reduced.

Stimulus policy	Immediate multiplier	Delayed multiplier
Base case	0.64	0.25
$\bar{\lambda}_k = 0.5$	0.66	0.28
$\bar{\lambda}_b/\bar{\lambda}_k = 0.25$	0.64	0.28
$\bar{\phi} = 5.5$	0.65	0.27
$\bar{\phi} = 7.5$	0.63	0.24
$\kappa_b = 0.0015$	0.64	0.25
$\kappa_b = 0.15$	0.64	0.25
$\nu_r = 0.05$	0.59	0.25

Table 9: Table displaying the discounted cumulative dynamic multiplier for listed scenarios for a fiscal stimulus in response to a financial crisis initiated by an MIT-shock of 5% to the diversion rate on corporate securities and a fiscal stimulus of 0.5% of quarterly GDP. Government debt is short-term, and is not subject to sovereign default risk. The stimulus is calculated using formula (32) over the first 1000 quarters.

Next, we look in Table 10 at the case where government debt is long-term and not subject to sovereign default risk. The difference between the alternative cases and the base case slightly increases with respect to the case where government debt is short-term. For example, the immediate multiplier for the case with $\bar{\lambda}_k = 0.5$ and $\bar{\lambda}_b/\bar{\lambda}_k = 0.25$ increases by 0.07 percentage points with respect to the base case for an immediate stimulus, and by 0.05 and 0.03 for a delayed stimulus, respectively. The intuition behind this result is that for lower values of $\bar{\lambda}_k$ and $\bar{\lambda}_b$, respectively, intermediaries' incentive compatibility constraint is relaxed, everything else equal, as a result of which capital losses on existing bond holdings decrease in case of a fiscal stimulus. Changes in the steady state leverage ratio have a relatively small impact on the multiplier. Therefore, we conclude from Table 10 that the conclusion that the result that the multiplier decreases when moving from short-term bonds to long-term bonds carries over for these alternative parameter values.

Finally, we look at the case with long-term debt and sovereign default risk in Table 11. We have

Stimulus policy	Immediate multiplier	Delayed multiplier
Base case	0.47	0.15
$\bar{\lambda}_k = 0.5$	0.54	0.20
$\bar{\lambda}_b/\bar{\lambda}_k = 0.25$	0.54	0.18
$\bar{\phi} = 5.5$	0.49	0.17
$\bar{\phi} = 7.5$	0.45	0.13
$\kappa_b = 0.0015$	0.36	0.08
$\kappa_b = 0.15$	0.49	0.15
$\nu_r = 0.05$	0.43	0.15

Table 10: Table displaying the discounted cumulative dynamic multiplier for listed scenarios for a fiscal stimulus in response to a financial crisis initiated by an MIT-shock of 5% to the diversion rate on corporate securities and a fiscal stimulus of 0.5% of quarterly GDP. The delayed stimulus has an implementation lag of four quarters. Government debt is long-term, has an average duration of 20 quarters, and is not subject to sovereign default risk. The stimulus is calculated using formula (32) over the first 1000 quarters.

extended this table with respect to the previous two tables by including parameters that affect the probability of sovereign default. From this table, we see that changes in parameter values have the largest effect on the multiplier when debt is long-term and subject to default risk. For example, a decrease in the diversion rate of corporate loans $\bar{\lambda}_k$ from its posterior mean of 0.64 to $\bar{\lambda}_k = 0.5$ causes the immediate multiplier to increase by 0.33 percentage points (= 0.19 - - 0.14), while the delayed multiplier increases 0.41 percentage points (= - 0.24 - - 0.65). The intuition why the change in the multiplier (with respect to the base case) is so much larger compared with the case with no sovereign risk is the fact that capital losses on existing bond holdings are much larger in the presence of sovereign risk. The impact of these capital losses on net worth is then amplified because the subsequent reduction in credit provision to the real economy leads to a lower price of physical capital (with respect to the base case). This, in turn, amplifies the capital losses on corporate loans. Therefore, a lower value of $\bar{\lambda}_k$ means that these capital losses on corporate loans are mitigated, as a result of which the multiplier increases. However, the conclusion that the multiplier decreases when sovereign debt is introduced continues to hold for the case $\bar{\lambda}_k = 0.5$. Specifically it decreases from 0.57 to 0.19 for an immediate stimulus, and from 0.20 to -0.24 for a delayed stimulus (compare Table 10 and 11).

When the steady state leverage ratio $\bar{\phi}$ decreases to 5.5, we see that the multiplier increases by

Stimulus policy	Immediate multiplier	Delayed multiplier
Base case	-0.14	-0.65
$\bar{\lambda}_k = 0.5$	0.19	-0.24
$\bar{\lambda}_b/\bar{\lambda}_k = 0.25$	-0.13	-0.68
$\bar{\phi} = 5.5$	0.03	-0.42
$\bar{\phi} = 7.5$	-0.33	-0.89
$\kappa_b = 0.0015$	-0.10	-0.52
$\kappa_b = 0.15$	-0.16	-0.69
$\nu_r = 0.05$	-0.21	-0.67
$\bar{p}^{def} = 0.0025$	-0.15	-0.66
$\bar{p}^{def} = 0.0075$	-0.16	-0.67
$\frac{d\bar{p}^{def}}{d\bar{b}} = 0.15$	0.09	-0.34
$\frac{d\bar{p}^{def}}{d\bar{b}} = 0.25$	-0.62	-1.29
$\bar{q}_b\bar{b}_{max} = 2.0$	-0.48	-1.10
$\bar{q}_b\bar{b}_{max} = 2.8$	0.01	-0.45

Table 11: Table displaying the discounted cumulative dynamic multiplier for listed scenarios for a fiscal stimulus in response to a financial crisis initiated by an MIT-shock of 5% to the diversion rate on corporate securities and a fiscal stimulus of 0.5% of quarterly GDP. The delayed stimulus has an implementation lag of four quarters. Government debt is long-term, has an average duration of 20 quarters, and is subject to sovereign default risk. The stimulus is calculated using formula (32) over the first 1000 quarters.

0.17 percentage points (= 0.03 - - 0.14) for an immediate stmulus, and by 0.23 percentage points (= - 0.42 - - 0.65) for a delayed stimulus (with respect to the base case). The reason is that a lower steady state leverage ratio means that intermediaries have more net worth, as a result of which their lending capacity is less affected by financial crises and capital losses from debt-financed fiscal stimuli. When the steady state leverage ratio increases to 7.5, we see that the multipliers decrease by 0.19 and 0.24 percentage points for an immediate and delayed stimulus (with respect to the base case), respectively. We see that the coefficient in front of households' quadratic adjustment costs from bond holdings κ_b has a relatively small effect on the multiplier, while the multiplier decreases when the fraction of constrained households decreases.

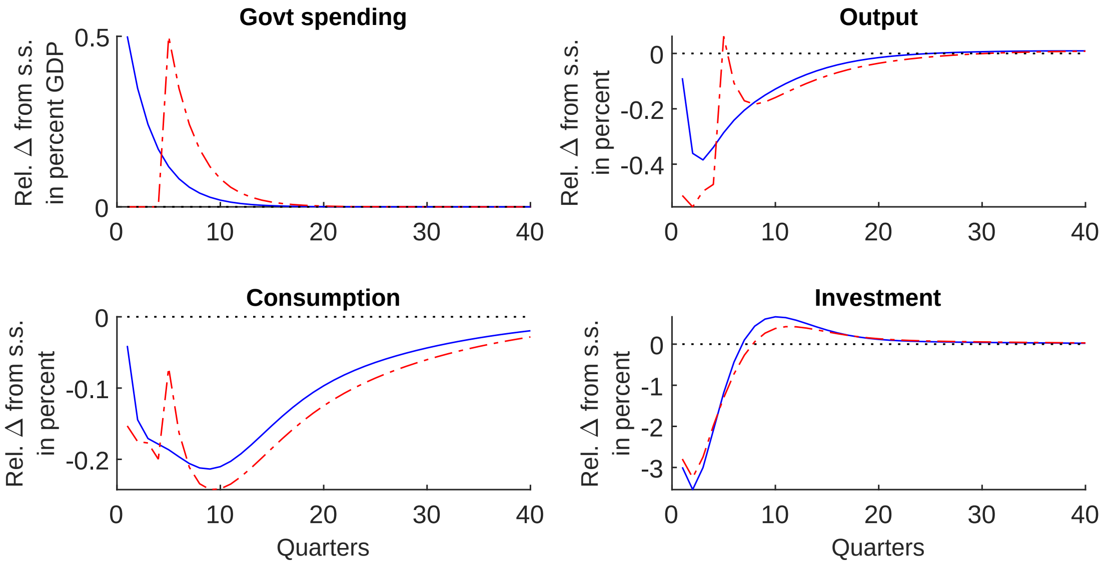
Compared with Table 9 and 10 (no sovereign default risk), we have performed additional robustness checks by varying the steady state default probability $\bar{p}^{def}$, the steady state first derivative of the default probability function $\frac{d\bar{p}^{def}}{d\bar{b}}$, and the steady state "maximum" level of debt $\bar{q}_b\bar{b}_{max}$. We hit these alternative calibration targets by adjusting α_b and β_b in equation (16). From Table

11, we see that changes in the steady state default probability $\bar{p}^{def}$ have a minor impact on the multiplier. However, changes in the first deriviative of the default function with respect to debt $\frac{d\bar{p}^{def}}{d\bar{b}}$ have a large effect: the multiplier increases by 0.23 percentage points (= 0.09 - - 0.14) for an immediate stimulus, and by 0.31 percentage points (= -0.34 - - 0.65) for a delayed stimulus (with respect to the base case) when the derivative decreases from 0.2 to 0.15. The multiplier decreases by 0.48 and 0.64 percentage points (with resepct to the base case) for an immediate and delayed stimulus, respectively, when the steady state derivative increases to $\frac{d\bar{p}^{def}}{d\bar{b}} = 0.25$. The intuition behind these quantitative large results is the fact that the derivative determines by how much the probability of default increases for an additional euro of bonds issued by the fical authority, and through that channel by how much bond yields will increase and bond prices decrease. The smaller the deriviative, the smaller the capital losses on intermediaries' existing bond holdings (as a result of an additional euro of stimulus), and the smaller the contraction in credit to the real economy. Similarly, the larger the derivative, the larger the capital losses, and the larger the contraction in credit provision to the real economy. Similarly, when the steady state "maximum" level of debt is equal to 50% of annual GDP, which is relatively far below the steady state level of government debt of 80% of annual GDP, an increase in debt issue by the government increases the probability of default by more than when the "maximum" level of debt is relatively close to the steady state level of debt. Therefore, we conclude that the key to the size of the multiplier is by how much the probability of default increases as a result of an additional euro of bonds issued by the fiscal authority, and not so much the level of the default probability from which changes occur.

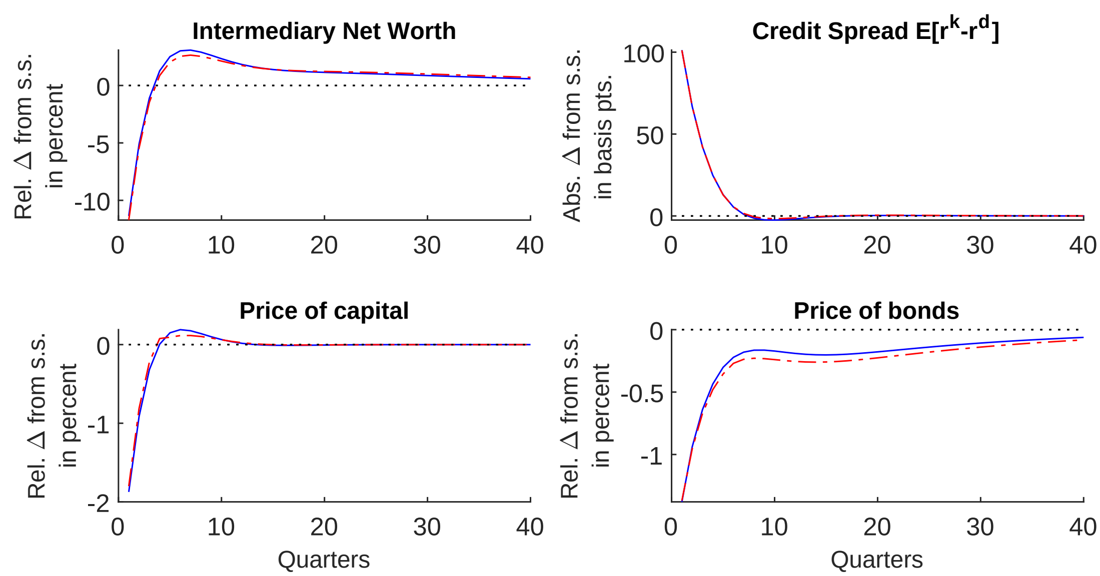
Finally, we compare the multipliers from the above-mentioned cases with the multipliers from the base case of the model version without sovereign risk. We observe that even for the case with $\frac{d\bar{p}^{def}}{d\bar{b}} = 0.15$, which features the largest multipliers upon variation of the default targets, the difference with the base case with no sovereign risk in Table 10) remains substantial. It is equal to 0.38 percentage points (= 0.47 - 0.09) for an immediate stimulus, and 0.49 percentage points (= 0.15 - - 0.34) for a delayed stimulus. Therefore, the conclusions from the main text carry over qualitatively to cases with alternative parameter values: the introduction of sovereign default risk leads to a substantial deterioration of the multiplier with respect to the case without sovereign risk.

F Additional figures

Financial crisis, sovereign default risk, long-term bonds: immediate vs. delayed government spending



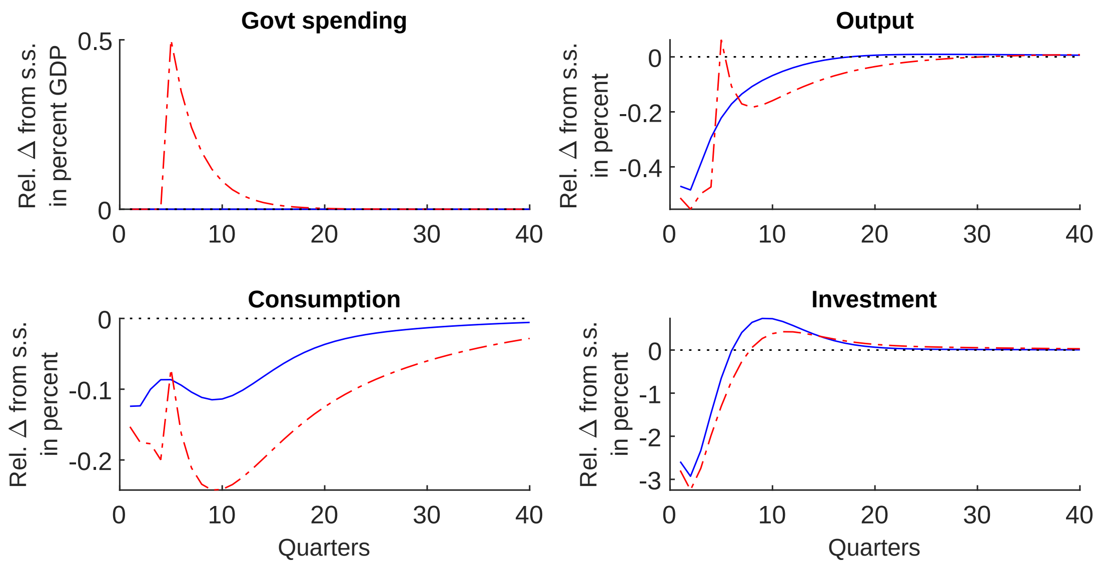
(a)



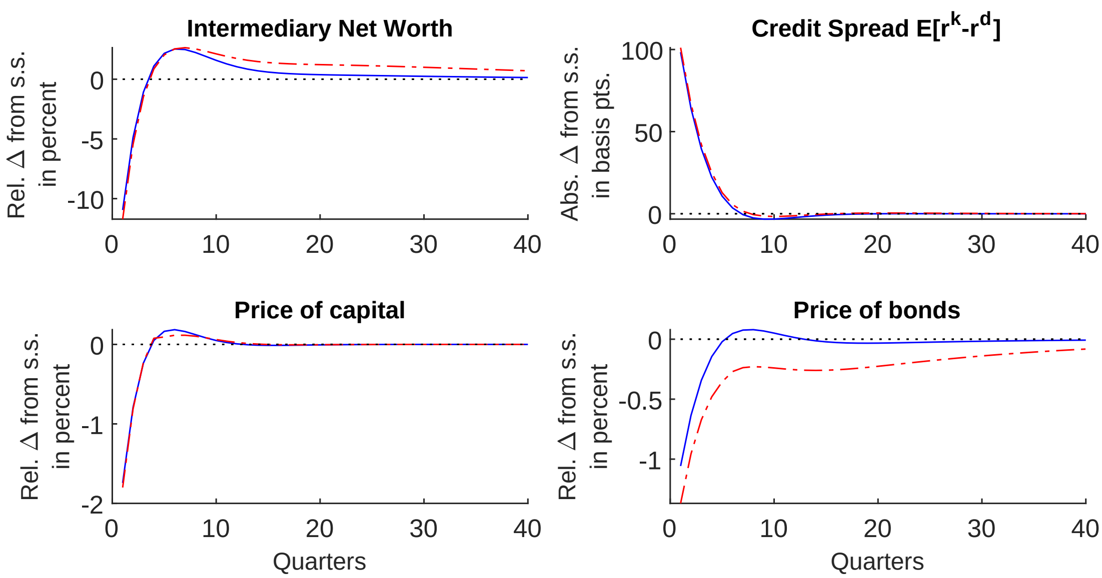
(b)

Figure 11: Plot of the impulse response functions comparing an immediate fiscal stimulus (blue, solid) and a delayed fiscal stimulus (red, slotted) in response to a financial crisis. The delayed stimulus is announced as the crisis hits, but implemented four quarters later. The size of the stimulus equals 1% of quarterly steady state GDP and is financed through additional debt issue. Bonds have an average duration of 20 quarters, and are subject to sovereign default risk. The financial crisis is initiated through an MIT-shock to the diversion rate of corporate securities of 2 percent relative to the steady state.

Financial crisis, sovereign default risk, long-term bonds: no policy vs. delayed government spending



(a)



(b)

Figure 12: Plot of the impulse response functions comparing no additional policy (blue, solid) and a delayed fiscal stimulus (red, slotted) in response to a financial crisis. The delayed stimulus is announced as the crisis hits, but implemented four quarters later. The size of the stimulus equals 1% of quarterly steady state GDP and is financed through additional debt issue. Bonds have an average duration of 20 quarters, and are subject to sovereign default risk. The financial crisis is initiated through a MIT-shock to the diversion rate of corporate securities of 2 percent relative to the steady state.

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